AN INVESTIGATION INTO MATHEMATICAL CONCEPT DEVELOPMENT AMONG KENYAN PRIMARY SCHOOL CHILDREN

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BY
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DECLARATION

"This thesis is my original work and has not been presented for a degree in any other University."

John A. Shihundu

"This thesis has been submitted for examination with my approval as university supervisor."

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Senior Lecturer,
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Abstract:

Kenya's problems relating to the demand for scientific and mathematical knowledge are more far-reaching, as elsewhere in the developing countries. No person can get far in scientific and mathematical thinking unless he masters the relevant concepts in science and mathematics. This study therefore, aimed at determining the extent to which Kenya's primary school children comprehend mathematical concepts. Secondary to this purpose, the study aimed at assessing the degree to which the same children rely on perceptual comparisons for their judgement in mathematical tasks.

Three separate tasks (classification, conservation and measurement) which represented three distinct mathematical concepts were selected for the investigation, which used the AVET (Audio-Visual Experimental Technique) as a testing instrument. This method combined the use of three senses (seeing, hearing and touching) to complete a test-task. There were 80 test - items administered to 675 children of Mombasa District, with a mean chronological age of 11.08 years and a standard deviation of 1.76. Data was analysed on the basis of four statement hypotheses: "that there is no difference in performance between the sexes, grades, socio-economic statuses and ability in the variables".

Mean - scores in the three tests (classification, conservation and measurement) were obtained for each hypothesis. The primary findings of the study were ($p < 0.05$) at 5% level of significance. These findings strongly support the existing theories (Beard, Piaget Lovell, Eshiwani, Copeland, Otaala.....) that children who fail to learn and master the relevant mathematical concepts at the right stages of intellectual development may be puzzled in learning later mathematical concepts.

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CHAPTER 1

INTRODUCTION

Concept formation in mathematics has been one of the most discussed issues in mathematics education in recent years. This has been partly due to the much publicised Piagetian research and partly due to the revolutionary changes that have taken place in the school mathematics during the past two decades. The importance of concept formation in mathematics is well documented by Lovell[1] (1971). In a recent article, Eshiwani[2] (1976) has observed that achievement in mathematics among the primary school children in Kenya is very poor. He has attributed this poor achievement to the lack of understanding the basic relevant mathematical concepts.

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[1]Kenneth Lovell: The Growth of Basic Mathematics and Scientific Concepts in Children. University of London Press, Ltd., 1971, pp. 11-24.

[2]George S. Eshiwani: "A Report to the National Committee on Educational Objectives and Policies". Republic of Kenya, April, 1976, p. 22.

CONCEPTS

The children are taught to memorize facts and to manipulate numbers without understanding the mathematical meanings behind these facts and manipulations. If performance in mathematics is to be improved in Kenya, then it is important that educators in Kenya understand which basic mathematical concepts give great difficulties to Kenyan children. Lovell[3] (1971) has identified three main mathematical concepts as being important to the understanding of mathematics in the primary schools. These concepts are: classification, conservation and measurement.

Classification serves as a basis for the development of mathematical concepts (Eshiwani 4, (1974), and therefore it should be the first mathematical idea taught to children. This concept is prerequisite to children's development of pre-logical thinking.

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[3] Kenneth Lovell: The Growth of Basic Mathematics and Scientific Concepts in Children. University of London Press, Ltd., 1971, pp. 11-24.

[4] George S. Eshiwani: "The Teaching of Mathematics to Primary School Children". Article of November 20, 1974, University of Nairobi, p. 1.

Number, for example, is one of the logical aspects of mathematics. According to Copeland[5] (1974), children can only grasp the number concept when they have mastered the concept of classification.

Children use different bases for classification. Research done in this area throws some light on this statement. For example, Liedtake and Nelson[6] (1973) have observed that at an early age children readily classify on the basis of colour, shape and size. On the other hand, Evans and Segall[7] (1972) have suggested that young children tend to judge as equivalent on the basis of striking but often incidental by perceptual properties such as colour,

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[5]Richard W. Copeland: How Children Learn Mathematics. Teaching Implications of Piaget's Research. Macmillan Publishing Co. Inc., 2nd Ed., 1974, p. 51.

[6]L.D. Liedtake & L.D. Nelson: "Activities, a Mathematics for Pre-School Children." The Arithmetic Teacher, 20, 7, Nov. 1973, 536-38.

[7]J.L. Evans & M.H. Segall: "Learning to classify by colour and by Function". A Study of Concept Discovery by Ganda Children, in Otaala's Piaget for Teachers, 1972, p. 9.

whereas older children are more apt to detect common attributes which super-ordinate to their exemplers. Lovell does not agree with either Lietdake or with Evans and Segall and he claims that the ability of children to classify seems to depend on the capacity to compare two judgements simultanously.

The concept of conservation, like that of classification is extremely important to mathematics learning. It is basic to all rational thinking. Until the child is able to conserve (Otaala[8], 1972), that is to bear in mind that whatever aspects of an object or phenomenon remained the same while other aspects underwent a change in appearance, he is not likely to grasp many kinds of verbal instruction. In Chemistry for example, "matter is neither created nor destroyed during a chemical change." Piaget, Inhelder and Szeminska[9] (1966), found in their studies that mere knowledge

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[8] Barnabas Otaala: Piaget for Teachers, Selected Readings, 1972 (Millie Almy: "The Usefulness of Piagetian Methods for studying Primary School Children in Uganda.") p. 4.

[9] Jean Piaget, Barbra Inhelder and Alina Szeminska: The Child's Conception of Geometry. Routledge and Kogan Paul, 2nd. Ed. 1966, p. 104.

of conservation of length when objects undergo a change did not imply any understanding of Euclidean matrices.

There is a question: "how do children then pass from qualitative conservation to the measurement of length?" Copeland[10] (1974) has suggested that measurement notions develop in relation to the basic concept of conservation. Children need knowledge of conservation of length, area and volume to help them to at least understand the concepts of measuring objects in one, two or three dimensions. Physics centres its studies on conservation of matter. For example: "a physical change is a reversible change." It would appear that the knowledge of conservation concepts in young children is a prerequisite to the understanding of the physical world, especially in physics.

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[10] Richard W. Copeland: How Children Learn Mathematics: Teaching Implications of Piaget's Research. Macmillan Publishing Co. Inc., 2nd.Ed. 1974, p. 247.

Before children can co-ordinate measurements in such a way as to fix a point in an area and others, they need to evolve a system of one-one correspondence with axes perpendicular. Piaget[11] (1966) has suggested that rectangular co-ordination depends on one-one correspondence principle. Lovell[12] (1971) on the other hand, has argued that in the measurement concept of length, the child is first to understand that the length of an object remains the same whatever changes occur in its position. This argument corroborates the previous arguments by other reseachers who suggested that measurement and conservation are closely inter-related.

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[11] Jean Piaget, Barbra Inhelder and Alina Szeminska: The Child's Conception of Geometry. Routledge and Kogan Paul, 2nd. Ed. 1966, p. 104.

[12] Kenneth Lovell. The Growth of Understanding in Mathematics, Kindergarten through Grade Three. Early Childhood Education Series, 1971, p. 102.

Measurement concept is a very important instrument for measuring all quantified matter. Mathematical concept development in measurement provides tools for practical activities in every day living. The teaching and learning of measurement concepts, (Carpenter[13], 1975) may help children to stabilize the concept of quantity. The ability among children to measure (Copeland[14], 1974), develops later than the number concept. It would appear therefore, that without correct foundation in the concept of conservation of length, area and volume, no child may achieve much in exercises requiring the concept or understanding of measurement in one, two or three dimensions.

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[13] Thomas P. Carpenter: "Measurement concepts of First and Second Grade Students". Journal for Research in Mathematics Education. January, 1975, pp. 11 - 12.

[14] Richard W. Copeland: How Children Learn Mathematics: Teaching Implications of Piaget's Research. Macmillan Publishing Co. Inc., 1974, p. 252.

Despite the wide application of measurement and its continuous appearance in the curriculum, (Bruni and Silverman[15], 1974), there is a great gulf between how the practical man measures, reports his measurements and judges their premises and the mathematician's conventions for dealing with measurement. The concept of mathematical model of space, (Smart and Marks[16], 1966) is basic to an understanding of the mathematics of measurement because the theory of measurement is developed within the mathematical model then applied to the actual physical measurement in space.

Though Kenya's primary school children can measure, it would appear that lack of proper and sound knowledge of measurement concepts render them less capable of estimating the measurements of motionless objects and later on to evolve an understanding of how to measure using measuring instruments with objects in static and motion conditions. A child whose concepts of measurement

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[15] J.V. Bruni and H. Silverman: The Arithmetic Teacher, 21, 6, October, 1974, 474-79.

[16] J.R. Smart, J.L. Marks and Nelson: The Arithmetic Teacher, 13, 4, April, 1966, 283-87.

are misunderstood or unmastered, may not measure accurately. Inadequacy in measuring would not favour the future engineers, draughtsmen and architects, whose services are indispensable in Kenya's mathematical, scientific and technical development.

THE PROBLEM

Although concept formation is recognised as an important area in mathematics education in Kenya and elsewhere in the world, no research has been done in the Kenyan context to investigate the difficulties which Kenyan children face in various mathematical concepts and to find out what factors influence the acquisition of these concepts.

The purpose of this study was to determine the extent to which Kenyan primary school children of grades three, four and five comprehend mathematical concepts of classification, conservation and measurement. A secondary purpose of the study was to assess the degree to which Kenyan school pupils rely on perceptual comparison or at least require perceptual support for the concepts of classification, conservation and measurement. Specifically, the study attempted to answer the following questions:

1. To what extent does educational level of Kenyan primary school children affect the acquisition of the following mathematical concepts: classification, conservation and measurement?

2. To what extent does the ability of Kenyan children affect their acquisition of the following mathematical concepts: classification, conservation and measurement?

3. To what extent does the socio-economic status of Kenyan children affect their acquisition in the following mathematical concepts: classification, conservation and measurement?

4. How do Kenya primary school boys' performance in mathematical concept tasks compare to girls' of a similar educational background?

DEFINITION OF IMPORTANT TERMS

AVET:

A teacher-made method (author's) in which three senses: hearing (audio), seing (visual) and touch (concreteness) are combined in the administration of an experimental class by exposing the candidates to the test materials visually. The diagrams of the test materials appear first in the booklet followed with the test-items. The invigilator draws the attention of the candidates to the material displays and follows with reading aloud the entire content of each test-item in the booklet before the class can write or answer. This method makes an advantage of the children's senses of learning all in the same time and process. The word AVET therefore is an abbreviation:

A	=	Audio
V	=	Visual
E	=	Experimental
T	=	Technique.

It means therefore, "Audio - Visual Experimental Technique". This method won the author* a prize in the Guinness Awards for Science and Mathematics Teachers in 1975. This study used 18 physical displays at the time of the experimental tests (see Appendix A) on this study.

Clinical method:

Piagetian method of investigation into children's concept formation in which they are interviewed in a carefully controlled way by the experimenter. In this method, the experimenter centres all the techniques and material questioning on the basis of the respondent's answers. The respondent is expected to discover some of the answers by himself out of the series of questions by the experimenter.

Conservation:

A concept to the effect that there is maintenance of a structure as invariant during physical changes of some aspects. For example, when some plasticene

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* J.A. Shihundu, a post-graduate student at the University of Nairobi, wrote an essay on Teacher - Pupil Approach to the Teaching of pre-Geometry Mathematics Among Young Children, 1975. Science Teacher 18, 5 & 6, June/July, 1975, 38.

from the ball into a banana remains invariant (conserved), i.e. there is no change in volume of plasticene.

Measurement:*

A concept in mathematical practice which associates a number with some unit of measure in order to describe some property of a person or thing. In this description, "things" are natural objects such as human beings, stones or houses. Properties of things or a person refer to characteristics such as colour, size, weight or intelligence. We normally measure properties of things which change positions. The change of position involves movements which must be linked to reference points, in the form of "co-ordinate axes". This study used spontaneous measurement. The intention was to expose children to measuring activities to serve the purpose of observing the ways in which the children's measuring behaviour as a whole develops (concept formation) in the course of the child's activity.

John Murray's description adopted from his:"The World of Measurement" (Donoran A. Johnson & William H. Glenn) in Exploring Mathematics on Your Own, Vol. 2, 1965. p.3. and Jean Piaget et al's description adopted from their "The Child's Conception of Geometry". Routledge and Kegan Paul, 1966, pp. 2 - 4.

Socio-economic status:

A term used to designate the per capita income of an individual's standing in the society. The study realized that the social standings of the pupils' parents or guardians tended to affect the manner and amount of acquring knowledge. The study classified or graded such standings (status) into lower, middle and upper, based on educational levels of the parents or guardians.

HYPOTHESES

The following null hypotheses were stated for this study:

Hypothesis$_1$ (Sex):

There is no difference in performance between boys and girls in mathematical classification, conservation and measurement concepts.

Hypothesis$_2$ (Grade):

There is no difference in performance between the pupils of grades three, four and five in mathematical classification, conservation and measurement concepts.

Hypothesis$_3$ (Socio-Economic Status by parent/guardian educational level)

There is no difference in performance between pupils of upper, middle and lower status in mathematical classification, conservation and measurement concepts.

Hypothesis$_4$ (Ability):

There is no difference in performance between pupils of above average, average and below average in ability in mathematical classification, conservation and measurement concepts.

CHAPTER 2

LITERATURE REVIEW

Much research has taken place in recent years in mathematical concepts development. Much of this research has been carried out in the western and oriental worlds. Most of this research literature is a replication of Piaget's school of thought, based on the theory that young children's intellectual (cognitive) development at least passes through four stages[17] (Sensori-motor, pre-operational, concrete operation and formal operational). The study of concept development in mathematics is very important in newly developing countries such as Kenya, where the late 20th century innovations are introduced even in primary schools. It has been suggested that no person can get very far in mathematical thinking Lovell[18] (1971), without first mastering the relevant mathematical concepts. In this study, the review of literature concentrated on: localization of the curriculum, and the mathematical concepts formation in classification, conservation and measurement among the young primary school children.

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[17] Paul H. Mussen, et al: Child Development and Personality. Harper International, 4th Ed., 1974, pp 309 - 318.

[18] Kenneth,Lovell: The Growth of Basic Mathematics and Scientific Concepts in Children. University of London Press, Ltd., 1971, pp. 11 - 24.

LOCALIZATION OF THE CURRICULUM:

"Curriculum in educational planning is very important. It acts or directs education in the same way a country's constitution does. In this understanding, localization of the curriculum in newly developing countries such as Kenya is an urgent need. The need for a Kenyan based curriculum is centred on the view that children of any nation learn best in their own environments. "Own environments" should be responsible for its syllabus.

Developing countries,especially in Asia and Africa have, much recently discussed the need to base curriculum development on local knowledge of concept development. For example, the seminars sponsored by UNESCO, UNICEF and CEDO (Bangkok, 1972 and Nairobi, 1974),[19] discussed about the needs to share all available information in the field of concept development; spread knowledge of "clinical method" (Piagetian method of experimentation) among the partner members; and plans for further research in this much needed area of mathematical and scientific concept development, based on the models of Piaget et al and Bruner. The Sri Lanka

[19] Bryan Wilson: "Three International Meetings." Science Teacher. 18, 3 & 4, Feb/March, 1975 pp. 21 - 23.

Report presented at the Bangkok seminar, discussed the investigation into Sri Lankan children's concept development. The relevant parts of this investigation are currently deployed by the Curriculum Development Centre of the Ministry of Education in Sri Lanka to develop new syllabus and teaching materials suitable to the culture and environments of the Sri Lankan primary school children. Another development at the Bangkok seminar was the discussion and adoption of the use of Piagetian "clinical method" as a mode of probing the children's thinking. To do this, some Asian English speaking children aged 5 - 13 years were used. Each participant discussed, by a "dialogue", with the child using varying experimental situations. The results sparked off much research in concept development at the primary school level of cognitive development among Asian children. From this research, new designs of instructional materials and innovative methods of teaching Asian children have taken place. Nairobi seminar reviewed the Bangkok seminar's deliberations. It was sad for the partcipants of Nairobi seminar to learn that there was very little research in mathematical and scientific concept development in African countries.

This situation was revealed by Ohuche and Pearson both participants at the Nairobi seminar, representing West Africa and sponsored by SEPA (Science Education Programme for Africa). The participants who came from all parts of Africa resolved at this seminar to undertake more effective means of collection, collation and dissemination of research relevant to concept development in Africa. A major lesson which emerged from the Bangkok and the Nairobi seminars was the awareness among the curriculum developers in mathematics, the need to consider localization of the curriculum in mathematical concept development, among the African and Asian primary school children. Related to the localization of the curriculum, was a consideration to modify the existing and future imported teaching materials to suit the varying socio-economic, cultural and cognitive developments within the material limits of African and Asian environments. This view attempted to question Piaget's fundamental theory on cognitive stages of child development. Up to now, there is still some consideration: whether or not to accept Piaget's claim that all children at least pass through four stages: (sensori-motor, pre-operational, concrete operational and formal operational) in cognitive development at a uniform rate of intellectual growth, without first considering certain factors (sex, grade, ability and environmental backgrounds). But there is no firm evidence to disprove the hypothesis that Piagetian stages of cognitive development are broadly applicable to children of all countries and that the precise nature of children's mental equipment and their consequent capacity to absorb new concepts in mathematics, is to a large extent, undetermined and are dependent on the children's previous experiences, which are a function of their socio-cultural environments. Correct conclusions about this widely publicised theory, might assist in the popularly held view by Piagetian replicators and other independent researchers regarding whether or not mathematical curriculum should be localized and the methods of teaching be based on foreign teaching and learning ideology.

CLASSIFICATION

Classification is a very important concept in fundamental mathematics. Many teachers of young children and parents teach counting (Eshiwani[20], 1974) as the first mathematical idea to children. This type of teaching emphasizes rote-memory and serves very little purpose. Since classification serves as a basis for the development of mathematical concepts, the first experience with mathematics should be in the area of classification. Experience has shown that children normally recognize physical objects on the basis of physical features such as colour, size and shape. Experiments by the author using children of 4 - 10 years on classification (colour, shape, size, class-inclusion, hierarchical and logical connectives) showed that the most elementary type of classification is that of sorting or grouping objects so that they are alike in some way, such as having the same colour, shape or size.

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[20] George S. Eshiwani: "The Teaching of Mathematics to Primary School Children". Part I, November 20, 1974, pp. 1 - 2.

Activities involving sorting or grouping objects (Eshiwani[21], 1974), should be presented to children in the form of mathematical games. When the child is able to consider more than one classification properly (Eshiwani) that is, if he is able to group in "another way" and sort both shape and colour, then he has performed a multiple classification task. In this regard, the child realizes that an object may belong to more than one class at the same time. This realization requires logical thinking or cognitive as well as perceptual structures. Eshiwani has described "hierarchical classification"[22] as difficult for younger children (under ten years).

Whereas children can sort objects in simple classification by perception (Eshiwani[23], 1974), they must use logic to understand the relation of the objects being classified to other objects. For example, the relation of dogs in the class of animals can be shown by means of Venn diagrams (relational diagram of sets: disjoint, union or intersection). The class of dogs is classified

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[21] Ibid, pp. 2 and 11

[22] Ibid, p. 16

[23] Ibid, pp. 17 - 20.

as a subset of the class of animals. Eshiwani has further suggested that since children must use logic to understand the relation of objects, there are certain words called "logical connectives" which children should know about (and, or, not, if...then etc). These words are conjunctions and disjunctions in the concept of classification. The knowledge or concept of the class-inclusion in classification is very important in the early stages of learning mathematics. A child who does not have the concept of class-inclusion relation (Eshiwani 1974), will usually experience difficulties in learning about a logical concept such as number. Such child may not easily understand the set of natural numbers (1, 2, 3...., 9) or a set of quadrilaterals, which includes squares, rectangles, rhombi etc.

There is corroboration in Eshiwani's main suggestions about the importance of inclusion-class in mathematical concept of classification with Copeland [24] (1974) who suggested that the counting concept as the first idea in mathematics was a rote-memory activity. Copeland has further suggested

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[24] Richard W. Copeland: How Children Learn Mathematics: Implications of Piaget's Research. Macmillan Publishing Co. Inc., 2nd Ed., 1974, pp. 51 - 52.

that the idea of number should grow out of an understanding of inclusion relation. In Copeland's opinion, classification serves as a basis, psychologically speaking, for the development of both logical and mathematical concepts. Based on this view, Copeland has quoted Piaget's recent work based on genetic data which revealed that development of number in children does not occur earlier than that of classes or of transitive relations at the age of 7 - 8 years. Copeland therefore concluded that the parallelism between the evolution of number, classes and seriation (ability to order) should be a first piece of evidence in favour of their independence as against the view that there is an initial autonomy of number. The objects which children recognize are based on certain physical properties such as colour, size and shape, or certain patterns of behaviour of such objects. In being able to recognize an object, the child has classified it into a certain category different from the many others, based on certain unique characteristics or properties. Copeland has further suggested that as new objects are discovered, they must be reclassified in relation to the objects already discovered.

