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METRIC-DEPENDENT DIMENSION FUNCTIONS

BY

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SUMMARY OF CONTENTS

Section 0 is a review of results in general topology and basic dimension theory which are used in the sequel.

In section 1, we study the relationships between the various dimension functions. We give a proof of a result mentioned by Nagami and Roberts (Nagami and Roberts, 1967) to the effect that on locally compact metric spaces, all the dimension functions studied here coincide. We prove a lemma (lemma 1.3) which shortens the proofs of a number of results.

In section 2 we study examples which show that different dimension functions can have different values on the same metric space. We give an example of a connected subset of I^2 which is a union of countably many (an more than one) disjoint non-empty closed sets which shows that a lemma used by Nagami and Roberts (lemma 2.3) cannot be extended to normal (infact metric) spaces. Nagami and Roberts also show that if A_i, $i \in N$ is a disjoint sequence of closed sets of I^n at least two of which are non-empty, then dim $(I^n - \bigcup_{i=1}^{\infty} A_i) \geq n-1$. They give a sketch of a Cantor 2-manifold for which this result is not true. We give a rigorous proof of this. Nagami and Roberts have given an example of a metric space (X,ℓ) with d_2 $(X,\ell) = 2$, d_3 $(X,\ell) = \mu$-dim $(X,\ell) = 3$ and dim $(X,\ell) = 4$. This has been the only known example where d_2 and d_3 differ. We generalize this to examples with $d_2 \leq n-2$, $d_3 = \mu\text{-dim} = n-1$ and dim $= n$ for any n, $n \geq 4$.

In section 3 we study results which show that a given metric-dependent dimension function can give different values for equivalent metrics on a set. We then study realization theorems, i.e. theorems to the effect that there exist equivalent metrics to a given metric that make a given dimension function realize given values. We prove a lemma (lemma 3.4) which generalizes a similar lemma by Goto (Goto, lemma 1).

In section 4 we study more characterizations of metric-dependent dimension functions, notably Lebesgue cover characterizations. We study a weak sum theorem for some metric-dependent dimension functions.