Classification is communicational concept. New concepts of higher order than those which the child already has (Skemp[25] 1971), can only be communicated by arranging for him to group together in his mind a suitable set of examples by classification. Skemp has therefore suggested that once a child has appropriately classified something, he has gone a long way towards knowing how to deal with it. The more ways in which the child can classify, the greater the variety of problems which he can solve. The more symbols he can attach to the same concept the more he can classify. In studying the growth of classificatory systems (Lovell[26] 1971), the child is given a collection of geometrical figures, letters of the alphabet and so forth, made of different materials and of different colours. Lovell documented well as an illustration, how a child plays with classification games. For example, a child

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[25] Richard R. Skemp: The Psychology of Learning Mathematics. Penguin Books, 1971 pp. 78 - 80.

[26] Kenneth Lovell: The Growth of Understanding Mathematics: Kindergarten Through Grade Three. Early Childhood Education Series, 1971, pp. 7, 8 and 23.

is told to put together things that belong together, or some suitable variation of such. From the age 2½ - 5 years, the child makes what Piaget calls graphic (diagrammatic/pictorial) collections. Between 5 and 7 years of age objects are now grouped by a child on the basis of similar properties and he can divide a group into sub-groups (subsets). From around 7 - 8 years of age the child is increasingly able to co-ordinate intention and extention, thus making way for true classification. At this stage (Lovell goes on), classification is recognised by the child as a set of systematic inter-relations, so that in: $A + A^1 = B$, implies: $A = B - A^1$, where A^1 is defined as not A (complementary). Lovell's illustration indicates clearly that primary school children should be able at least, to carry out what may be termed "additive" composition of classes and that they can elaborate a set of nested classes where each one is included in the next larger one, which in turn is included in the next larger one, and so on. For example, the pupil can subsume red and blue squares under the class of squares, red and blue circles under the class of circles and squares under the class of shapes.

From the activities of classificatory games (Copeland[27], 1974), emerges the definitions connected with colour, shape and size and that from a psychological point of view, perceptual structures may be sufficient to solve simple classification problems. By this procedure, children will usually sort by shape before colour, followed with sorting by size. Children of about four years of age begin classification arbitrarily, such as first by colour and then shape and last of all by size, which is expected to be somehow difficult in practice. At the age of 5 - 9 years, geometric display of the object is no longer the prime consideration by the child. Classification is now made by some property, such as colour or shape. At the age of eight years, a child is able to classify the object into subsets based on the property of shape.

A "class" cannot be constructed, (Copeland[28], 1974) by perception but only by logic, for it presupposes a series of abstractions and generalizations from which it derives its meaning. To properly classify objects, their relations to other objects already studied must be known. Such relations can

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[27] Richard W. Copeland: How Children Learn Mathematics: Teaching Implications of Piaget's Research. Macmillan Publishing Co. Inc., 2nd Ed., 1974, pp. 52-55.

[28] Ibid, pp. 55-59.

be shown by Venn diagrams. According to Copeland, to solve classification problems, it is necessary to realize that a class involves two kinds of properties or relations: properties that are common to the given class (intersection) and the other classes to which it belongs (cats are animals as rats are) and properties that are specified to the given class that differentiate it (disjoint): such as colour and shape. These latter properties are conveyed by quantifiers (adjectives) e.g. red, blue or triangular, square objects. The common quantifiers are logical connectives such as: all, some, none, one, and, or, etc.). The quantifiers help in classification by multiple tasks and their implications. In Copeland's opinion, children experience difficulties when they use "OR" (Union of sets), which is a disjunction. Logic of implication involves the use of negations: "if not... then..." seriation (ordering) develops earlier than classification due to the fact that a relation such as size can be perceived, while a "class" as such cannot (Copeland[29] 1974).

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[29] Ibid, pp. 63, 65 - 69 and 80.

One wonders whether it might be interesting to devote the first year of school (Copeland/ Jerome Bruner[30] 1974), to a series of exercises in manipulating, classifying and ordering objects in ways that highlight basic operations of logical addition, multiplication, inclusion, serial ordering and the like. Such early science of mathematics pre-curriculum might go a long way toward the kind of intuitive and inductive understanding that could give embodiment later in the formal courses in mathematics and science.

Hierarchical classification, involving logical inclusion (Povey [30] 1962), is not possible before about the age of 7 or 8 years and that the simplest operation is concerned with classifying objects according to their similarity and difference. Such classification is at least accomplished by including sub-classes with larger and more general classes, a process which implies logical inclusion. Such classification which seems very simple at first is not acquired until around seven or eight years of age.

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[30] Ibid, p. 78

[31] R.M. Povey & E. Hill: "Can Pre-School Children Form Concepts?" Education Research Journal: 17, 3, June 1975, pp. 180-191.

Much recently Nairobi[32] (1974) seminar sponsored by the Unesco and Unicef, held the view that among the tasks used in the research of concept development, classification is widely used in Africa. The seminar also pointed out that most of the later studies in Africa, were not conducted for the purposes of investigating any aspects of Piaget's theory, but mainly to assess children's abilities to abstract as judged by such criteria as flexibility in shifting bases of classification, use of super-ordinate concepts and type of basis (colour, form, function etc.) preferred for classification. The seminar made a further observation by suggesting that classification tasks by themselves have limited use in diagnosing the development stage achieved, according to the Geneva (Piaget et al, 1964) school of thought, since concrete operational structure of classification is attained only when class inclusion is mastered. The Nairobi seminar reviewed some research literature carried out in Africa, using at least classificatory tasks: Hendrikz(1966), Etuk (1967), Fjellan[33], (1969) and Otaala (1971a).

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[32] Unesco-Unicef: "The Development of Science and Mathematics Concepts in Young Children in African Countries". Report of a Regional Seminar, Sept. 1974, Nairobi, pp. 47 - 48.

[33] Ibid, pp. 48 and 55.

Fjellan selected wild animals in Kenya with which Akamba children and adults were familiar. Fjellan developed appropriate tests to examine the formal characteristics of the Akamba children's sorting by colour of the wild animals. Fjellan's results only concluded that colour was not as an important criterion in the children's classification and that the children were able to form classes on the basis of more abstract characteristics.

The Nairobi[34] seminar discussed and categorised the study in Africa to fall under three main types(of Piagetian type) on classification: (a) free sorting of an array of objects with little or no constraint on the number or type of the bases of classification available, (b) sorting with constraints, where the objects presented possess only a limited number, after three or four, dimensions available as bases for grouping; (c) addity problems, where one item must be excluded from a larger group to which it is dissimilar. Two conclusions emerged from these categories:

(i) performance in classification depends on familiarity with the material to be used and the bases of classification.

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34 Ibid, p. 48.

(ii) The use of abstraction increases with age.

The seminar also observed that where constraints had been imposed, colour seemed to be preferred to other attributes as the basis for sorting, but the finding might not be generally valid, since the cases usually involved test materials with which children were unfamiliar. In particular, the seminar referred to the example of Gay and Cole (1967) and (1971) when they discovered the problems which the Kpelle of Liberia encountered with classifying using unfamiliar objects. The Gay-Cole investigation created another fresh investigation of culturally relevant methods of classification. A different contribution on classificatory schemes came from the Bangkok [35] (1972) seminar sponsored by the Unesco, Unicef and CEDO in Thailand. The seminar suggested that concepts by observations (blue, five, triangle) are commonly taught by representing positive and

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[35] Unesco, Unicef, Cedo: "The Development of Science and Mathematics Concepts in Children". Report of a Regional Seminar, May/June, 1972, Bangkok, pp. 64 and 65.

negative instances of the concept, and are tested by asking the child to display classificatory behaviour. After the child begins to consistently perform correct classification,instances can be made more complex by introducing more irrelevant information. This procedure may lead the child to generalize the concept of classification to a wide variety of classificatory situations.

Beard[36] (1963 - 64) investigated and documented a research backing in mathematical concepts among Ghanaian children. One of her test-items was on combining shapes (item iii). In this test, children were required to show how simple shapes could be fitted together to make complex ones, using several identical triangles. Beard suggested that such test required a number of abilities, recognition of equivalence of size and shape, skill in drawing, and capacity to see in the mind's eye how shapes can be turned and fitted together as part of a large whole. In her conclusion, Beard,[37] (1963 - 64), confirmed that richer environment (of English and Ghanain children) provided

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[36] Margaret R. Beard: "An Investigation into Mathematical Concepts among Ghanaian Children. Teacher Education in New Countries; 9, 1, May 1968, 7.

[37] Ibid. 9, 2, Nov. 1968, 144.

more advantage in concept actual learning than a poor environment and that mother tongue could aid in quick learning of mathematical concepts. Lack of rich environment and absence of mother tongue are perhaps some of the causes mainly operating against the expected concept development among the African children. Beard in fact stated clearly that the knowledge of many concepts is still usually taken for granted in teaching. For example, children who fail to learn the relevant mathematical concepts in their daily lives are puzzled by arithmetic.

CONSERVATION

Although there has been a gradual recognition that practical work should be provided to aid mathematical learning, (Beard[38] 1963 - 64), especially in primary schools, the knowledge of many concepts is still usually taken for granted in teaching. For example, no special exercises are provided to ensure that children know: weight or area are "conserved" when only shape is altered, leaving the quantity constant. This results into children of this experience to depend on examples given by books or teachers and will not be able to deduce how to do problems of a slightly different kind.

"Conservation" (Siegel[39] Hooper and Flavell, 1968) is the cognition that certain properties (quantity, matter, length, etc.) remain invariant (are conserved) in the face of certain transformation (displacing objects, or object parts in space). The earliest stages in the development of conservation and measurement concepts are characterized by

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[38] Ibid, p. 144.

[39] I.E. Siegel, F.H. Hooper & Flavell. Logical Thinking in Children, Research Based on Piagetian Theory. Holt, Rinehart & Winston, Inc., 1968, pp. ix and 295-96.

a complete inability to conserve or apply measurement process and a total dependence on one-dimensional perceptual judgement. Conservation and measurement concepts develop gradually as children become less dependent on a single immediate dominant dimension. Carpenter[40] (1975), in outlining the above named experiences has suggested (through Bruner, Olver and Greenfield, 1966 and Piaget , 1960), that reliance on the perceptual aspects of an event is the single most important factor contributing to most conservation errors. Carpenter has further suggested that conservation errors occur because the immediate perceptual prpperties of the conservation problems over-ride the logical properties that imply conservation and that conservation would occur if the factors that contribute to this "perceptual seduction" were removed from the conservation problems.

One of the factors that enable children to abandon centring on single dominant dimension (Carpenter[41] 1975), is the recognition that an increase

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[40] Thomas P. Carpenter: "Measurement Concepts of 1st and 2nd. Grade Students." Journal for Research in Mathematics Education, Jan. 1975, p. 3.

[41] Ibid, p. 4.

in one dimension is compensated for by a decrease in another. For example, when water is poured from a short, wide container into a tall,narrow one, the increase in height is compensated for by the decrease in width. From this argument it is fair to state that until the child is able to "conserve" (Otaala[42] 1972), that is,to bear in mind that whatever aspects of an object or phenomenon remain the same (invariant) while other aspects undergo a change (transformation) in appearance, he is not likely to grasp many kinds of verbal instruction. The child can learn to count and to recite the multiplication table (rote-memory) but as long as he has no notion that the number of ten or so objects is the same regardless of whether they are arranged in two sets of five sets of two or spread over the surface of a table or bunched together in a pile, he will have difficulty in reasoning systematically as well as following (understanding) adult reasoning.

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[42] Barnabas Otaala: Piaget for Teachers. in Millie Almy's: "The Usefulness of Piagetian Methods for Studying Primary School Children in Uganda," 1972, p. 4.

Copeland's[43] (1974), Piaget's[44] (1966) and Otaala's[45] (1972), investigations conservation tasks have some outstanding suggestions which could be used for further research work in conservation concepts. For example, Otaala investigated in 1972, the conservation abilities of unschooled Iteso adults. By using Piaget's interpretation of conservation concept, Otaala has inferred that "conservation" refers to the ability to understand that certain properties of objects remained invariant (are conserved) in the face of external transformation. Otaala concluded by contending that a child who cannot understand conservation would have difficulty with the problem of conserving a liquid when transformed by use of differently shaped or sized containers. For example, if a child estimates that there is more liquid in the taller, narrower container, because the level has been increased, he centres his thought or his attention on the relation between heights of the containers and ignores their widths. Otaala has suggested therefore that conservation exists in

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[43] Richard W. Copeland: How Children Learn Mathematics: Teaching Implications at Piaget's Research, Macmillan Publishing Co. Inc., 2nd Ed., 1974, pp. 292-293.

[44] Jean Piaget: The Child's Conception of Geometry, Routledge and Kogan Paul, 2nd. Ed., 1966, pp. 90-91 and 104.

[45] Barnabas Otaala: Piaget for Teachers: (1972) "A Preliminary Investigation of conservation Abilities of Unschooled Iteso Adults." pp. 20 - 21.

children from about the age of eight years. He has further suggested that in the third stage of development (concrete stage) a child assumes conservation for each of the transformations the quantity undergoes. Whatever change the child observes (Otaala[46] 1972), he knows that if the amounts were originally equivalent, they must remain equivalent. Some investigations have claimed (Greenfield 1966), that the development of conservation is related to some formal schooling. Greenfield used Wolof children (unschooled) and found that age was the biggest determinant for conservation ability.

(Otaala[47] 1972), has used Piaget's objecting factors to the belief that experience of objects, in physical reality is obviously a basic factor in the development of cognitive structures. For example, the conservation of the substance in the case of changing the shape of a ball of plasticene:

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[46] Ibid, "A Preliminary Investigation of the conservation Abilities of Unschooled Iteso Adults." pp. 20 - 21.

[47] Ibid, "Cognitive Development in Children": Development and Learning (Piaget) pp. 6 - 7.

"We give this ball of plasticene to a child who changes its shape into a sausage form and we ask him if there is the same amount of substance as there was before. We also ask him if it now has the same weight and thirdly if it now has the same volume." The findings which have been the same every time this experiment has been done, show that first of all, there is conservation of the amount of substance. At about eight years old, a child will say: "there is the same amount of plasticene." Only later does the child answer that the weight is conserved and still later, that the volume is conserved. The conclusion drawn from this objecting factor is that no experiment, no experience can show the child that there is the same amount of substance. He (child) can weigh the ball and that would lead to the conservation of weight. He can immerse it (ball) in water and that would lead to the conservation of volume. This conservation of substance is simply a logical necessity (Piaget). The child now understands that when there is transformation something must be conserved because by reversing the transformation it can come back to the point of departure and once again have the ball.

Piaget[48], Inhelder and Szeminska(1966) asked and answered a very important question showing that measurement and conservation are related mathematical concepts. The question was: "without measurement and metric co-ordinates, how can a child identify the positions which are vacated by their original objects and re-occupied by others?". The answer lies in the conservation of length when objects undergo change of position, ... that conservation is measured only if the site of an object maintains a constant size (i.e. the distance relations) when it left empty, and the size of a site which was previously empty is not altered when it is occupied by an object. Piaget et al have therefore suggested that conservation of length when objects undergo a change of position does not yet imply understanding of Euclidean metrics. Piaget et al have yet posed other relevant questions: "How do children pass from qualitative conservation to the measurement of length? Will the conservation of length be delayed in this situation or will this special situation and measurement in general prove

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[48] Jean Piaget, Inhelder and Szeminska: The Child's Conception of Geometry, Routledge and Kogan Paul, 2nd. Ed. 1966, pp. 90 - 91 and 104.

tractable as soon as conservation has been achieved? More generally, what is the process leading from conservation to measurement? "Piaget et al have maintained that the over-all effect of their above questions and their suggested answers is that for each newly filled site there is a corresponding site which is newly empty and vice versa, which implies conservation alike of the distance between objects and of the length of objects when moved.

Piaget[49] , (1966),again posed this question about the study of the understanding of distance and length: "do little children think of an area as a stable attribute which may be conserved even while the shape of an object is altered?" This question must be answered, Piaget maintained, before the study of metric relations is involved. To find out how this kind of conservation is constructed Piaget adopted the usual method of showing children an area which is made up of sub-areas organised in one way and then altering the apparent structure of the whole, while the children look on: The other question Piaget asked was: "Does the whole remain invariant in

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[49] Ibid, pp. 261 - 273 and 278.

spite of the re-arrangement of its parts?" The conclusion drawn by Piaget et al is that there is conservation of area as soon as there is operational grouping in addition and subtraction of area. A further conclusion is that in any domain whatever, the discovery of conservation implies the construction of a logical or sub-logical "grouping" or else a mathematical "grouping" of operations which is why the study of these invariants and their appearance is important.

A further suggestion from Piaget[50] is that from the stand point of geometry, area and volume are related closely but from that of psychology, the question of volume raises many problems of its own, hence the need for the study of these concepts (area and volume) by means of various discussions which precede them. Piaget made an observation when they suggested that an allowance must be made for the fact that children cannot be asked about conservation of volume without introducing physical objects like bricks or cubes filled with sand etc. which applies particularly to the pupils at the level of concrete intelligence.

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[50]Ibid, pp. 354 - 55.

Piaget[51], Inhelder and Szeminska carried out some experiments about the concept development of the conservation and measurement of areas. They suggested that to discover what sort of notions little children might posses with regard to the conservation of areas, two complimentary techniques might be used: the experimenter might present an area composed of several separate sections and modify the arrangement of these parts to see whether or not the child would consider that the whole remained constant. In method two, the child is simply shown two rectangles recognised as congruent and the experimenter cuts a portion off one and moves it to another part of the same figure. Thus the experimenter cuts the rectangle diagonally in half and puts the two sections together in the shape of a triangle, or he cuts off the four corners and puts them against the sides to produce an irregular polygon etc. The questions always asked are: "Are these the same size?" "Is there the same amount of room?" etc. From these experiments, there is a clear indication that children make judgements about an area as such, but there is a clear development from non-conservation through partial and intuitive conservation.

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[51] Ibid, pp. 273 - 74.

Piaget[52] et al have further suggested that whatever preliminary questioning is made, the enquiry always finishes with questions about the conservation of volume. For example, the experimenter starts from the original model, say of $36cm^3$ (or more as the case may be), that shape having been faithfully reproduced by the child out of unit-cubes, with or without the help of an adult. The adult now builds various other constructions out of these same bricks by changing the form of the bottom or base layer. The child watches the adult and is asked (child): whether there is as much room in the new "house" as there was in the old, or whether there is more or less and: could one use the self-same bricks which now go to form the new house in order to re-make one like the original and exactly the same size. This question of conservation of volume is not easy to be certain about, as is clear from the experiments carried out by Piaget and his associates. In this case, the experimenter must try to discover whether the child relies solely on the conservation of the actual number of bricks being used, or whether he thinks in terms of their total volume and its conservation also. The fact

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[52] Ibid, pp. 357-58

that some children accept the first kind of conservation but reject the second, proves that the two notions (much roomness and self-sameness) are distinct.

Piaget et al have in the final analysis of conservation of volume suggested that by looking at such a concept from the physical point of view, conservation is based on the invariance on a quantity of matter instead of that of volume in the sense of the amount of space taken up by solid or liquid matter. Modification of form in three dimensions entails variation both in length and in area while the volume remains constant.

The Unesco - Unicef Nairobi[53] (1974), seminar suggested, based on previous research, that about eight different conservation tasks have been used in concrete conservation and that over half of the studies employed, no more than two (tasks) at a time, and few of these used any other Piagetian task. The choices of (tasks) are fairly well distributed, with conservation of liquid quantity

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[53] Unesco - Unicef: "The Development of Science and Mathematics Concepts in Young Children in African Countries". Report of a Regional Seminar, Sept. 1974, Nairobi, p. 45 (d and f).

being most popular. This choice (liquid quantity) presents problems of interpretation, since success on tasks presumably related to the same cognitive structures are not achieved at the same time, and may be attained in different orders in different cultures. The seminar further suggested that great ranges in the criteria by which successful conservation is judged could also be found. The seminar formed the opinion that many researchers have not required explanations for conservation answers, and those that have, exhibited great variation in the degree of consistency that they required in order to judge children (subjects) as conservers.

Copeland[54] (1974),has much recently suggested that sometimes ago, it was taken for granted that there was no change in the length of a ruler as it was moved along an object being measured. The big question was therefore: "Do children conserve the length of an object-changes as it is moved?"

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[54] Richard W. Copeland: How Children Learn Mathematics: Teaching Implications of Piaget's Research. Macmillan Publishing Co. Inc., 2nd. Ed., 1974, pp. 292 - 93.

Copeland explained that as the ruler was moved to a position the children thought that the space from which the ruler had been moved remained the same.

Copeland[55] has again discussed the concept of conservation when he suggested that conservation is also fundamental to the development of measurement concepts. He further suggested that in order for the child to measure, he (child) must develop the concept of conservation of length. Copeland has even gone further when he suggested that also fundamental to and a prerequisite for the understanding of measurement is the conservation or invariance of distance and length.

Much recently Copeland[56] contended that many adults have taken conservation for granted that there is no change in the length of a ruler as it is moved along an object being measured. The question Copeland asked is whether children conserve the length of an object when it is moved or whether they (children) think that the length of an object

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[55] Ibid, pp. 248 - 49 and 252

[56] Ibid, pp. 255 - 57.

changes as it is moved. Copeland tried to answer the above question by also considering whether children think that the space from which the ruler has been moved remains constant as it is moved in new position. To investigate the above question, Copeland used Piaget's model when two straight sticks 5 centimetres long were used. The children involved in this experiment were asked if the sticks are the same length. One stick is then moved forward (away from the other stick). The children are again asked if the sticks are the same length. From the children's responses, Copeland concluded that at the early stages of a child's education, he (child) thinks that the stick which has been moved is longer since (children) look only at the extreme end of the stick or its end point which is not farther away. The children do not consider the other end point in that length is constant.

Children at this stage of cognitive development consider both end points of the stick simultaneously and the interval between, or length of the stick. Other children on the other hand, equate movement with "longer" (if a stick is moved, it grows longer). Children's concept in this mode of thinking is that objects grow or contract when moved forward (grow) or backward (contract). In this case, the children cannot measure because they do not "conserve" Euclidean length of an object as rigid when such object is moved. Mathematically, these children are still working at the more primitive topological level, which allows an object to be stretched rather than having to remain rigid as in Euclidean geometry, Copeland has observed and suggested.

Copeland[57] drew some conclusions from the above observations and has suggested that at stage 3 (concrete operational), children have the concept that sites or positions in space occupied by the sticks (referring to previous experiment with sticks) are the same length. At this very stage, children normally realize that moving an object does not change the space it occupies. The concept of conservation of length of an object as the object is moved in relation to the sites of Euclidean space, becomes logically necessary. Thus, the necessary concepts of change of position, conservation, and an external reference system as a prelude to measurement do not appear for many children until the age of seven to eight years old or until sometimes during the second or third grade of school.

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[57] Ibid, pp. 258 - 59.

Copeland also observed and remarked that many teachers (of primary school) do not realize this conceptual development and yet they attempt to teach measurement before they teach conservation. Copeland[58] tried to describe the difficulties which confront children with conservation of length of an object as it is moved when he (Copeland) suggested that as an object is moved away, children less than seven years old usually think, it becomes longer. The question asked by Copeland is "Can children then conserve the length of subunits or do they (subunits) somehow change length too"? To investigate the conservation of length of sub-units, Copeland has suggested that, Piaget used two rows of match sticks placed side by side and the same length. One row is altered by one or more of the match sticks at an angle to the others so that the row is no longer straight. The children are asked if the two rows of match sticks are the same. Copeland has suggested that Piaget had concluded the investigation by saying that the idea of necessary conservation (in order to measure), which entails the complete co-ordination of operations

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[58] Ibid, pp. 259 - 60 and 262.

of subdivision and order or change in position, is accomplished at stage three (7 - 8 years)of age.

Copeland[59] again illustrated through Piaget's method how to investigate the concept of conservation of area. The method involved two procedures. The first procedure involved two identical rectangular arrangements of six blocks. This arrangement was then altered for one rectangle by moving the two blocks at one end and placing them on top or below the others. The children were then asked if the rectangles were still "the same size" or had "the same amount of room". The second method acted as a check on the first one. It involved the use of two congruent rectangles cut out of card-board. One rectangle was then cut into two pieces and the pieces were re-arranged so as to have a different shape from the other rectangle. The experimenter observed and concluded that children from five to six years of age think that the area or amount of room changes as the shape changes. The reason perhaps is that the children of this age are fooled by perception on which they (children) base their answers. "One configuration is larger because it looks large or because in method two it has been cut." Copeland has therefore suggested that the

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[59] Ibid, pp. 292 - 294

concept of conservation of area requires the concept of grouping of parts to form a whole and the realization that the arrangement or grouping (addition) of these parts does not affect the area (sum). By comparing the stages of development in children for the concepts of length and area, Copeland has concluded that the conservation of area appears at the same time as the conservation of length though one might think area more difficult.

When studying the conservation of volume, Copeland has[60] suggested the concept of conservation or invariance as applied to volume is found to develop later than as applied to such ideas as number, length and weight. To investigate the understanding of volume, Copeland used a set of 36 building blocks. These blocks are first placed on a 3 x 4 block base. The children are then asked if the house containing the "same amount" of room could be built on a smaller island as could be shown on a 3 x 2 block base. Children under 7 years do not think that the building could be made taller without increasing volume. In this case, there is no form of conservation of volume with a change in shape. However, from 8 years, children

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[60] Ibid, pp. 299 - 301.

begin to understand conservation of volume. At this stage (from 7 years old) a child is able to transform a 3x3x4 house and reconstruct it on a base of 2x2. He (child) begins by making it 2x2x4. After some hesitation, he realizes that he must build it higher to 2x2x7. Comparing two houses, he then adds another story making it 2x2x8. He thinks he is right but is unable to explain why. The child is asked to copy a 3x3x4 model which he does. The experimenter then alters the model to 2x2x9. The child is again asked if there is still the same amount of room, he says that it is because it is the same bricks. The model is further transformed to 2x18x1. In this arrangement, the length at first makes the child think that there is more, but then he remembers that there is the same amount of room because the same number of bricks is involved. The child is again misled with a model of 1x36x1 because it is very long. He agrees that it is the same bricks but still thinks that there is more room. Children in concrete operational stages are capable of mastering the idea of conservation of volume, Copeland has concluded. The type of conservation referred to is of the interior volume (volume determined by the boundary surfaces of the blocks) because the children's notions are still topological as well as Euclidean. Hence the most difficult concept is the conservation of volume of liquids because the interior volume of bricks cannot be equated with the volume of water occupied or displaced by the bricks. At the formal thinking, children are now able to measure volume by seeing its relation between length in three dimensions (length x width x height) conservation of volume at this 4th stage of development, is extended to include that of occupied or displaced volume. A 12 - year old asked to build a house that is 3x3x4 on a 2x3 base, says he needs 9x4 or 36 bricks. "Why?" This is so because it is high and each row is 3 and that is 4x3 or 12 and multiply that by 9 because that is the area on top. This multiplication through the child's thinking is of course wrong, because he wants to multiply by 9 rather than by 3 but he is close to the correct procedure. At last children in this stage 4, (formal thinking) come to discover for the first time that "volume" is not just the interior "contained" by some three dimensions object such as a brick,but that space exists in its own right whether occupied by the brick or not occupied by it.

Copeland[61] has suggested from Piaget's point of view that this change in concept is regarded as the child having understood the relation between boundary lines (or areas) and its volume, when children - inevitably deepen their sense of the conservation of volume. The children discover for the first time that it is not merely the interior "contained" which is invariant but the space occupied in a wider context. The relation between the lengths of sides and volume remains invariant when the whole shape is transformed, and this is essential truth which is seized upon at stage 4 and expressed in the form of mathematical multiplication: 3x3x4=2x3x6=2x2x9=36 etc. Because interior volume can be measured and calculated from now on, its invariance (conservation) now extends to the surrounding space.

Modgil [62](1974),with authority on Piagetian Research, in 1974 suggested that the plethora of studies relating to conservation acquisition in young children indicates its popularity in cognitive developmental research. He says, this is due to the

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[61] Ibid, pp. 302 - 303

[62] Sohan Modgil: Piagetian Research, A Handbook of Recent Studies, NFER Publishing Co. Ltd., 1974 p. 31.

central role it has played in the research and theorizing of Piaget and the consequent interest of teachers. Modgil has said that Piaget uses evidence of sequential cognitive progress through three stages of equilibration in achieving a conservation. Initially, Modgil goes on, the child adopts a "strategy" of responding to a single dimension even in the face of continued transformation. Secondly, judgements flactuate between both dimensions, either on the same problem or on varied problems. Finally, both dimensions are related simultaneously and their transformational relations are realized.

Modgil[63] has further suggested through Elkind (1967) that there is existence of two kinds of conservation: identity and equivalence conservation. Modgil has described the investigation carried out by Hooper (1969a and 1969b) on the two kinds of conservation. Hooper administered tasks of one identity and two equivalence conservation problems to young children. Hooper concluded that equivalence conservation appeared later than identity conservation. Further results revealed that 75% of the children failed both identity and equivalence tasks

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[63] Ibid, pp. 56 - 58.

for conservation of discontinuous quantity and 13¾% were operative on both tasks. 11¼% passed identity and were non-operative on equivalence, whereas no child passed equivalence but failed identity. Modgil quotes other researchers, for example: Northman and Gruen (1970) who did not support the hypothesis that identity conservation, being a simple task, was attained developmentally prior to equivalence conservation. These researchers on the other hand, support Piaget and Inhelder (1956) when they(Northman and Gruen) suggested that identity and equivalence conservation emerge simultaneously, as postulated by Piaget et al.

To investigate a case for identity and equivalence conservation in children, Modgil has described Elkind's (1967) experiment in which the experimenter advanced two kinds of conservation. In the equivalence conservation task, two identical beakers (S_1 and S_2) are filled with equal amount of liquid. The contents of S_1 are then poured into a third differently shaped container (V). The child is then asked whether or not the amounts of liquid in (S_2) and (V) are still equivalent. On the other hand, if the child was asked to compare the contents of (V) to itself in the original beaker (S_1) the task would be of identity conservation. Further, that identity conservation is a necessary but not sufficient condition for equivalence conservation. Transitivity is the critical mental operation which the child must perform in an equivalence task but not in an identity task.

Modgil[64] using Piaget et al (1941) has suggested that children attain conservation of substance at about the age of seven years, conservation of weight at 9 years and conservation of volume at about 12 years. Modgil has described the investigation of Lovell and Ogilvie (1960 and 1961) about Piaget's theory on conservation concepts of substance, weight and volume using children of junior school age. The over-all results indicated the presence of the three stages in the development of the concept of substance. But Modgil himself has maintained that his evidence and others, does not always agree with that of Piaget, nor does it enable the Modgil's school of thought to prove or disprove the assumptions that the child arrives at the concept of conservation because he is able to argue logically in concrete situations.

Mogdil[65] described Uzgiri's (1964) support for Piaget et al (1941) with regard to the sequence

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[64] Ibid, pp. 63 - 65

[65] Ibid, pp. 65.

and the time of the attainment of substance, weight and volume. Modgil pointed out that studies like those of Beard (1957), Hyde (1959) and Lovell - Ogilvie (1960) suggest that children who demonstrated conservation ability of any particular concept with the plasticine balls did not show conservation of the same concept when confronted with a different material or vice-versa. Uzgiris therefore examined systematically the effect of varying the materials used to test for the conservation of substance, weight and volume. Over-all results indicated that when different materials were used, there was a significant difference in the conservation responses of children. Uzgiris has therefore commented: "Individual's past experience may well underlie situational differences and account for the observed inconsistency across the various materials. It may well be that when a schema is developing, specific contacts with the environment will lead it (schema) to accommodate more in certain areas than in others, producing situational specifity in terms of specific past experiences of the individual". From this conclusion, (Uzgiris) emerged the study by Price Williams,[66] Gordon and Ramirez(1969), which included in its objective an

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[66]Ibid, p. 65.

investigation into different kinds of environment which may enhance or inhibit conservation of operativity. The general results indicated an early age for the acquisition of the conservation of weight and volume.

Modgil[67] quoting Piaget et al (1965) on conservation of length said that Piagetian school of thought has shown that the age for the acquisition of conservation of length is between seven and eight years of age. Braine (1959) in Modgil's demonstration has shown that by using non-verbal techniques, it is possible to lower some of Piaget's age norms by more than two years. Modgil has made a condition through Smedslund (1963) that if Braine's conclusions are valid, it might be reasonable to hypothesize that the use of essentially non-verbal techniques would allow the child to reveal his acquisition of the conservation of length at an age somewhat younger than seven or eight years. Modgil however, has gone back to Smedslund (1965) who commented on Braine's reply regarding the development of transitivity of length. According to Braine, children normally become capable of making transitive inferences of the types:

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[67] Ibid, p. 115.

$A > B$. $B > C$:$\supset$: $A > C$ and $A < B$. $B < C$: $\supset$ $A < C$. (where $>$ means greater than and $<$ means less than) around the age of five years.

Modgil[68] analysed the results of the previous experiments and comments. He concluded that an analysis of the studies on this topic reveal that the sources of a child's understanding of the conservation concept might depend on:

(a) the child's level of development at the beginning of the training to conserve;

(b) the training method employed at the time of teaching the child to conserve;

(c) the particular tasks used;

(d) the amount of training;

(e) the criteria used to evaluate the success.

Taloumis[69] (1975), described much recently that Piaget (1960) in a research concerned with how a child develops an understanding of the concepts of area conservation and area measurement, concluded that a child's construction of both concepts, develops

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[68] Ibid, p. 126

[69] Thalia Taloumis: "The Relationship of Area Conservation to Area Measurement as Affected by Sequence of Presentation of Piagetian Area Tasks to Boys and Girls in Grades 1-3." Journal for Research in Methematics Education, 6, 4, Nov. 1975, 232-40.

gradually through stages and that conservation concepts precede measurement concepts. The appropriate ages at which different levels of concepts were acquired were indicated through Flavell's (1963) discussion of the fact that Piaget's studies fail to provide information as to the backgrounds, sex intelligence levels or testing experiences of the children. Hence, Taloumis has suggested that in order to investigate certain aspects of Piaget's theory of cognitive development in area, the study should standardize procedures by administering to the same child a modified version of some area conservation tasks and two area measurement tasks by using a standard questionnaire form. Taloumis used three instruments: two index cards, congruent fields of grass and the congruence of two gardens and the congruence of two plots of ground for flowers. The questions centred on "same amount of ground" "more ground". The results of the study indicated that area conservation and area measurement are affected by grade and by sequences of presentation of Piagetian area tasks to boys and girls in grades 1-3. A 3x2x2 ANOVA (Analysis of Variance) indicated that there was a main effect for grade and sequence but not for sex, at 5% level of significance.

Grade ... F = 30.37

Sex ... F = 1.35

Sequence ... F = 7.43

As there was a main effect for grade and sequence, a Multiple Regression for conservation was used to establish the cause of the main effect through correlations:

Multiple Regression Analysis

Grade	Sex	Sequence	Mean
1	1.43	9.64	0.82
2	0.52	2.24	0.68
3	2.66	3.98	0.25

Results[70] of the experiment indicated that sequence of presentation affects the scores of the 2nd. group of area tasks. Secondly, that area conservation performance and area measurement performance are equally predictable if the scores on one of these is known. On plotting the means of the scores for conservation and measurement on a graph within the four sex sequence groupings, the results indicate that children's performances in area conservation and area measurement show no differentiation in the amount of increase from grade to grade.

[70] Ibid, p. 241.

Performance on both groups of tasks appears generally parallel to each other. Taloumis has further suggested that the results do not agree with Piaget's theory that, measurement occurs spontaneously at least for grades 1 - 3. In this regard, Taloumis has advised that for the classroom teachers, the study has this to offer:

1. lends support to programmes that include instruction on area measurement in the primary grades;
2. gives no evidence that boys and girls need differentiated instructional approaches.

MEASUREMENT

Carpenter [71] (1975), carried out a study in which he suggested that the purpose of measurement concept is to identify some of the aspects of the measurement process that young children naturally attend to and those that they ignore or are unable to make use of. He specifically attempted to assess the degree to which young children rely on perceptual comparisons or at least require perceptual support for their measurement operations. Another factor which Carpenter investigated was whether measurement comparisons involving equal quantities were of the same difficulty as those involving unequal quantities, and finally, he attempted to identify some of the misconceptions regarding the measuring process that result from those and other factors.

By providing the background of his study, Carpenter pointed out that his study (1972) gave results which conflict with the conclusions of literature by Piaget et al (1960) regarding young children's conception of quantity and measurement. Carpenter has said that early, Piaget et al had suggested that the earliest stages in the development of conservation and measurement concepts was

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[71] Thomas P. Carpenter: "Measurement Concepts of 1st - 2nd. Grade Students". Journal for Research in Mathematic Education, January, 1975, p. 3.

characterized by a complete inability to conserve or apply measurement process and a total dependence on one dimensional perceptual judgement. Piaget's contention was that conservation and measurement concepts developed gradually as children became increasingly less dependent on a single immediate dominant dimension.

Carpernter[72] investigated a case in which he attempted to determine whether conservation and measurement errors were a function of perceptual dependence or simply resulted from children centering on the last cue available to them. He quoted Piaget's (1972) views when he (Piaget) had asserted that one of the factors which enabled children to abandon centring on a single dominant dimension was the recognition that an increase in one dimension was compensated for by a decrease in another. For example, when water is poured from a short, wide container into a tall, narrow one, the increase in height is compensated for by the decrease in width. When quantities are measured with different units, an increase in unit size is compensated for by a decrease in the

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[72]Ibid, pp. 3 - 4.

number of units. By contrasting measurement problems in which it was possible to visually distinguish this compensating relationship between unit size and the number of units with problems in which it was not, Carpenter attempted to determine whether the factor of compensation significantly contributes to young children's conservation and measurement judgements. Traditionally, the problems reported in research on children's conception of conservation and measurement have involved comparisons of equal quantities that are subsequently transformed to appear unequal. Some reseachers (Saltz, Siegel, 1967) have noted a tendency of young children to over-discriminate and look for differences where they (differences) do not exist. From these observations, it can be hypothesized that the tendency to seek inequality is a significant factor contributing to conservation and measurement errors.

Carpenter's[73] results indicate that although children have a number of misconceptions regarding the measurement process and often misapply measurement operations, measurement has some meaning for the majority of students in all grades. Carpenter concluded, using Bearison's (1969) study, by suggesting that the teaching of measurement may help to stabilize a child's conception of quantity.

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[73] Ibid, pp. 11 - 12.

A further conclusion, most important is that measurement concepts begin to appear in young children earlier than Piaget et al (1960) concluded. This conflict appears to be due to the fact that Piaget et al employed less structured measurement tasks than used in Carpenter's study. Similarly, Carpenter has concluded that the conflict with the conclusions of Piaget (1952, 1960), Bruner et al (1966) and others regarding the role of perception in conservation and measurement judgements appear to be due to the fact that in all the studies on which the investigators based their conclusions, the distracting cues were visual. Their (investigators) conclusions that children depend on the immediate perceptual qualities of an event also appear to be a function of this lack of experimental variability.

At the Bangkok[74] (1972) seminar organised by the Unesco and Unicef, the participants suggested that measurement is a necessary and vital activity of both children and adults in every-day living. The seminar further suggested that one of the most important and urgent concept for investigation was

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[74] Unesco - Unicef: "Report of a Regional Seminar:" Bangkok, May/June, 1972, pp. 24 and 27.

the child's understanding of the invariance in length. The author has made an opinion arising from Bangkok's school of thought that research in concepts of conservation and measurement require immediate and urgent attention in Kenya as it touches on the nation's way of making decisions in their daily living.

Lovell[75] (1971),on the other hand, has suggested that the fact that area is conserved in no sense ensures that the child can measure area. Lovell further suggested that it is usual to employ a square of side one linear at first in measuring unit. However, Lovell has contended, the child finds it difficult at first in measuring area, and it is to a consideration of these difficulties that researchers now turn to. Lovell has described the experiment conducted by Piaget, Inhelder and Szeminska. The study (Piaget) used two techn - iques to investigate the difficulties of measuring among the young school children. The two techniques used were: measurement by superposition and measurement by unit: iteration (repetition) Research studies by Piaget et al (1960) and Lovell - Ogilvie (1961) have revealed that an understanding of

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[75] Kenneth Lovell: The Growth of Understanding in Mathematics; Kindergarten thro' Grade Three. Early Childhood Series, pp. 108 and 177.

volume in the three senses (internal volume), capacity (volume as "occupied space" - room) and complementary volume (amount of liquid displaced by an immersed object) is not understood by pupils who are below eleven years of age. Lovell has concluded the study by suggesting that certain kinds of activities would normally promote the pupil's thinking in regard to the concept of volume within the age range. For example, Lovell has suggested through his investigation that the teacher could help the child to appreciate that the internal volume or capacity, say a box, is a measure of the material that would fill it.

Copeland [76] much recently has suggested that measuring as an activity or operation involves physical objects and is a very concrete type of activity. Accordingly, children like measuring as it involves the usage of measuring instruments. Copeland has questioned the manner in which children learn to measure and suggested that its roots are in perceptual activity, but the concepts are not completely developed until sometime between the age

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[76] Richard W. Copeland: How Children Learn Mathematics, Teaching Implications of Piaget's Research, 2nd. Ed., 1974, pp. 247 - 248.

of eight and eleven years. Copeland has also suggested that measurement notions develop in relation to the basic concept of conservation. Copeland has further suggested that the concept of conservation is also fundamental to the development of measurement concepts.

The question raised by Copeland was: "How does a child then approach a problem situation that requires measurement?" Copeland described Piaget's investigation which bears on the question: Piaget used a tower of blocks of various shapes. The child is asked to make a tower the same as the investgator's or as tall as the investigator's. Then the child later on is asked to build the tower on another table, which is lower, so as to avoid visual transposition attempt to place them together visually). Copeland[77] through Piaget, has concluded that measurement is a synthesis of the operations of subdivision into parts and of substitution of a part upon others. The ability to measure therefore develops later than the number concept because it is more difficult to divide a continous whole, such as an object being measured, into interchangeable subunits than it is to count a set of objects that are separate and discrete from each other, such as beads or blocks.

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[77] Ibid, p. 252.

Copeland[78] has further suggested that attempts at actual measurement are investigated by using two strips of paper pasted on a piece of cardboard with each strip made up of segments at various angles. The children then are asked if the strips of paper are the same length and then asked to verify their answers using a movable strip of paper. Copeland has suggested that "until length of an object can be conserved as an idea, measurement is meaningless". Second, the child must understand the concept of subdivision since the object to be measured must be subdivided into sub-units the same length as the measuring instrument (ruler). The child then moves the measuring unit along the object being measured, marking off or subdividing the object into units the same length as the measuring unit or ruler. The lengths of these sub-units must also be conserved for meaningful measurement. Third, the child must realize that a distance between two objects is conserved when other objects are placed between them (objects).

Copeland[79] has summarized the concept of area measurement when he suggested that conservation of area develops in the child at approximately the

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[78] Ibid, pp. 263, 269 and 270.

[79] Ibid, p. 311.

same time as conservation of length. Therefore, conservation is a prerequisite for an understanding of measurement in either one or two dimensions. Children at age nine in general are ready for measurement in two dimensions using the method of superposition, (inter-change of positions). Copeland[80] has suggested in regard to volume measurement that since measurement of volume is not usually taught until the junior high school level, children should be ready for this concept at that time. At whatever age volume measurement is taught, it would be most worthwhile that there be readiness experiences in addition to formula. In any case, the formula for volume measurement should not be taught until the children have had experience with three-dimensional objects (length, width and height) and are beginning to understand the necessary concepts for the formula.

Piaget[81] et al have made several investigations regarding the measurements of length, area and volume. For example, Piaget has suggested that

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[80] Ibid, p. 312.

[81] Jean Piaget et al: The Child's Concept of Geometry, Routledge and Kegan Paul, 1960, pp. 26 - 29.

the study of spontaneous (motion) measurement raises a problem due to the idea of transforming the motor element inherent in action to make it conform with a co-ordinate system; while the construction of measurement proceeds in the main from perceptual comparisons. Piaget maintained that for a long time children are content with visual comparison, and only later do they think of putting objects alongside each other to check their (objects) estimate, while the concept of a measuring rule which could provide a common measure arose still later. Piaget et al have asked: "how is perceptual measurement related to operational measurement?" To answer this important question, Piaget's school of thought used two straight sticks in order to compare their (sticks) lengths A and B, to decide which was greater. By engaging visual measurement experiment Piaget et al concluded that perceptual measurement was inexact and merely approximative and was subject to illusions or systematic errors. Piaget et al, have further suggested that so long as a child has no notion of structuring his (child) space by means of co-ordinate axes, he is in no position to limit visual comparison in any way (that in some ways, visual comparison is more accurate in young children than it is in older children).

Piagetian[82] school of thought has divided mensurational stages of development into three stages whose summary is as follows:

Stage 1. (4 - 6 years) marked with direct visual comparison where transfer of activity is purely visual (perceptual).

Stage 2. (5 - 7 years) marked with the first appearrance of change of position leading to the manual transfer as an anxiliary to visual transfer and elementary use of changes of position.

Stage 3. (7 years onwards) marked with transitional development between levels 2 and 3 leading to gradual discovery of an independent middle term. This important stage also marks the idea of operational common measure which necessitates the idea of transitive congruence without iteration (repetition) of units. The last part of this stage marks the most famous operational common measure which leads to the evolution of a metrical system by unit iteration.

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[82] Ibid, pp. 31, 40, 44, 50, 56 and 62.

Piaget[83] et al have made some outstanding conclusions about their (Piagetian) investigations. The conclusions emphasized the presentation of changes of position and problems of measuring. Some of Piagetian[84] conclusions are that at stage one representation of changes of position consists simply in recalling the object's own actions. This amounts to a set of bare sensori - motor schemata. Possible reference objects are centred on the pupil's own movements, and on the arrival points alone. As a result, the child (pupil) instead of viewing his own movements in terms of co-ordinated reference objects, he shows a systematic egocentric distortion of space. At level 3 (stage) the child can reconstruct his own changes of position adequately in conformity with the laws of mathematical "groups" and in terms of external reference objects. These in turn are systematically co-ordinated. Hence, the child's conception of space includes himself as one moving object among many. Changes in his own position are integrated within an objective spatial group and therefore comparable with other forms of movement. Another conclusion is that little children at say stage one, possess an exaggerated confidence in visual

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[83] Ibid, pp. 64

[84] Ibid, pp. 64 - 65.

comparison; their measuring may be summed up with the words: "I look and I see." This faith is undermined when they (children) come to notice a difference in base levels. As a result, the two perceptual fields are brought together by manual transfer (stage 2). Piaget has suggested that because the child's body transfer is inaccurate, sooner or later he rejects it, but the image born of imitation gives rise to the symbolic object. This object is symbolic in that it is still a copy of the things being compared, but is a common measure independent of the child's own body.

Piaget[85] et al have suggested that the knowledge gained in spontaneous measurement is a clue to knowing the idea of linear measurement. Linear measurement is a problem which concerns the relations between synthesis(spontaneity)and differentiation (change of position) and co-ordination (three dimensions) of complementary qualitative groupings. Investigations of these attributes (linear measurements) follow the same experimental procedure as is already mentioned for spontaneous

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[85] Ibid, pp. 116 - 117.

measurement only punctuating for linearity. For example, the pupil is asked to judge between strips of paper in a variety of linear arrangements involving right angles, acute angles, obtuse angles etc. but the stripes of paper are pasted on cardboard sheets in linear experimental activities. Words like: (equal, longer, shorter) are used to answer the questions. The pupil is required to verify his answers by assigning him to phrases like: what makes you say so? how do you know? and eventually by measuring comparatively the strips of paper using a geometrical instrument e.g. a thread, ruler, tapemeasure etc.

Piaget[86] et al have further suggested that true measurement of distance and lengths begins when the pupil recognizes that any length may be decomposed into a series of intervals which are known to be equal because one of them (interval) may be applied to each of the others in turn. Generalized subdivision, according to Piaget, therefore gives rise to measurement because it enables the pupil to think of a unit as forming part of any number of wholes, that is, as an elementary common part. As such, although the operations of measurement exactly parallel those involved in the child's

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[86] Ibid, p. 149.

construction of number, the elaboration of the former is far slower and unit iteration is, as it were, the coping stone to its construction, because at level 3 (stage) measurement is still qualitative.

Piaget's[87] caution in the geometry of area is that the operational construction of areas and solids (volume) raises a new psychological problem over and above those encountered in measurement in two or three dimensions. To define a point in an area or in a solid, the pupil takes two or three measurements and co-ordinates them by reference to a two or three dimensional framework. By using a two- or three - entry table, he establishes one-one correspondence between seriated points in each of two or three dimensions. The fact of measurement transforms relations of order of position into metric relations, but the operations involved are of logical multiplication. Piaget et al have further suggested that by contrast, the calculation of area or volume implies mathematical multiplication of metric relations. For example, the area of a rectangle

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[87] Ibid, p. 260

measuring 2 units by 3 units is given by the mathematical formula 2x3=6 (square units) and square units are units of area, not lengths in two dimensions. Likewise, solids are the product of mathematical multiplication of three metric variables (length x width x height), the resulting being a given number of cubic units. From these observations, the logical question to ask is: "What is the relation between metric co-ordinate system and Euclidean notions of area and volume?" To answer this question, Piaget et al undertook a number of enquiries which involved the operations of addition, subtraction and simple sub-division in the field of areas, followed by considering the problem of doubling an area or a volume.

Piaget[88] et al have suggested that to study the measurement of area, they (Piagetian) used two techniques: One of the techniques consisted in presenting the pupil with a limited number of square unit cards which he must then move by successive iteration from one part of the surface being measured to the next. The other method involded the use of

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[88] Ibid, pp. 292 and 296 - 97.

cards to cover the area to be measured. In another experiment measurement of areas involved unit iteration. In this method, the child is shown a number of shapes which are equal in area but which differ markedly in shape. One (A) is a square which can be composed out of nine smaller squares. The others (B) and (C) are irregular together with a pencil. The pupil is free to examine the material and draw on it, the cards being plain, white, without dotted lines. When the experiment is through with squares (A), (B) and (C) the experimenters pass on to two more shapes which are more heterogeneous and not equal (D and E). The child is offered a choice of three counters to measure the latter shapes. One of these shapes is a quarter of (D) meaning (E) is worth three and a half such squares. The second is a rectangle worth two such squares, so that two would fit into (D). Finally, there is a triangle equal to a square cut diagonally in half. If the triangle is used as the unit, (D) is worth eight units and (E) is worth seven.

Piaget[89] et al, have suggested that measuring areas can be undertaken in two different ways, involving different operations. For example: San and Mag began by applying the unit square to the

89 Ibid, p. 301.

perimeter which tells them (San and Mag) the larger square measures 3 units along either axis. It is only when they have measured the area direct that they know that the square is composed of nine unit squares. However, children in upper concrete stage of operation simply measure length in two or three dimensions and area by successive applications of an areal unit inside a larger area. Piaget has advised that what children cannot do before stage 4 (formal thinking) is to make a direct transition from length to area by a process of arithmetical multiplication (or by reverse transition by arithmetical division).

The Piagetian[90] school of thought has suggested on measurement of volume that from the stand point of geometry, area and volume are closely related, but from that of psychology, the question of volume raises many problems of its own, therefore the need to study it. However, the notion of volume corresponds more closely with the physical structure of objects because there can be no macroscopic object without three dimensional characteristics. Piaget et al observed that the equality of volume is expressed by the words: "as much room".

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[90] Ibid, p. 354 and 356.

Piaget,[91] Inhelder and Szeminska have drawn some important conclusions regarding the measurement of length, area and volume. For example, they (Piagetian group) have suggested that the possibility of measurement (by unit iteration) depends on the construction of lengths, areas and volumes in forms which allow of elementary conservation. This suggestion depends too, on qualitative operations of subdivision and change of position which one intimately linked with conservation. Thus, the relation of measurement to the sub-logical operations of sub-division and change of position is an exact parallel to that of number in regard to logical operations of nesting class hierarchies and seriations. Piaget's group further observed that the construction of number appears at the same time as that of the logical operations at which number is a synthesis at stage 3 of development, but that of measurement appears later than its constituent operations (upper 3rd stage).

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[91] Ibid, p. 397.

Carpenter[92] and Ruth Lewis carried out a study whose purpose was to find the degree to which young children recognize the importance of maintaining a standard unit of measure in a measurement operation and how they (children) acquire the knowledge that the number of units measured is inversely related to the size of that unit. Carpenter's study is an extension of his earlier work (1975) on the development of measurement concepts in young children, in which some conclusions emerged, namely: that first and second grade students do not yet recognize that using a smaller unit of measure increases the number of units in the measurement of a fixed quantity. In this earlier study, children were asked to measure equal quantities with different-sized units of measure. Results revealed that over 50% of the pupils maintained that the quantity that measured more units was greater, in spite the fact that before measuring they (pupils) had visually determined that the quantities were equal.

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[92] Thomas P. Carpenter: "The Development of the Concept of a standard Unit of Measure in Young Children", Journal for Research in Mathematics Education. No. 7, 1, January, 1976, p. 53.

Carpenter[93] has further suggested that in measurement studies the distracting cues are numerical, the result of measuring with different sized units of measure. Children fail to conserve or fail to interpret correctly the results of measurement operations because they (pupils) centre on a single dominant dimension, For example, the height of the liquid in the conservation problems or the number of units in the measurement problem. These children fail to recognize the compensating relationship between the height and width of the containers in the conservation problem or between the unit size and number of units in the measurement problem. Carpenter and Lewis have also suggested that unless measurement concepts are generalized from conservation process rather than learned through specific measurement activities, one would not necessarily expect the same developmental sequence for measurement that occurs for conservation.

Carpenter's[94] results obtained from the 2x2 repeated measures of ANOVA (Lindquist, 1953) indicated that the prediction problems are significantly easier

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[93] Ibid, p. 54.

[94] Ibid, p. 56.

than the comparison problems, but there is no significant difference in difficulty between the linear and the liquid problems. Carpenter et al, have suggested that on the linear prediction problem, all the errors occurred because pupils maintained that the strips must measure the same number of units.

Taloumis[95] much recently investigated the relationship of area conservation to area measurement as affected by sequence of presentation of Piagetian area tasks to boys and girls in grades one through three. Taloumis investigated Piaget's (1960) theory of how a child develops an understanding of the concepts of area conservation and area measurement, when Piaget concluded that a child's construction of both (conservation and measurement) concepts develops

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95 Thalia Taloumis: "The Relationship of Area Conservation to Area Measurement as Affected by Sequence of Presentation of Piagetian Area Tasks to Boys and Girls in Grades 1 - 3" Journal for Research in Mathematics Education, 6, 4 , Nov. 1975, pp. 232

gradually through cognitive stages, with conservation concepts. Taloumis has used Flavell (1963) to criticize Piaget's studies, when he (Flavell) suggested that Piaget's studies fail to provide information as to the backgrounds, sex, intelligence levels, or testing experiences of the children. As a result, Taloumis replicated Piagetian studies by modification, using three area conservation and two area measurement tasks coupled with a standard questionnaire form to every individual child.

In the measurement tasks, Taloumis[96] asked the child to compare in area two regions at a time. On two occasions cut-outs (unit-squares and two - unit rectangles) were made available to the child to cover a pair of non-congruent regions and to make the comparison in area. When the third pair of regions was presented for comparison in area, half-unit squares were also available. In this exercise, the area concept was that the areas of two non-congruent regions may be compared by determining the number of unit-squares covering each of the regions. The interview questions were: "Does this have the same amount of space as this?" "Does one of them have more?"

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[96] Ibid, pp. 236 - 37

In the area measurement task, Taloumis told the child to cover the triangular region with the first pile of cut-outs (unit-squares and two-unit rectangles) and then to use any shape from the second pile of cut-outs (half unit squares). The expectation was that once the triangle was covered, the child was asked to compare the triangle to a second region in area. The interview questions were: "Does this have the same amount of space as this one?", "Does one of them have more space?"

In summarising the results[97] of the study (Taloumis) it was found that area conservation and area measurement are affected by grade and by sequence of presentation of Piagetian area tasks to boys and girls in grades 1-3. Toloumis used a 3x2x2 ANOVA (see the plans below).

- PLAN (A')

Source of variation in Measurement Scores:

Source of Variation	df	MS	F - Ratios
Grade	2	2742.6	39.92
Sex	1	1.93	0.03
Sequence	1	331.52	4.83

[97]Ibid, pp. 236 - 38.

PLAN (B)

Means for Measurement by Main Effect Groups

	Categories	n	SD	Means	SE
Grades:	1	56	8.69	42.77	1.16
	2	56	8.68	50.57	1.16
	3	56	7.32	56.73	0.98
Sex:					
Boys	1	84	10.45	50.13	1.14
Girls	2	84	9.61	49.92	10.05
Interaction					
CM	1	84	10.16	51.43	1.11
MC	2	84	9.71	48.62	1.06

* CM = Conservation - Measurement

MC = Measurement - Conservation

Further interpretation of Taloumis data indicated that the Pearson Product - Moment Correlation Coefficients were not homogenous, therefore Taloumis computed a Multiple Regression Analysis. The data obtained from this test indicated that: (1) correlations for sequence with conservation and conservation with measurement were statistically significant in grade one; (2) correlation for conservation with measurement was statistically significant for grade two; (3) no correlation was statistically significant for grade three; (see the plans below):-

PLAN (C)

Multiple-Regression for C and M Grade One

Variable	No.	Regression Coefficient	Computed - Value	Partial Correlation Coefficient
Conservation:				
Sex	1	1.43	0.73	0.10
Sequence	2	9.64	4.80	0.56
M	4	0.82	7.01	0.70
Measurement:				
Sex	1	-0.44	-0.26	-0.04
Sequence	2	-7.80	-4.43	-0.53

PLAN (D)

Multiple-Regression for C & M: Grade Two

Conservation:				
Sex	1	0.52	0.29	0.04
Sequence	2	2.24	1.27	0.18
M	4	0.68	6.57	0.68
Measurement:				
Sex	1	-0.74	-0.42	-0.06
Sequence	2	0.68	6.57	0.68
C	3	0.68	6.57	0.68

PLAN (E)

Multiple - Regression for C & M: Grade Three

Conservation:				
Sex	1	2.66	1.58	0.22
Sequence	2	3.98	2.33	0.33
M	4	0.25	2.15	0.29
Measurement				
Sex	1	-1.56	-0.80	-0.11
Sequence	2	-3.96	-2.01	-0.27
C	3	0.33	2.15	0.29

Taloumis[98] data on Multiple- Regression Analysis with the t - values indicated that: (1) the area tasks given second in either sequence had a higher score for grades one and three, (2) the two sets of composite scores (data in two rows) were highly correlated.

In summarizing Taloumis study, the out-standing result is that sequence of presentation affects the scores of the second group of area tasks. If area conservation tasks are administered first, the scores on area measurement tasks are increased. The implications for future research arising from Taloumis study are: (1) training in area measurement may

98 Ibid, pp. 240 - 41.

improve a child's performance in area conservation; (2) learning takes place across Piagetian tasks, given in sequence, which should be further investigated or taken into account in future studies. Consequently, Piaget's theory of cognitive development, that the ability to measure quantities is dependent on acquired concepts of conservation, appears not completely tenable. The reverse seems tenable. The scores show that significant learning takes place within the period of assessment of children, and that there seems to be transfer of learning in either direction or in both. The results show that area conservation does not necessarily come before area measurement, and that area measurement facilitates area conservation.

The second important result in Taloumis study is that area conservation performance and area measurement performance are equally predictable if the scores on one of these is known. Further still, the study reveals that on plotting the means of the scores for conservation and measurement on a graph within the four sex and sequence groupings, the result indicates that children's performances in area conservation and area measurement show no differentiation in amount of increase from grade to grade.

Performance on both groups of tasks appears generally parallel to each other. These results do not agree with Piaget's theory that measurement occurs "spontaneously" at least for grades one through three.

Taloumis[99] (1976), has made some recommendation for the classroom teacher arising from his study:

(1) the study lends support to programmes that include instruction on area measurement in the primary grades

(2) the study gives no evidence that boys and girls need differentiated instructional approaches.

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[99] Ibid, p. 241.

CHAPTER 3

METHODS AND PROCEDURE

In February, 1975, the author carried out an investigation for the Guinness Awards for Science and Mathematics Teachers. He wrote an Essay on pre-geometry learning among grade three pupils of Nairobi and Bulubulu primary schools. In this investigation, the author developed a new method he called AVET* (Audio - Visual Experimental Technique) which he later used in this study.

DESCRIPTION OF THE SAMPLE

The sample of Kenya's primary school children was drawn from ten fully maintained schools in Mombasa Municipality. The Municipal Education Officer selected the schools to give a fair distribution from different zones and to include the right age of the children's cognitive development. In each school, a sample of 72 children was expected,

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* AVET method won the author a special prize of the Guinness Awards for Science and Mathematics Teachers, 1975 (see Science Teacher; 18, 5 & 6, June/July 1975, 38).

The author considered factors such as: viewing, perception, hearing, language etc., which sometimes pose difficulties to candidates in answering questions which restrict the use of paper and pen only. AVET aimed at reducing these difficulties in the test items. The subjects were exposed to fair treatment of perception, hearing and language at the test-time.

twenty four (24) from each of the grades three, four and five. In the final analysis the sample became 675 instead of 720. Among the 675, the following distribution was realized: Grades three (226) four (220) and five (229). Sex; boys (333) and girls (342). Socio-economic status based on the educational level of the parent or the guardian of the pupil was: lower (300), middle (281) and upper (94). There was a distribution based on ability as recorded in school's 1975 examination performance: below average (310), average (212) and above average (153). The sample was selected at random and was fairly representative of the population with a mean chronological age of 11.08 years and a standard deviation of 1.76.

The sample distribution was as follows: In Mombasa 63 pupils participated in the experiment. Alibhai Panju, Buxton and Sparki each presented 72 pupils, while Mtongwe and Tudor presented 68 pupils each. In Mvita 65 pupils participated and Fahari there were 62 pupils who participated. In the remaining two schools, Makupa 66 and Makande 67 pupils respectivefully took part in the experiment. The choice of the zone for this study came from the invitation by the Municipal Education Officer, Mombasa. Participation of the pupils and the staff (Research Assistants) was therefore voluntary and formed part of the schools' routine learning situation. The author, however, recruited his own (2) research assistants for coding purposes. There were in all sixty-two research assistants who received prior training from the author in the administration of the test. These were six teachers of mathematics in grades three, four and five (two per grade) from each school. There were ten schools, so sixty teachers were recruited for duties in various administrative sections of the test, which took place in October, 1975.

DESCRIPTION OF THE TESTS

The author had prepared relevant testing materials in advance of the study. (multi-colour blocks and cut-outs) which he adjusted using other improvised materials to suit the testing situation of the experimental zone. The actual preparation of the items was designed from the experience gained at the Guinness Awards Competition Essay on teacher-pupil approach of teaching. Full details of the items appear in (Appendix A) of this study. There were 80 items distributed as follows: classification (45), conservation (21) and measurement (14).

The rationale for setting the items in the ratio of 45:21:14 was to stress the importance of classification followed by conservation and measurement to young children in tender classes (grades 3-5).

Classification

The selection of classificatory items arose from the author's actual experiences with testing primary school children at Nairobi and Bulubulu primary schools, using Dienes logical blocks and Cuisenaire rods (Copeland[100] 1974). Similar testing was an assignment to the author's M.Ed. (Master of Education) degree candidacy by the Department of Educational Communications and Technology early in 1975. Throughout the testing period as above, the author realized that Kenya's primary school children could not classify objects in their physical state as expected from their cognitive development.

In realizing the above deficit (shortage in classification concept), the author prepared personally 45 items on classification concept. In testing the pupils in classification, the author's purpose was to establish whether or not, Kenya's primary school pupils were capable of carrying out

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[100] Richard W. Copeland: How Children Learn Mathematics: Teaching Implications of Piaget's Research, 2nd. Ed., 1974, pp. 64 - 65 and 337.

classification tasks at their concrete operational stage of cognitive development in mathematical concept development. Two other purposes were for the author to use classification performance as a means of assessing to what degree Kenya's primary school pupils rely on perceptual judgement for their classification of physical objects as a sign of development in pre-logical thinking. The last purpose was that the author wanted to determine the extent to which Kenya's primary school pupils comprehend classification concepts in mathematics. The test - items in classification consisted of (45) items: colour (6), shape (8), size (5), multiple-classification (9), class-inclusion (6), hierarchical classification (3) and logical connectives (8).

Conservation

It is the author's experience that most of Kenya's primary school children lack the initial common sense in their daily practical living. For example, pupils of almost all ages in primary schools cannot infer that two quantities of equal amount (liquid or solid) in identical containers or shapes or models will remain invariant (conserved) when such quantities undergo any physical transformation. In 1974, teacher students at Shanzu Teachers College, in Coast Province, Kenya were asked among other questions in a mathematics examination, to show how the meniscuses (levels of liquids in transparent containers) would appear in different containers of different shapes and sizes. This was a puzzling question and was forwarded to the author for advice. The author tried few experiments with some of the students in their Physics Laboratory. He concluded that the students had not conceived the concept of conservation at the right time.

The author saw a need to investigate into the fact that conservation was a necessary condition for all rational thinking. He therefore chose conservation concept to determine the extent to which Kenya's middle primary school pupils comprehend conservation concepts in mathematics and the degree to which pupils rely on perceptual judgement for their conservation tasks in the physical state of the objects which undergo external transformations. Conservation test-items consisted of 21 items prepared by the author. Based on the experience of the above-named situations, the author prepared test-items as follows:

length (9), area (8) and volume (4). He used the mathematical trichotomy (equalto, greater than and less than) equivalent to: "the same as", "more than" and "fewer than" to compare the various invariants when objects underwent some type of external transformations. The author followed Piaget's[101] (1966),model of item - content with modifications to suit Kenya's situation.

Measurement:

One time in Nairobi University's Physics Laboratory, the author while on his physics experiments met with challenging ideas on measurement involving densities and oscillations of various materials. The author was in particular, surprised by the recurrence of human error in measurement results to obtain the maximum required data for practical purposes in density and oscillation. He questioned the cause for limitation despite his practical experiences with measurement applications in his live experiences.

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101 Jean Piaget et al: The Child's Conception of Geometry. Routledge and Kegan Paul, 2nd. Imp., 1966, chapters: v, xi and xiv.

"What really happens if it is the primary or secondary school student?" The author carried out some investigations with primary and teacher training students to discover the cause of a limitation in the interpretation and use of measurement, at least from the lowest level of learning. The author concluded that there was lack of understanding in the concept of measurement in mathematics.

The author described measurement concepts as the fundamental instruments for all practical mathematics and science. He therefore included measurement in the study and prepared 14 items. The major purpose for the selection was to determine the extent to which the primary school pupils in Kenya comprehend measurement concepts and the degree to which these pupils rely on perceptual judgement for their measurement tasks in practical mathematics. "Do children understand the actual meanings and applications of: "to measure", "a measure" and "measurement?" The 14 test-items included: distance and length (8), area (4) and volume (2). The author followed, for contents' sake, Copeland's [102] (1974), model, but modified it to suit the Kenya's primary school pupils in Coast Province.

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[102] Richard W. Copeland: <u>How Children Learn Mathematics:</u> Teaching Implications of Piaget's Research, 2nd. Ed. chapters: 14 and 16.

DESIGN OF THE STUDY

This study started on October 6th, 1975 by a pilot study see Appendix B for details of the pilot study. This start was a recommendation by the project's supervisor at the University of Nairobi as a pre-requisite to the main study, for all candidates in the PTE (Primary Teacher Educators) programme of the University of Nairobi.

The purpose of the pilot study was to assist the author in selecting materials and instruments which would be appropriate and meaningful to the main study. Further to this purpose, the study was used to determine the possibility of predicting the results of the main study. The pilot study took place at Likoni Primary School, P.O. Box 96028, Mombasa, and is one of the fully maintained schools as those used in the main study. The sample of the pilot study was fairly representative consisting of 72 experimental pupils selected at random as follows: grades three through five (24) pupils each, boys (36), girls (36). The age range was from 9 years to 17 years with socio-economic status questionnaire(8) (see Appendix C) determined on the basis of parental or guardian's educational levels. There was also a consideration of ability of the pupils classified in below-average, average and above-average groups as recorded in pupils' school registers. Each ability grouping consisted of 8 pupils: boys (4) and girls (4).

The author administered, assisted with six research assistants (mathematics teachers of grades 3 - 5), three tasks in classification, conservation and measurement concepts (see Appendix B) of this study. The research assistants and the pupils participated in the pilot study voluntarily (on request by their Education Officer, as initiator of the study). The author analysed the data of the pilot study for finding item difficulty and test reliability co-efficient. The author analysed the items and established:

(a) item difficulty

(b) reliability co-efficient of the test.

(a) Item Analysis (N = 72)

Task	Mean	S. D.	I.Difficulty
Classification	17.0	7.0	0.27
Conservation	7.6	4.5	0.86
Measurement	7.7	7.2	0.70

(b)	Scale	Analysis	(N = 72)
Task	Mean	S.D.	Rel.Coeff.
All(3)	31.0	12	0.98

The analyses, based on Tuckman[103] (1972) models reflected some weak areas especially in conservation and measurement tasks.

Based on the results of the pilot study, the author improved certain parts of the main study. For example, he improved the displaying technique where the blackwall was covered with white sugar paper to provide sufficient and true reflection of the materials. A further improvement was to recruit one of the research assistants especially for the duty of reading aloud the items in the test-booklet to the experimental class. The criterion of recruitment for this special duty was a teacher who possessed fluency in spoken and written English combined with mathematics linguistic experiences. The headteachers assisted in selecting the research assistants of this calibre also specialized in their respective duties as follows:

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[103] Bruce W. Tuckman: Conducting Educational Research. Harcourt Brace Jovanovich Inc., 1972, Ed. pp. 138 - 140.

i. Presentation of test-item materials (RA_1) ... 1

ii. Custody of working tools and timing (RA_2) ... 1

iii. Display of test-item materials (RA_{3-5})...3

iv. Vocal reading of test-items (RA_6) ... 1

Total (RA_{1-6})...6

The original design aimed at using 720 pupils in the experimental sample, though in the final stages only 675 pupils were available. By original structuring the design was as below:

DESIGN OF THE STUDY

		ABILITY			
GRADE	SEX	BELOW AVERAGE	AVERAGE	ABOVE AVERAGE	TOTAL
3	Boys	40	40	40	120
	Girls	40	40	40	120
4	Boys	40	40	40	120
	Girls	40	40	40	120
5	Boys	40	40	40	120
	Girls	40	40	40	120
TOTAL		240	240	240	720

The tests used in the study were Z - test and ANOVA (one way Analysis of Variance) at 5% levels of significance.

PROCEDURE

On Wednesday, 15th October, 1975, at 9.00 a.m. all the 60 research assistants from the ten experimental primary schools assembled with the author at Serani Primary School (one of the schools in the Municipality of Mombasa, chosen as a venue for its geographical zonal convenience) to receive instruction from the author on the administration of the questionnaire and the test, as follows:

1. Each experimental school would give six mathematics teachers from grades three, four and five already trained as research assistants.

2. All the six (RA_{1}-RA_{6}) at every experimental school would administer the completion of the questionnaire on socio-economic status (Appendix C) which should be ready before the day of the test. RA's were allowed to assist pupils in giving the correct information.

3. Proper seating plan would be secured at each experimental school, supervised by the RA's (probably the school hall) away from the regular classrooms of grades three-five.

The seating plan would be made so as to provide sufficient space for each pupil, to sit facing the front of the class where all the experimental displays would appear.

4. There was required enough supply of sharpened pencils, sharpeners, white and yellow blackboard chalk, drawing pins, clips, water, sand, light and pieces of rubber.

5. Each group of the RA's from each school should take seven copies of the time programme for the visits of the author. The seventh copy should be given to the head-teacher but each RA to keep one copy for use.

The time - programme was copied to the Municipal Education Officer in his capacity as the chief administrator of the schools involved in the experiments.

TIME TABLE
OCTOBER, 1975

Date	Exp. School	Time
Friday 17	Mtongwe	9.00 a.m.
Tuesday 21	Mombasa	9.00 a.m.
Wednesday 22	Alibhai Panju	9.00 a.m.
Thursday 23	Buxton	9.00 a.m.
Friday 24	Sparki	9.00 a.m.
Friday 24	Mvita	2.00 p.m.(by request)
Saturday 25	Makande	9.00 a.m.
Monday 27	Tudor	9.00 a.m.
Tuesday 28	Fahari	9.00 a.m.
Wednesday 29	Makupa	9.00 a.m.

EXPERIMENTAL SCHOOLS

The author always arrived at the experimental school by 8.00 a.m. except on Friday afternoon when he arrived at Mvita at 1.00 p.m. He usually stayed at each school up to about 4.00 p.m. including the time for marking the test with his RA's. The author carried with him:

1. Student Identity for himself (University of Nairobi).
2. Research Permit from Office of the President.

3. Test-item materials (see Appendix A).
4. Printed out (80) test-item booklets sealed in envelopes.

The author greeted the headteacher on arrival and requested for his RA's to meet in the prescribed hall for the test. Jointly the RA's and the author rehearsed over their duties and checked every thing to see that all was well for the test. RA_2 invited the experimental pupils to the hall for handing in the questionnaire duly completed as a form of registration for the test.

When the author was satisfied that the working atmosphere and the instruments for the test were proper in the experimental hall, the research assistants (RA_1-RA_6) moved to their relative assignments in readiness to start the experimental test. The author collected questionnaire formats duly completed by the experimental pupils. He followed by giving relevant instructions to the pupils about the test as follows:

1. That the pupils assembled to write a test as they normally did in other tests.

2. Each pupil was required to work at his normal speed but should try to answer all items in the spaces provided in the test-items booklet, using pencil only.

3. The test consisted of three procedures:

   i. displays of the test-materials in front of the class for them to see before answering any question,

   ii. listening to the vocal reading of the test-item for every question while the class listened, before answering the questions,

   iii. actual answering of the test-items using the combined experiences of seeing the displayed test-materials and vocal reading of the test-items.

4. The booklet contained 80 test-items distributed in three parts: A, B and C. All working personnel and the pupils would deal with one question at a time before moving on to another question.

5. The test would take place in the same hall throughout the testing period at each school. The time allowed would be three hours with breaks of 20 minutes and 10 minutes between parts A, B and C.

6. RA_2 would start and stop all the writing of the test at each experimental school.

When all was quiet, the author signaled by hand, the RA_2 to start the test. The pupils first completed certain identification formalities (names, school, class, age and sex). Actual timing started when pupils were ready to write the actual test-items starting with part A on classification (see Appendix A for instruction and procedure). The test instrument was the author's AVET, throughout the test period.

THE TEST - ITEMS:

Classification:

There were 45 test-items on classification and were distributed as follows:

(1) By colour (6): The experimental pupils were instructed that there was a set of six members in different colours, all on one string and displayed

in front of the class. The six members were numbered cardinally (1 - 6) and the diagram representing the entire set was included in the test-booklet together with the test-items. The pupils were required to use their experiences in audio-visual cues to answer the test-items. Questions were of the nature: "What colours are the following members?" The colours consisted of (yellow, red, black, blue, green and grey). Candidates were required to fill in booklet spaces the correct classification by colour in English.

(2) By shape (8): As in colour exercise, the candidates were shown a set of eight members of different shapes all on one string in front of the experimental class. The members were numbered cardinally (1-8) with their diagram included in the test-item booklet. The candidates were required to use the same experiences as used in colour to name the shapes of specified members in the set. The shapes consisted of (square, rhombi, rectangle, circle, loop, triangle, L - shape and crest). Questions were: "What shapes are the following members?" Candidates were required to fill in the spaces provided in the booklet for their classification by shape.

(3) <u>By size (5):</u> Using the same technique as in the previous test-items, the candidates were shown a set of 18 members in different sizes, all on one string in front of their class. The members were numbered cardinally (1 - 18) and their diagram was included in the test-item booklet. The candidates were required to name the sizes of specific members in the test. Questions were: "Which two members are the same in size?" (Choose two members from each group of the set). Candidates were required to fill in the book-space the correct sizes for classification by size.

(4) <u>By Multiple classification (9):</u> The candidates were shown a set of twenty-four members in different colours, sizes and shapes, all on one string, in front of their class. The members of the set were cardinally numbered (1 - 24) and their diagram was in the test-booklet. The candidates were required to use their experiences of the experimental procedures to count the number of specified members and to name the multiple classes of such members in the set. Questions were:

(a) "How many triangles are red, blue ...."

(b) "Name two members which have the same shape but have different size. (Members were paired: shape/colour, size/shape, colour/size etc.).

Candidates were required to fill in the booklet-space for their multiple classification.

(5) <u>By Class - inclusion(6):</u> The candidates were shown a set of twelve members numbered cardinally (1 - 12) all on one string in front of the class. The members consisted of four sets in triples. The members were all of square shape but coloured according to the triples which differed in sizes arranged in descending order. The arrangement of the triples by colour was: first triple (largest squares) red, second triple (next largest) green, third triple (third largest) blue, fourth triple (last largest) black. One colour (yellow) was included in the question as a falsified case to test whether the candidates would notice that this colour (yellow) was not at all represented in the membership of the set. The testing purpose was aimed at helping the candidates understand that one colour could include many members of the same shape and that the same shape could include several colours and several sizes. Therefore, the largest set with its size, shape and colour created the other sets which may be called the subsets of the large one. Candidates were therefore required to classify by class-inclusion using colours. Questions were: "Which members are red, green, yellow (falsified), blue and black?"

(6) <u>By Hierarchical classification (3)</u>: Candidates used the same set as in item 5 above. They were required to use the hierarchical order of the set based on the same shape (square) but in different sizes and colours to logically find out which are the subsets of the large set, based on the transitive relation. Questions were: Are there: "more red members or more green members?" "less blue members or less black members?" "The constant answers in this exercise would be: "same" or "equal", meaning the number of members in each subset was fixed by colour, therefore the comparison by colour even considering the shape and size would not make the difference to the number of members in the triples.

(7) <u>By Logical Connectives (8):</u> As in the other items, the candidates were shown four separate sets: A, B, C and D in different colours, shapes and sizes on four different strings in front of their class. There were 8 members in each set numbered cardinally (1 - 8). The diagrams of the sets were in the test-booklet. Candidates were required to combine all their experiences in the previous items (all answered items) to spot out how members in separate sets are

related (disjoint, intersection and union of sets) using conjunction ("and") and disjunction ("or"). To check whether candidates knew how to relate sets using Venn Diagrams (set relationship by diagramatic representation), they first answered questions (3) using the words: "all", and "some" including a negation "not" with "all". The first lot of questions required the candidates to match given statements with Venn diagrams (see Appendix A). Questions were: "All"... are not ... (disjoint), "some"... are ..., "some" are... (intersection) and "all"... are... (sub-set or class-inclusion). The second lot of questions concerned direct application of "and" and "or" (previously explained as inter-section and union of sets without using symbols). Questions were: "Which members are ... "and"...?", "Which members are..."or"....?"

Conservation:

There were 21 test-items distributed as follows:

1. Length (9): The experimental class was shown two strings A and B each measuring one metre long. There were five blue and four red squares on string A and 2 blue and 2 red squares on string B. The diagrams of the materials were in the test-booklet (see Appendix A). Candidates were expected to notice that as long as the two strings A and B maintained the same length (one metre) or (100 cm) long, spacing of the squares on each string would not alter the length of the strings, distance or length would be "conserved". Questions were: Which string is longer than the other: "A or B?" or "B or A?" Which string has more/ less squares than the other "A or B?" Which string is longer than the other, "A or B?" etc. There was a question on conversion from metre to centimetres (1 m = 100 cm).

Another exercise on length involved transformation of three ropes A, B and C, which were equal in length (30 cm each). The ropes were moved in five different positions: (1) all three ropes were parallel to one another in a vertical position, (2) only two ropes were parallel as in (1) but the third one (A) touched the rest, (3) ropes B and C were parallel to one another in horizontal position with rope A forming a transversal (4) ropes C and A were parallel to one another in vertical position with rope B making a semi-circle between ropes C and A (5) the three ropes A, B and C joined to form an equilateral triangle. The entire set of the positions was in the diagrams and was in the test-booklet. The sets were shown to the candidates who were required to use the different positions of the ropes to notice that in every case, the lengths of the ropes A, B and C were invariant, despite the changes (transformations). Questions were multiple type (see Appendix A). For example: In diagrams 2 and 4, C is equal to B in length, C is shorter than B in length etc. There were five questions on use of squares on two ropes of equal lengths and four questions on three ropes of equal lengths which changed position in five different transformations.

(2) Surface Area (8): The candidates were shown a large drawing of two homes A and B on manila paper. The homes stood on equal surface area. There were two houses T and S of equal size in the two homes.

The fields in the homes were covered with green grass. There was one cow in home A and two cows in home B. The cows ate all grass forming circular equal portions. The diagram of the whole operation was in the test-booklet. Candidates were required to find which home remained with more grass than the other when both had different number of cows which ate equal amounts of grass. The interpretation of this transaction was to conserve the equality of the homes in their surface area (amount of surface occupied) neglecting the assets (amount of insideness occupied). This experience is relative to conservation of surface area concept.

Conservation of area was also reached by subtracting or adding the number of cows (based on the idea that the amount of grass eaten represented sub-sets of the total surface area of each home, when houses also occupied equal subsets of the same equal area of the homes). Questions were multiple - choice, centring on homes A and B in terms of surfaceness occupied. A further conservation task was by use of three fields measuring 18 by 13 (units) and two equal 13 x 9 (units) as sub-sets of the bigger field.

The drawings of these fields numbered 1-3 were shown to the class. (see Appendix A). By mathematical logic, the surface area of field (1) was: 18x13 sq. units = 234 sq. units = surface are of fields (2) + (3) = 2(13x9)sq. units = 2x117 sq. units = 234 sq. units. Candidates were required to notice the relation of field 1 and(fields 2 + 3) in terms of amount of surface occupied. The logic here was that fields (2) and (3) were sub-sets of field 1 in surface area. The meaning attached to this reasoning is: "no matter how many times an object is separated into smaller and smaller sub-sets, their total amount of surface (area) occupied will add up to the same amount as the whole one, that is, surface area is conserved despite any external transformation of the original whole. Questions were again of multiple-choice, centring on equality and inequality of the sub-sets of field 1, relative to fields (2) and (3). (see Appendix A).

(3) Volume (4): Candidates watched two research assistants (RA_3 and RA_5) on the author's instruction, make two balls from plasticene materials.

The volume of the two balls was assumed to be equal (balls made of the same size). Candidates again watched RA_3 reconvert the ball into three bananas of equal size (same volume).

Candidates were expected to notice that the volume of the three bananas was equal to the remaining ball held by RA_5. This interpretation would suggest that no matter how many bananas would be formed from the ball reconverted (formerly equal in size with the other), the material of plasticine (amount of room occupied) would be invariant (volume of the ball and bananas leads to the concept of conservation). The diagram in this operation was in the test-booklet (see Appendix A). Questions were multiple - choice, comparing the amount of plasticine of the original ball with its by-product (the bananas).

Using the same procedure of experimentation, the RA_3, and RA_5 performed another experiment under the author's direction. The experiment involved three bottles A, B and C, and water from the Indian Ocean. RA_3 and RA_4 used medium "Tusker beer bottles" of equal size, while RA_5 used a "Fanta orange bottle". The difference between the bottles was size (width and height). RA_3 and RA_4 poured water (using a funnel) into bottles A and B half-way full, so that bottles

A and B carried equal amount (volume) of water. Water from bottle B was transferred into bottle C ($RA_4 \rightarrow RA_5$). As bottle C was narrow and tall, the height of water in it rose higher than it was in the control bottle A. The diagram of the operation was in the test-booklet. Candidates were expected to notice that water in bottles A and C was equal in volume. This transaction aimed at helping the candidates to conceive the concept of conservation of volume (amount of room occupied). These questions as others were multiple-choices centring on a question: "Which bottle (A and C) occupied more room?"

In the next experiment, the apparatus in the previous experiment (bottles A, B and C) were used but only tilted them through an angle of 60° so that they remained parallel to one another as they were at 90°. Candidates were required to study the meniscuses of the water in bottles A and C and try to show how bottle B's menicus would appear if it contained the same volume of water as in the start of the experiment. The logic involved was that any loose content in separate containers would form meniscuses relative to the combined rotational and parallel transformations of the containers. The volumes of the contents would also be conserved despite any external transformations. The diagram of the whole operations was in the test-booklet (see Appendix A) for questions. "Loose content" is used in the study to mean continuous matter (liquids, sand, flour, sugar, grain etc.) which requires whole measuring in terms of its volume (amount of room it occupies).

Measurement:

There were 14 items on measurement distributed as below:

(1) Length (8): The same technique as in other tests (AVET) was used in measurement tasks on length. Candidates were shown a large diagram of two villages named 1 and 2 drawn on manila paper. The two villages were separated equally by a road (see Appendix A). There were six paths: A, B, C, D, E, F which connected the two villages on both sides of the road. The diagram of the arrangement was in the test-booklet. Candidates were expected to notice that the villages were separated by a road forming equidistant (equal loci) relation to the paths which joined the villages. Questions were multiple-choice, centring on estimated measurements inferred from

perceptual judgement. For example, "which path is equal to path D in length?" D is opposite and equidistant to C from the road, so C is the correct length equal to D. Further estimation of measurement by perceptual judgement centred on subsets. For example: "How many times is path A in path B ? (actually equal, therefore, no inequality involved).

In another experiment the candidates were shown two equal strips of paper A and B made on manila paper. The two strips of paper were transformed into 4 diagrams. All the strips measured 30 cm. long. (see Appendix A) for the arrangement, ending up in diagrams 3 and 4 where in diagram 3, a third strip of equal length (30 cm.) C was imposed on strips A and B. In diagram 4 strip A was separated into sub-sets 1 and 2. Candidates were expected to watch the movement and positions of the strips of papers A, B and C, whose lengths were equal (30 cm. each). All the arrangements were in the test-booklet. Candidates were required to find the exact measurement of B in cm. using the concept of conserved equality of measurement. Questions which were centred on multiple-choice were used.

Trichotomy ("equal to", "bigger than" and "smaller than") was also used in the test. Abstraction was used (similar to tradition) when candidates were expected to multiply the sides of each field to be able to respond correctly on given answers (see Appendix A). For example: How many square units are in field (1) (multiple choice). Further test on quantity of surface occupied centred on addition and subtraction of the sub-sets (smaller rectangles in relation to the bigger rectangle). For example: Field (1) = (field 2 + field 3).

(3) Volume (2): Four bottles A, B, C and D of different sizes (different carrying capacity) and pure sand from the Indian Ocean were used in this experiment. Candidates watched RA_3, RA_4, RA_5 and the author perform an experiment in front of the class. Bottles A and D (Tusker beer type) were of equal carrying capacity. Bottle B was a strawberry jar (short and wide) and bottle C was tall and narrow (Fanta type). Equal amounts of pure dry sand were poured into bottles A and D using a funnel. Sand from bottle A was transferred into bottle B and sand in bottle D was transferred into bottle C. (see Appendix A). The class was expected to notice conservation of volume in all bottles containing

pure sand (A = D, then A   B and C $\simeq$ D) Therefore from A = D, any transfer of A to B = transfers of D to B. By inference, B = C. This is what the candidates were supposed to note first (conservation of volume) in various containers of different shapes and sizes. The level of sand in bottles differed by compensatory process in horizontal and vertical three-dimensions. Shortness was compensated for by wideness, and highness was compensated for by narrowness. The operation was in the diagrammatic form and was in the test-booklet. Two questions based on inequalities were asked using "more than", and "less than" to compare the amount of room occupied by sand in bottles A and D and transferred to a situation of B and C. For example: sand in bottle D is more than sand in bottle B. The actual answer centred on equality of the amounts of sand in the relevant bottles (B and C).

This experiment, among others, suggested a support in Piagetian theory that conservation precedes measurement concepts and Copeland's that the two (conservation and measurement) are related concepts.

THE SCRIPTS

There were 675 scripts each attached to the duly completed questionnaire on socio-economic status of the child's parent or guardian (see Appendix C). The author and the RA's (six per school) marked the scripts in the afternoon of the day the tests took place. The marking took place at the same experimental school. The author provided sufficient copies of the marking schemes (see Appendix A) to the RA's. The scheme was formed by answering as relevant, all the 80 items in the test-item booklet. Each correct answer received one point and wrong answer zero. All marking was made in the presence and supervision of the author (had to coach markers on procedure and concepts expected in the answer - scripts). There were 60 - 72 scripts at each marking centre and six RA's divided among themselves at least ten scripts each which took about two hours to complete marking. The author collected all marked scripts and left the school having completed the day's work. Distribution of each marking session's work was as below:

1. actual marking ...6 (the RA_1-RA_6)
2. recording ...2 (other - maths teachers)
3. control of stationery..1 (any member of staff)

4. general purpose ....1 (any member of staff)

Total ... 10

The RA's accorded the author maximum co-operation and attention during this most trying hour. Good marking would at least guarantee relevant data aimed at producing the results for the purpose of this study. The only offer to these markers was sometimes transport or communal soft drinks paid from the research pocket money. In rare cases the author supplied substantial lunches to some of the RA's. Services were mainly voluntary on the request of the headteachers through their Muncipal Education Officer.

DATA CODING:

The author recruited two special research assistants (on the basis of their work integrity and dedication) to help with coding exercise. The main exercise was to fill the data into the computer 80 - column data sheets. The author had indicated in the pilot study that he would use the computer services for the item analyses.

The first task was to go over what the RA's and subsidiary staff had done on the scripts. When everything required on the scripts was approved by the author, the next task was to design how the coding would be done (see Appendix D of this study). The author consulted with his supervisor who approved the coding data format. From that time onwards, the specially recruited RA's bent on the job which took one month to complete. The exercise usually started from 8.00 a.m. to 1.00 p.m., resuming at 2.00 p.m. to 5.00 p.m., resuming from 7.30 p.m. to about mid-night or early in the morning. When coding was completed, the author again consulted his supervisor. When the process was approved by the supervisor, the author took the raw data to the University of Nairobi's Computer Science Institute at Chiromo for data analyses as follows:

1. General: By SFPE Programme, data analysis provided information on: means, standard deviations, number of pupils taking part in the tests in various independent variables and moderators (sex, grade, socio-economic status and ability), and percentages of such parametric test results.

2. Specific: Programmes $SO4_A$ and $SO4_B$ were run to obtain results of the item-analysis by one-way ANOVA (Analysis of Variance). There was an assumption that there would be effects in the item-interactions, so an XSD3 Programme for Multiple Regression Analysis was run to supply information on correlations among the items and tasks. Computer services delayed a bit but the results eventually came in good time and approved by the supervisor for data analysis (see chapter 4) of this study.

CHAPTER 4

ANALYSIS OF DATA

Results of the pilot study (see Appendix B of this study) showed that there were contradicting views in performance by Kenyan primary school children between their sex, grade, socio-economic status and ability. For example, the popularly held view that boys are better academicians than girls was not always true (girls appeared to perform equally well as boys). Similarly, different educational levels and different socio-economic statuses did not create or form an over-all criterion as the basis for either superior or inferior performance. Only ability revealed an advantage among the grades (see basic assumptions in chapter one of this study).

Based on these pilot study findings, the author predicted similar findings in the main study. The idea of creating chapter four therefore was to analyse the data of the study under five different statistics: scale and item analysis to establish a difficulty index on the 80 test-items for the classification, conservation and measurement mathematics concepts; one-tailed Z- statistics to find if the mean-scores for boys and girls differed significantly and three F-tests (univariate analysis of variance) to find if the sources of variation on the classification,conservation and measurement tests by grade, socio-economic status and ability were significant. All the test-items (80) were tested through four null phypotheses (see Tables 5 - 14) at 5% level of significance.

SCALE AND ITEM ANALYSIS

Tables 1 - 3 show the results of scale and item analyses of the three mathematical tests in classification, conservation and measurement concept development among 675 Kenya's primary school pupils of grades three, four and five. The author applied Kuder-Richardson Formula 20 to eighty items with a view to obtaining the pupils' responses on each individual item-score. The over-all results showed a grand mean-score of 32.02, a standard deviation of 14.16 and a reliability co-efficient of 0.92. The following results were obtained in the three tests (classification, conservation and measurement):

CLASSIFICATION: Table 1 shows the results of scale and item statistics in the classification test. The results showed a mean-score of 17.317, a standard deviation of 8.841 and a reliability co-efficient of 0.91. There were seven concepts in the classification test which showed the following results:

TABLE 1

SCALE AND ITEM STATISTICS IN THE CLASSIFICATION TEST (N = 675)

Mean = 17.317 Reliability = 0.91

Std. Dev. = 8.841 No. of Items = 45

ITEM	MEAN	S.D.	PERCENT CORRECT.
1	0.827	0.379	82.67
2	0.769	0.422	76.89
3	0.769	0.422	76.89
4	0.130	0.337	13.04
5	0.670	0.471	66.96
6	0.693	0.461	69.33
7	0.062	0.242	6.22
8	0.521	0.500	22.15
9	0.412	0.493	41.19
10	0.459	0.499	45.93
11	0.261	0.439	26.07
12	0.437	0.496	43.70
13	0.111	0.315	11.11
14	0.102	0.303	10.22
15	0.566	0.496	56.59
16	0.591	0.492	59.11
17	0.582	0.494	58.22
18	0.603	0.490	60.30
19	0.624	0.485	62.37
20	0.612	0.488	61.19
21	0.488	0.500	48.44
22	0.372	0.484	37.19
23	0.394	0.489	39.41
24	0.422	0.494	42.22
25	0.188	0.391	18.81
26	0.044	0.206	4.44

(Table 1 Con'td)

27	0.080	0.271	8.00
28	0.156	0.363	15.56
29	0.641	0.480	64.15
30	0.584	0.493	58.37
31	0.613	0.487	61.33
32	0.564	0.494	56.44
33	0.554	0.497	55.41
34	0.364	0.482	36.44
35	0.467	0.499	46.67
36	0.397	0.490	39.70
37	0.212	0.409	21.19
38	0.231	0.422	23.11
39	0.169	0.375	16.89
40	0.182	0.386	18.22
41	0.004	0.067	0.44
42	0.018	0.132	1.78
43	0.138	0.345	13.78
44	0.144	0.351	14.37
45	0.126	0.332	12.59

1. Colour: All items received good responses except item four which tested the pupils' knowledge and application of the grey colour. This colour is commonest in use as a paint in Mombasa because of its resistance to rust on iron goods. The responses formed only 13.03% on this item, perceived as white. The best responses were on item one (82.67%) which was a primary colour (red). Performance as a whole was satisfactory.

2. Shape: Results were only fair with extremely poor responses on items seven (a rhombus) which was perceived as square; fourteen (a loop) which was perceived as a semi-circle or rain-bow; thirteen (L-shaped) which was perceived as a circle. The best responses were on item eight (52.15%) a square and the worst responses were on item seven (6.22%) a rhombus.

3. Size: Classification by size received good (41.21%) responses. Inability to classify the correct sizes by matching shapes of the same size centred on pupils' lack of the concept of equality and similarity. For example in item fifteen, three squares consisted of two equal squares with one large square. Pupils were required to choose the pair of squares which were equal in size. Worst performance was (2.42%).

4. Multiple Classification: Pupils were required to combine their knowledge of colour, shape and size to do harder classificatory exercises. Good responses came from item twenty only (61.19%) which centred on counting the number of circles in the set of twenty-four members. Items 26, 27, 25 and 28 which required pupils to choose which members had the same property but differed in other property, received very poor responses. For example in item 26, (4.44%) pupils were required to select members which were the same in size but different in shape in a set of twenty four members. The poor responses might have been caused by the weak responses in simple classification items.

5. Class-Inclusion: Results were good. Item 34 which received only 36.44% correct responses was difficult. This item which tested the concept of empty set required the pupils to respond to the fact that when a set consisted of squares of different sizes and all of the squares were removed, from the string, there would be no members left. This result should index the first class of counting called zero. The rest of the items were a repeated exercise on colours using the same shape (square), so responses were encouraging perhaps through the pupils experience with classification in previous exercises.

6. Hierarchical Classification: Responses were only fair. Item 37 (21.19%) received poor responses when the use of the conjunction "or" was used for the first time to stand for union of sets. The results in this exercise showed clearly that pupils would be faced with great difficulties when they classify by logical connectives.

7. Logical Connectives: Derived from the idea of hierarchical classification, the test suffered the weakest responses when the pupils were required to use the conjunctions: "and" and "or". Items on Venn diagrams also received poor responses (see Appendix A, items 38 - 40). Items 41 - 45 received the poorest responses, especially items 41 and 42 on the use of "and". These items received 0.44% and 1.78% respectively, showing the highest item-difficulty in the classification test. Perhaps one cause of the inability to classify by use of these conjunctions ("and" and "or") was due to lack of correct instruction by the teachers during the early school years (grades one and two).

<u>CONSERVATION:</u> Table 2 shows the results of scale and item statistics in the conservation test. The results showed a mean-scores of 8.446, a standard deviation of 4.073 and a reliability co-efficient of 0.76. The following results were obtained in the conservation test:

1. <u>Length:</u> Good responses came from items 46 (78.96%) and 47 (76.44%). Poorest responses came from items 49 (6.07%) and 50 (10.81%). Items 46 and 47 tested pupils on conservation of length when there were two strings of equal length but possessed different number of squares. By use of "more" and "less", involving a counting concept, pupils gave good responses. When the exercise changed to comparison of length using the words "longer" and "shorter" pupils moved into a difficult area where they gave poor responses (item 49). Pupils conceived that the string with more squares on it was longer than that with fewer squares, (item 48). When the exercise used the word "shorter" in item 49, the response was similar to that in item 48, meaning shortness was in the fewer number of squares on the one string than the more number of squares on the other string. The remaining items received fair responses when

TABLE 2

SCALE AND ITEM STATISTICS IN THE CONSERVATION TEST (N=675)

MEAN = 8.446 Reliability = 0.76

STD. DEV.=8.073 No. of Items = 21

ITEM	MEAN	S.D.	PERCENT CORRECT
46	0.790	0.408	78.96
47	0.764	0.425	76.44
48	0.212	0.409	21.19
49	0.061	0.239	6.07
50	0.108	0.311	10.81
51	0.295	0.456	29.48
52	0.396	0.489	39.56
53	0.313	0.464	31.26
54	0.385	0.487	38.52
55	0.404	0.491	40.44
56	0.357	0.479	35.70
57	0.250	0.434	25.04
58	0.350	0.477	34.96
59	0.390	0.488	38.96
60	0.578	0.494	57.78
61	0.453	0.498	45.33
62	0.350	0.477	34.96
63	0.490	0.500	49.04
64	0.587	0.493	58.67
65	0.401	0.491	40.15
66	0.517	0.500	51.70

the pupils were to respond to the invariance of length when the ropes of equal lengths were only transformed into different positions in different arrangements and used the words equal, shorter and longer to compare the invariances.

2. Area: Responses in area exercises were satisfactory but not very encouraging considering the experiences already in the pupils' mathematics. Good responses came from item 60 (57.78%) which concerned the conservation of area where a whole field was compared to its sub-sets in the form of rectangles. Pupils perhaps did the mechanical calculations before they could make the comparisons or merely made judgement by perception of the sizes involved on the figures. When the exercise changed to the use of equality, by comparing the whole and the two rectangles which were equal in size except arranged differently, responses deteriorated. The poorest responses came from item 57 (25.04%) when the pupils were required to find the area of grass in the two homes A and B by considering the amount of grass eaten by the cows in each home. Other items concerned the addition and subtraction concepts for use in the conservation of area.

3. Volume: Responses were satisfactory. Item 64 (58.67%) netted the highest responses. It concerned with the conservation of the amount of plasticine when two balls of equal volume had one of them converted into three bananas and pupils were required to conceive that the volume of the remaining ball and that in the three bananas was the same (invariant). The same approach and response was used for the amount of water in different bottles of different sizes and shapes. Poorest responses came from item 65 (40.15%) when the bottles used in item 64 were tilted through an angle of 45^{o} and the pupils were required to conceive that the way the container of a continuous substance is placed does not change the content (that is, the volume is conserved). Item 66, which was thought to pose some difficulties received good responses (51.70%). The exercise required the pupils to use the concept of parallelism gained in item 65 to indicate the level and shape of water in the empty bottle. Pupils responded better in the conservation of volume than the conservation of length and area.

MEASUREMENT: Table 3 shows the results of scale and item statistics in the measurement test. The results showed a mean-score of 6.213, a standard deviation of 3.151 and a reliability co-efficient of 0.67. The concepts which the pupils responded to were on length, area and volume.

1. Length: Items 67 which tested the concept of equality in length received the highest number of responses (60.89%). Item 71 which tested the concept of estimating the actual length of a substance passed through different transformations received the lowest responses (25.33%). Items 70 and 73 also received low responses. The concepts for the test in these items consisted of estimations, comparisons and perception of the given fields for information on the measurement of length. Poor responses might be attributed to the pupils inability to relate the whole of a length to its constituent sub-sets or a transformation of a whole and the conservation of its spontanous measurement.

2. Area: Results in this test were only fair. All responses fell below 50.0%. Items 75 and 78 received the lowest responses (35.11% and 36.44%). Pupils were unable to relate the area

TABLE 3

SCALE AND ITEM STATISTICS IN THE MEASUREMENT TEST (N=675)

Mean = 6.213	Reliability = 0.67
Std. Dev. = 3.151	No. of items = 14

ITEM	MEAN	S.D.	PERCENT CORRECT
67	0.609	0.488	60.89
68	0.550	0.498	54.96
69	0.573	0.495	57.33
70	0.339	0.474	33.93
71	0.253	0.435	25.33
72	0.524	0.500	52.44
73	0.387	0.487	38.67
74	0.341	0.474	34.07
75	0.351	0.478	35.11
76	0.441	0.497	44.15
77	0.403	0.491	40.30
78	0.364	0.482	36.44
79	0.541	0.499	54.07
80	0.529	0.500	52.89

TABLE 4 SUMMARY: (N = 675)

TEST	NO. ITEMS	MEAN	S.D.	RELIABILITY
CLASSIFICATION	45	17.32	8.84	0.91
CONSERVATION	21	8.45	4.07	0.91
MEASUREMENT	14	6.21	3.15	0.67
TOTAL	80	32.02	14.16	0.92

of the large whole field to its sub-set fields which formed part of the large field by creating them in rectangular shapes. Items 78 concerned with the relation of the large field to the two small fields by considering their areas using either addition or subtraction concepts. The highest number of responses came from items 76 and 77 (44.15% and 40.30%), where comparison and actual mechanical manipulation were expected. The responses suggested that pupils could rely favourable on perceptual judgement for their concept of area measurement. Multiplication concept was a factor responsible for good responses in item 77.

3. Volume: Results were fair. Items 79 and 80 received favourable responses (54.07% and 52.89%). Pupils watched with keen interest the experiment performed on volume of sand in different bottles of different sizes and shapes. The knowledge of conservation of volume was a great asset to good responses in the measurement of volume.

SUMMARY: A summary of the scale and item statistics in table 4 shows that pupils formed their mathematics concepts ranging from classification to conservation and down to measurement when judged from their mean-scores, standard deviations and reliability co-efficients. Based on this pattern of performance, it was evident that there was need to investigate the source of variations. To do this, test of significance at 5% levels of significance were applied to mean-scores to find if the differences in the means were significant in this study (see Tables 1 - 4).

HYPOTHESES

The results of scale and item statistics showed that some differences were present in the performance between sex, grade, socio-economic satus and ability, over classification, conservation and measurement tests. Based on these results, tests of significance were applied to determine whether means differed significantly between the sexes, grades, statuses and abilities. To do this, tests of significance were applied to the following null hypotheses at the 5% levels of significance.

Hypothesis one: (Sex)

"There is no difference in performance between boys and girls in mathematical classification, conservation and measurement concepts." Table 5 shows mean-scores in the three tests by sex. Means were tested against chance means by using a one-tailed Z- test at 5% level of significance. The results showed that the values of Z observed were: 0.53 in the classification, 0.40 in the conservation and 0.48 in the measurement tests. The critical value of Z- test at 5% is 1.64. The results in the three tests showed $p < 0.05$ ($0 < p < 1.64$). This shows that there is no significant difference in performance between boys and girls ($p < 0.05$) in the classification, conservation and measurement tests.

TABLE 5

MEAN-SCORES IN THE THREE TESTS BY SEX

(N = 675)

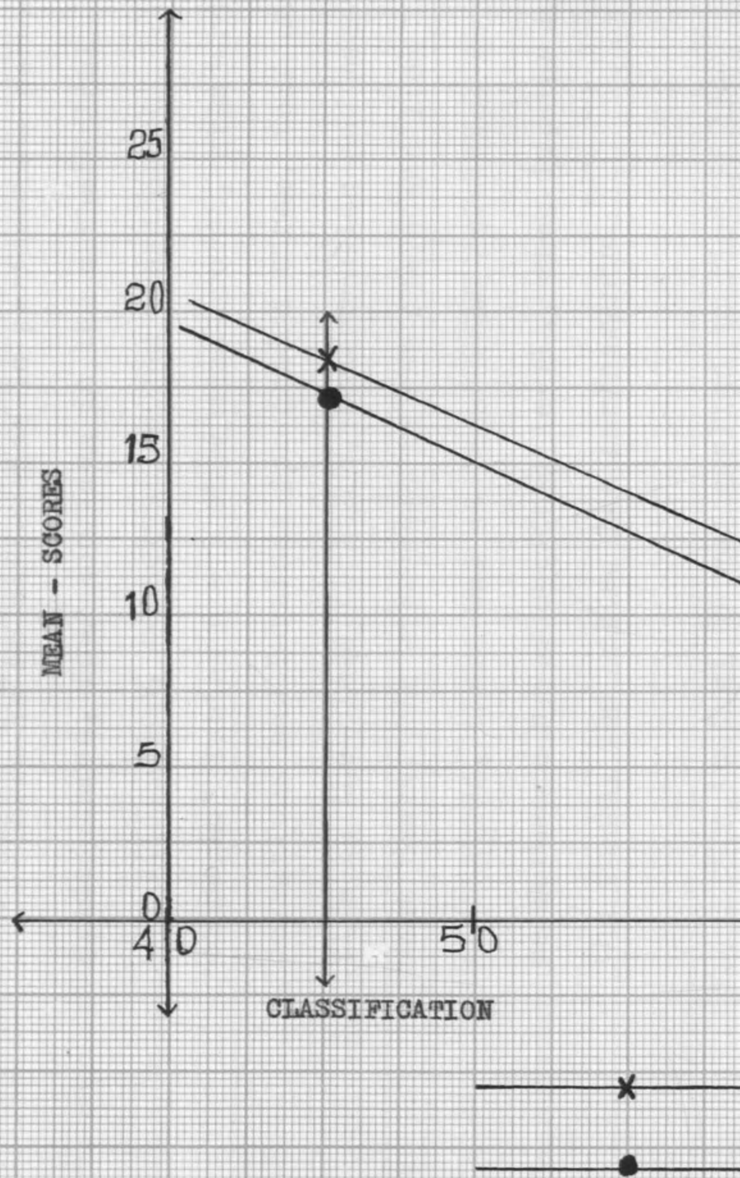
(i) CLASSIFICATION

	BOYS	GIRLS	Z-VALUE*
No. of Pupils	333	342	
Means	17.35	17.29	
Std. Deviations	8.80	8.90	
			0.53
(ii) CONSERVATION			
No. of Pupils	333	342	
Means	8.60	8.30	
Std. Deviations	4.34	3.80	
			0.40
(iii) MEASUREMENT			
No. of Pupils	333	342	
Means	6.46	5.95	
Std. Deviations	3.22	3.10	
			0.48

* at $p < 0.05$

Graphical representation was also applied to find if there was any interaction between the performance of boys and girls in the three tests. By plotting the score-points against the mean-scores, the boys' graph showed a slight advantage over the girls', but there was no interaction in the graphs, they were parallel to each other for the sexes. (see graph 1) of this study.

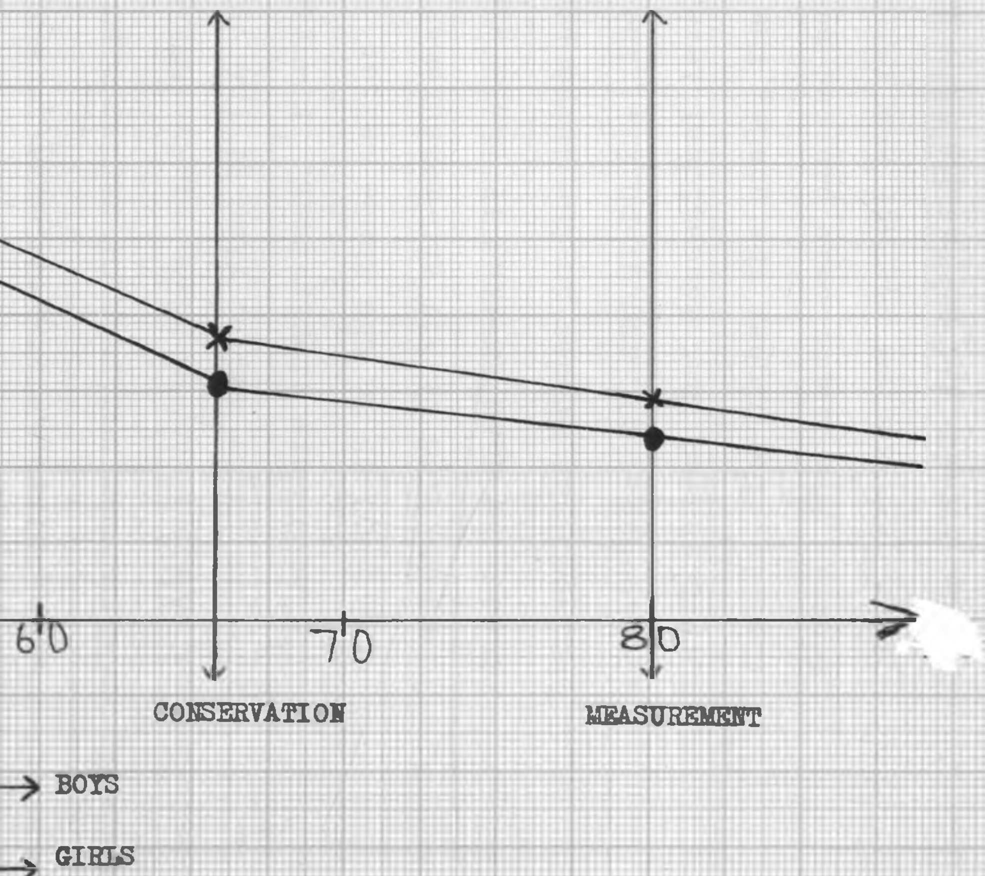
Based on these findings, the author accepted the null hypothesis that there is no difference in performance between boys and girls in mathematical classification, conservation and measurement concepts.


25
20
15
10
5
0
MEAN - SCORES
40
50
CLASSIFICATION

GRAPH 1

MEAN SCORES IN THE THREE TESTS BY SEX

(n_1=333, n_2=342)



Hypothesis Two: (Grade)

"There is no difference in performance between grades three, four and five in the mathematical classification, conservation and measurement concepts." Tables 6 - 8 show mean-scores, standard deviations and F-values for grades three, four and five in the classification, conservation and measurement tests. As discussed in the scale and item statistics, there was at least some advantage in performance by educational levels, that is, the higher the grade, the better the performance.

F - test at 5% was applied to find if the means differed significantly. To do this, a Univariate analysis of variance was applied to find if there were any sources of variations in the tests and to find if the variations (if any) were significant. F - results at 5% were $p < 0.05$ in the three tests: classification (1·50) conservation (1.53) and measurement(1.50) at ($0 < p < 19.50$). There was need to find if there was any interaction between the grades in the three tests by graphical representation (see graph 2). The results on the graph showed that educational levels had an advantage, that is, the higher the grade, the better the performance. The graphs were plotted by comparing the grade mean-scores against the scores of the three tests. There was no interaction among the grades.

TABLE 6

MEAN-SCORES BY GRADE

(CLASSIFICATION)

GRADES	3	4	5
No. of Pupils	226	220	229
Means	13.20	17.45	21.26
Std. Deviations	8.05	8.80	7.77

UNIVARIATE ANOVA TEST OF SIGNIFICANCE

FOR SOURCES OF VARIATION ON TESTS BY GRADE

(N = 675)

	SS	DF	MS	F
Between Sample	99.7	2	49.85	...
Within Sample	22293.3	672	33.03	...
Total	22393.3	674	82.88	...
Results	...	...	...	1.50

$p < 0.05$, $(0 < 1.50 < 19.50)$

TABLE 7

MEAN-SCORES BY GRADE

(CONSERVATION)

GRADE	3	4	5
No. of Pupils	226	220	229
Means	6.68	8.21	10.42
Std. Deviation	3.10	3.95	4.19

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY GRADE

(N = 675)

Source of Variation	SS	DF	MS	F
Between Sample	21.65	2	10.825	...
Within Sample	4854.87	672	7.09	...
Total	4876.52	674	17.915	...
Results	...	...	...	1.53

$p < 0.05$, $(0 < 1.53 < 19.50)$

TABLE 8

MEAN-SCORES BY GRADE

(MEASUREMENT)

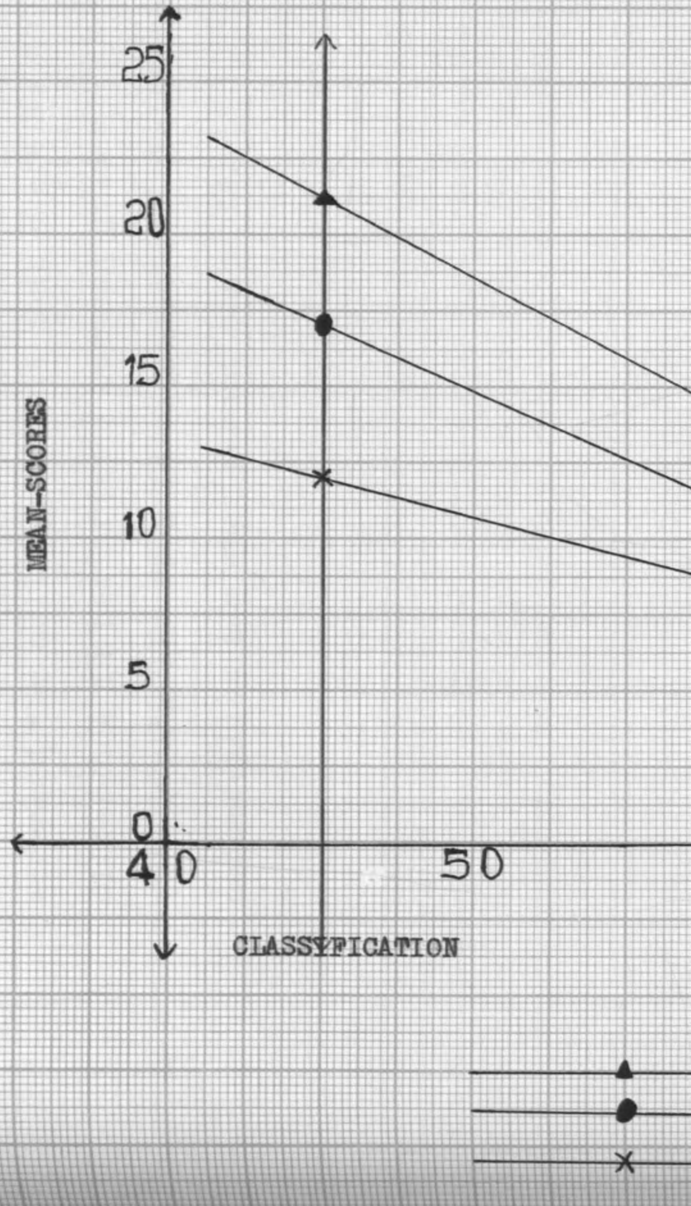
GRADES	3	4	5
No. of Pupils	226	220	229
Means	4.95	6.20	7.52
St. Deviations	2.70	3.07	3.12

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY GRADE

(N = 675)

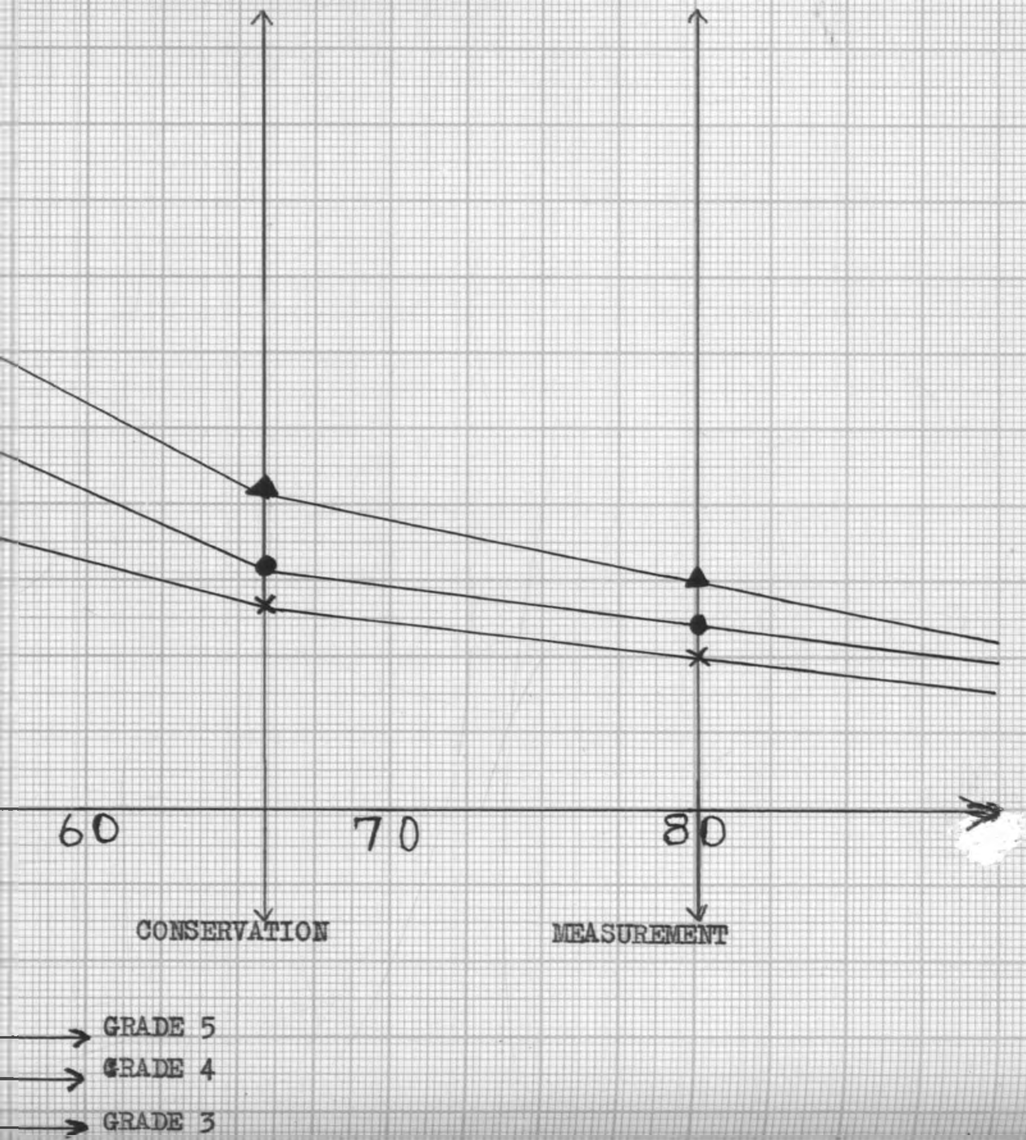
Source of Variation	SS	DF	MS	F
Between Sample	12.66	2	6.33	...
Within Sample	2825.07	672	4.22	...
Total	2837.73	674	10.55	...
Results	...	...	...	1.50

$p < 0.05$, $(0 < 1.50 < 19.50)$



25
20
15
10
5
0
MEAN-SCORES
40
50
CLASSYFICATION

GRAPH 2:
MEAN-SCORES IN THE THREE TESTS BY GRADE
(n_1=226, n_2=220, n_3=229)



On the basis of these results, the author accepted the null hypothesis that there is no difference in performance between grades three, four and five in mathematical classification, conservation and measurement concepts.

Hypothesis Three: (Socio-Economic Status)

"There is no difference in performance between pupils of upper, middle and lower socio-economic statuses in mathematical classification, conservation and measurement concepts". Tables 9 - 11 show mean-scores, standard deviations and F-values for socio-economic statuses in the classification, conservation and measurement tests. Mean scores showed that upper-class group had an advantage over the other two groups (middle and lower) in performance.

F - test at 5% level of significance was applied to find if the means differed significantly. To do this, a univariate analysis of variance was applied to find if there were any sources of variation in the tests and to find if the variations (if any) were significant. F - results at 5% were, $p < 0.05$ in the three tests: classification (1.49) conservation (1.57) and measurement (1.67 at ($0 < p < 19.50$). This observation prompted a further investigation for any inter-actions between the different socio-economic groups used in the tests. Three graphs (see graph 3) were obtained by plotting the test-scores against the

TABLE 9

MEAN-SCORES BY SOCIO-ECONOMIC STATUS

(CLASSIFICATION)

STATUS	LOWER	MIDDLE	UPPER
No. of Pupils	300	281	94
Means	15.26	18.53	20.26
Std. Deviations	8.76	8.62	8.34

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY SOCIO-ECONOMIC STATUS (N=675)

SOURCE OF VARIATION	SS	DF	MS	F
Between Sample	218.4	2	109.2	...
Within Sample	49752.6	672	74.0	...
Total	49971.0	674	183.2	...
Results	...	...	...	1.49

$p < 0.05$, $(0 < 1.49 < 19.50)$

TABLE 10

MEAN-SCORES BY SOCIO-ECONOMIC STATUS (CONSERVATION)

STATUS	LOWER	MIDDLE	UPPER
No. of Pupils	300	281	94
Means	7.94	8.67	9.38
Std. Deviations	3.77	4.17	4.4

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY SOCIO-ECONOMIC STATUS (N = 675)

SOURCE OF VARIATION	SS	DF	MS	F
Between Sample	52.2	2	26.1	...
Within Sample	11156.8	672	16.6	...
Total	11209.0	674	42.7	...
Results	...	...	...	1.57

$p < 0.05$, $(0 < 1.57 < 19.50)$

TABLE 11

MEAN-SCORES BY SOCIO-ECONOMIC STATUS

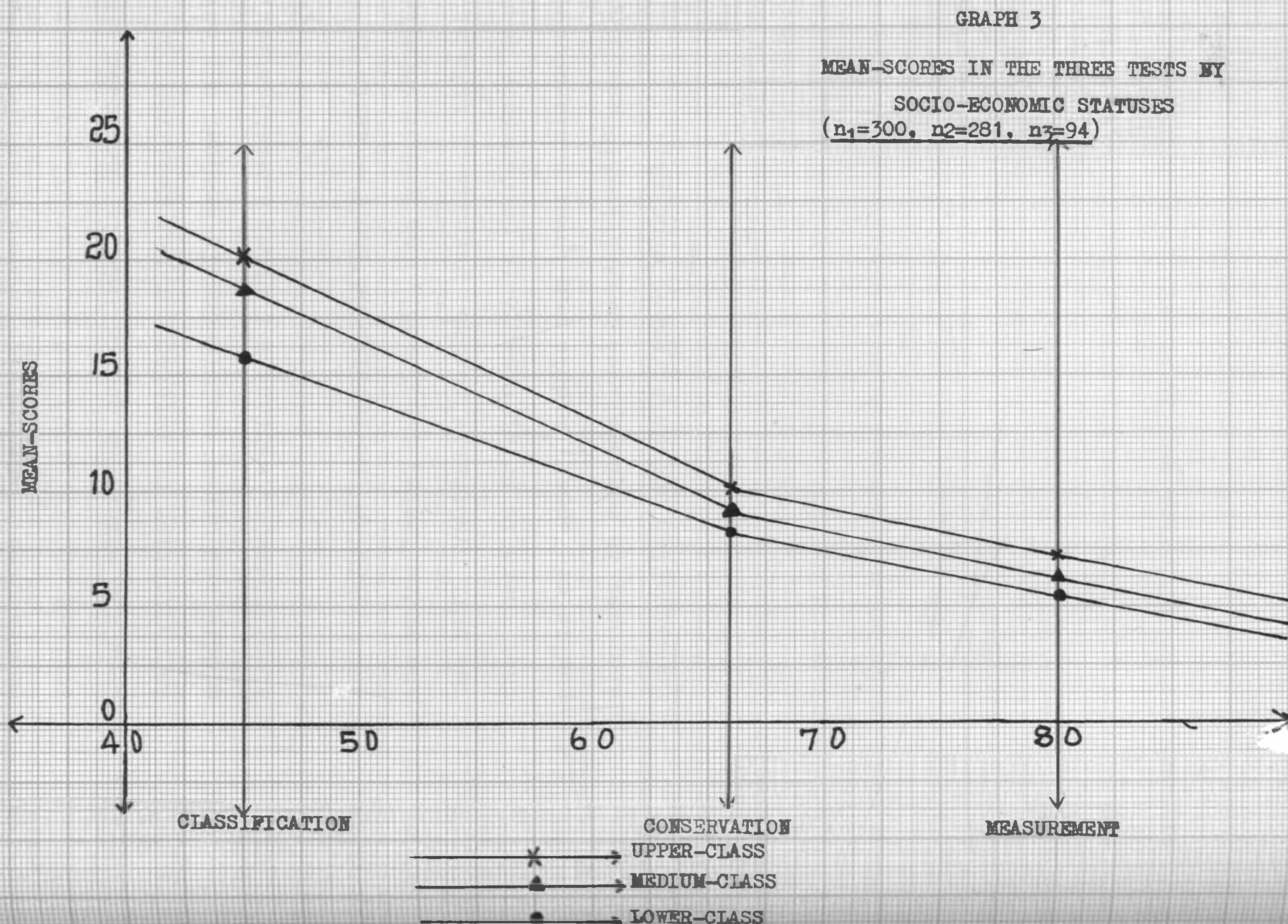
(MEASUREMENT)

STATUS	LOWER	MIDDLE	UPPER
No. of Pupils	300	281	94
Means	5.65	6.57	6.95
Std. Deviations	3.02	3.08	3.47

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY SOCIO-ECONOMIC STATUS (N = 675)

SOURCE OF VARIATION	SS	DF	MS	F
Between Sample	30.91	2	15.46	...
Within Sample	6625.09	672	9.84	...
Total	6656.00	674	25.30	...
Results	...	...	...	1.67

$p < 0.05$, $(0 < 1.67 < 19.50)$


GRAPH 3
MEAN-SCORES IN THE THREE TESTS BY
SOCIO-ECONOMIC STATUSES
(n1=300, n2=281, n3=94)
MEAN-SCORES
25
20
15
10
5
0
40
50
60
70
80
CLASSIFICATION
CONSERVATION
MEASUREMENT
UPPER-CLASS
MEDIUM-CLASS
LOWER-CLASS

mean-scores. The results showed that the upper socio-economic group had an over-all advantage over the other two groups (middle and lower groups). An indication of interaction was realized for middle and upper socio-economic groups, but not significant to warrant a further investigation.

Based on the results of the F - tests in the three tests, ($p < 0.05$), the author accepted the null hypothesis for the purpose of this study, that there is no difference in performance between pupils of upper, middle and lower socio-economic statuses in mathematical classification, conservation and measurement concepts.

<u>Hypothesis Four:</u> (Ability)

"There is no difference in performance between pupils of above-average, average and below-average in ability in mathematical classification, conservation and measurement concepts". Tables 12 - 14 show mean-scores, standard deviations and F - values for ability in the classification, conservation and measurement tests. Mean-scores show that the above-average group had an over-all advantage over the other two groups (below-average and average) in performance.

F - test at 5% was applied to find if the means differed significantly. By use of a univariate analysis of variance an investigation was made to find if there was a source for variation and how significant (if any) was the source. F - results at 5% were ($p < 0.05$) in the three tests: classification (1.41) conservation (1.63)and measurement (1.53) at ($0 < p < 19.50$). The results led to a further investigation to find if there were any interactions between the three groups of different abilities (above-average, average and below-average). Three graphs (see graph 4) were obtained by plotting the test-scores against the mean-scores. The results showed that the above-average group had an over-all advantage over the other two groups (average and below-average groups). There was no indication of any interaction between the three groups in the three tests.

TABLE 12

MEAN-SCORES BY ABILITY

(CLASSIFICATION)

ABILITY	BELOW-AVERAGE	AVERAGE	ABOVE-AVERAGE
No. of Pupils	310	212	153
Means	11.16	20.91	24.81
Std. Deviations	7.29	5.55	6.63

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY ABILITY (N = 675)

SOURCE OF VARIATION	SS	DF	MS	F
Between Sample	131.61	2	65.8	...
Within Sample	31334.39	672	46.6	...
Total	31466.00	674	112.4	...
Results	...	...	...	1.41

$p < 0.05$, $(0 < 1.41 < 19.50)$

TABLE 13

MEAN-SCORES BY ABILITY

(CONSERVATION)

ABILITY	BELOW-AVERAGE	AVERAGE	ABOVE-AVERAGE
No. of Pupils	310	212	153
Means	6.10	9.32	12.00
Std. Deviations	2.98	3.03	4.20

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY ABILITY (N = 675)

SOURCE OF VARIATION	SS	DF	MS	F
Between Sample	35.70	2	17.85	...
Within Sample	7366.30	672	10.96	...
Total	7402.0	674	28.81	...
Results	...	...	...	1.63

$p < 0.05$, $(0 < 1.63 < 19.50)$

TABLE 14

MEAN SCORES BY ABILITY

(MEASUREMENT)

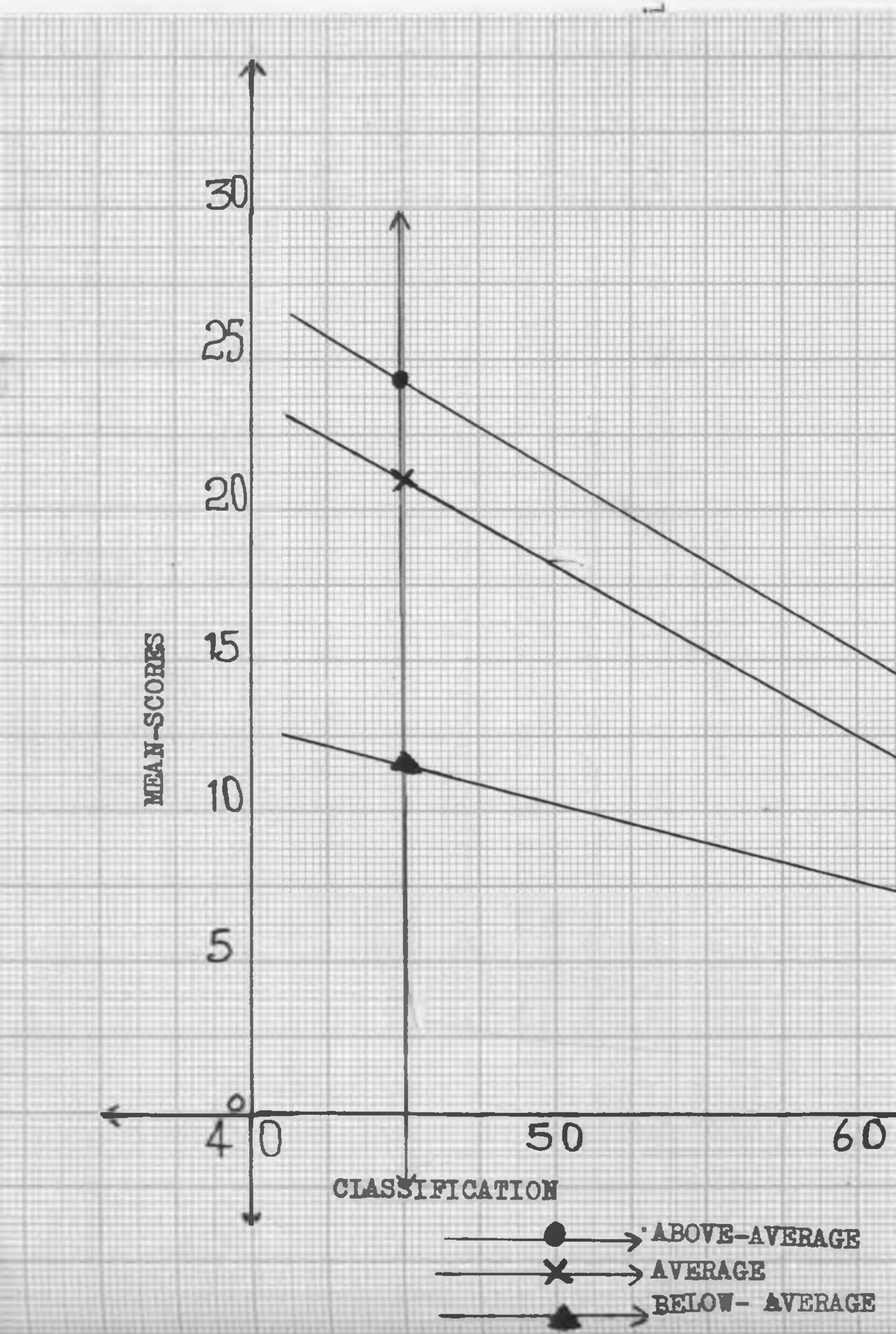
ABILITY	BELOW-AVERAGE	AVERAGE	ABOVE-AVERAGE
No. of Pupils	310	212	153
Means	4.42	6.97	8.8
Std. Deviation	2.60	2.52	2.80

UNIVARIATE ANOVA TEST OF SIGNIFICANCE FOR SOURCES OF VARIATION ON TESTS BY ABILITY (N= 675)

SOURCE OF VARIATION	SS	DF	MS	F
Between Sample	22.30	2	11.15	...
Within Sample	4871.70	672	7.25	...
Total	4894.00	674	18.40	...
Results	...	...	...	1.54

$p < 0.05$, $(0 < 1.53 < 19.50)$

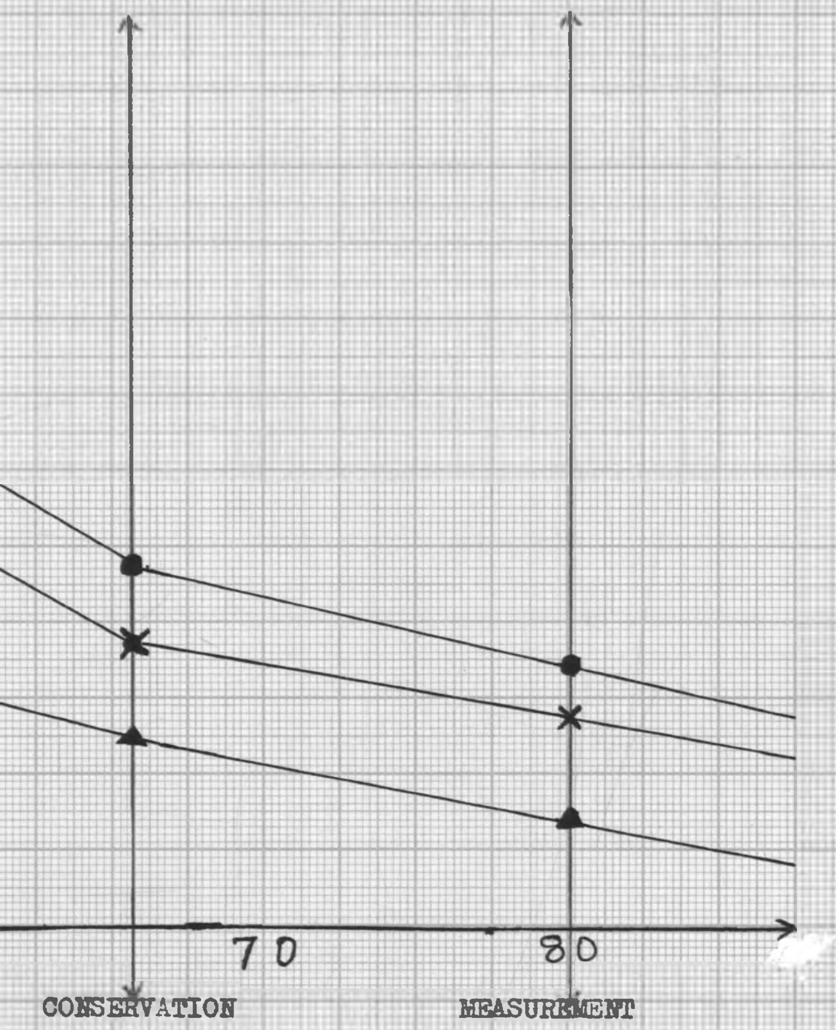
ion,


30
25
20
15
10
5
0
MEAN-SCORES
40
50
60
CLASSIFICATION
ABOVE-AVERAGE
AVERAGE
BELOW- AVERAGE

GRAPH 4

MEANS-SCORES IN THE THREE TESTS BY ABILITY

(n_1=310, n_2=212, n_3=153)



Judged from the results of the F - tests in the three tests, ($p < 0.05$), the author accepted the null hypothesis for the purpose of this study, that there is no difference in performance between pupils of above-average, average and below-average in ability in mathematical classification, conservation and measurement concepts.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Concept formation in modern mathematics has been most discussed in mathematics education in recent years. The discussion has mainly been due to much publicity in Piaget's and his replicator's research investigations cent ring on the theory that no child can get very far in mathematical thinking unless he first masters the relevant mathematical concepts at his own stages of cognitive development. Piagetian research and the revolutionary changes that have taken place in the school mathematics during the past two decades, prompted the author to carry out an investigation into mathematics concept development among 675 Kenya's primary school pupils of grades three, four and five in Mombasa District.

To investigate into mathematics concept development, the author used three mathematical tests (classification, conservation and measurement) with four null hypotheses to find if there is difference in performance between sexes, grades (three, four and five), socio-economic status and ability. The results of the null hypotheses tested at 5% levels of significance were as follows:

<u>Hypothesis One</u> (see Table 5): "There is no difference in performance between boys and girls in the mathematical classification, conservation and measurement concepts".

i. Classification ($Z = 0.53$, $df = 2/672$, $p < 0.05$)

ii. Conservation ($Z = 0.40$, $df = 2/672$, $p < 0.05$)

iii. Measurement ($Z = 0.48$, $df = 2/672$, $p < 0.05$)

Null hypothesis was accepted.

<u>Hypothesis Two</u> (see Table 6): "There is no difference in performance between grades three, four and five in the mathematical classification, conservation and measurement concepts."

i. Classification ($F = 1.50$, $df = 2/672$, $p < 0.05$)

ii. Conservation ($F = 1.53$, $df = 2/672$, $p < 0.05$)

iii. Measurement ($F = 1.50$, $df = 2/672$, $p < 0.05$)

Null hypothesis was accepted.

<u>Hypothesis Three</u> (see Table 7): "There is no difference in performance between pupils of upper, middle and lower socio-economic statuses in the mathematical classification, conservation and measurement concepts."

i. Classification ($F = 1.49$, $df = 2/672$, $p < 0.05$)

ii. Conservation ($F = 1.57$, $df = 2/672$, $p < 0.05$)

iii. Measurement ($F = 1.67$, $df = 2/672$, $p < 0.05$)

Null hypothesis was accepted.

Hypothesis Four (see Table 8): "There is no difference in performance between pupils of above-average, average and below-average in ability in the mathematical classification, conservation and measurement concepts".

i. Classification ($F = 1.4.$, $df = 2/672$, $p < 0.05$)
ii. Conservation ($F = 1.63$, $df = 2/672$, $p < 0.05$)
iii. Measurement ($F = 1.53$, $df = 2/672$, $p < 0.05$)

Null hypothesis was accepted.

The over-all findings in this study tended to support what Piaget has shown when he stated that all children learn mathematical concepts very slowly and inadequately. This is not because they cannot learn to do mathematics of a mechanical kind or that they fail to learn relationships between common quantities, but rather, they do not understand the relationship involved. When children learn a relationship in one specific situation, they do not see how it may be generalized to similar situations. For example, the children learn about fractions without applying this to subsets in the set itself or learn about area, volume, capacities, but cannot conserve or measure or classify quantities in the most concrete form. The children may agree that two squares of paper are equal in size, but if one of the square - papers is cut and the pieces are re-arranged, the children are not certain that the areas are equal, or they agree that two quantities of water are equal, but if one of them is poured into smaller or larger containers with varying shapes, the children believe that the smaller containers hold more water. The same difficulties are found in making classifications, in relating a whole with its parts, in matching two series of any kind and in the understanding of any measurement by a spontaneous approach.

It would appear from these findings that children think intuitively, dominated by perceptual judgements which tend to exaggerate the importance of one feature. For example, the length of a rectangular area as compared with a square one, or the number of small glasses compared with a single large one. The child or adult who thinks intuitively in any situation is likely to be incapable of mentally reversing an action. The individual does not see that a piece of paper altered in shape can be altered again so he cannot deduce that its area is invariable. Such a character, needs many practical experiences of classifying, conserving and measuring, using colours, shapes, sizes, lengths, areas and volumes, before the actions become internalized so that mental actions can replace physical ones.

It would appear, judging from the findings of this study, that during childhood (between 7 and 12 years), children's thinking becomes increasingly logical and is gradually freed from perceptual judgements. However, a child's logical thinking is applied to concrete objects and is limited to situations which are easily represented in practical ways and which are close to the child's own experience . That means that unless the child is exceptionally gifted in modern mathematics concept development, he does not survey many possibilities mentally, making hypotheses from which he draws conclusions, nor does he use implication in his argument. When such thinking goes on unchecked among the individuals at the early stage, verbal thinking becomes the order of the day through the adolescent stages. This order of thinking acts against logical thinking in mathematics.

This study stemmed mainly from Piagetian earlier investigations which did not consider the effects of environment and methods of teaching on the kind of thinking for which children are capable, but Piaget's belief that children learn through their actions, in all their variety, implies that both (environment and teaching) must influence the level of children's reasoning. In this study, the author found that environment plays an important part in the mathematical concept development among the young children. It may be correct to suggest here that changes in the methods of teaching by use of the environment could facilitate the development of children from intuitive to logical thinking.

The idea of investing in the quality rather than quantity of teachers has always been one of the main aims in the training of teachers. The problem of the inhibition of learning mathematics concepts leads to the question of appropriate personal qualities in the teacher and his training. The knowledge and skill necessary for teaching modern mathematics concepts can be acquired by process of time, in that everything depends on the correct modern mathematics experience, where emphasis on theory is likely to do more harm than good. Whereas a physician works with human psychic formations which are already set and limited, the teacher of mathematics works on material which is plastic and open to any impression and will have to keep before him the obligation of forming the young psyche not according to his own personal ideals, but rather according to the state of mind and possibilities inseparably bound up with the child.

In view of the previous observations, the author has strongly suggested that the best way of understanding how children form mathematical concepts comes from the understanding of one's own childhood and that the idea of teachers to resist change in mathematical syllabus, content and methods of teaching should be put right in Kenya's primary school mathematics innovations. Teachers must accept change and should do their best to understand how their children learn modern mathematics before they concern themselves with methods of teaching the subject. Above everything else, Kenya should find scientific, mathematical and technological aims, objectives syllabus-contents and methods of teaching modern mathematics to suit her varying environments, socio-economic backgrounds and needs. To achieve this goal, Kenya should consider modifying whatever she has imported from the foreign countries. Special research is called for in the area of effects caused by the environments and social classes in the methods of teaching and learning modern mathematics among Kenya's primary school pupils.

The final analysis of this study showed that concepts which are important to mathematical concept development (classification, conservation and measurement) have not themselves been understood properly by both the teachers* and their pupils. The immediate cause responsible for the inability to form and learn mathematics concepts and failure to apply them is due largely, to lack of practical experience. Though there has been a gradual recognition that practical work should be provided to aid mathematical learning in primary schools, the knowledge of many concepts is still usually taken for granted in the teaching and learning the subject. This is due partly to the shortage of primary school modern mathematics teachers who qualify to teach the subject and partly due to poor environments inhabited by most of Kenya's primary school children. For example, no special attention, through practical approach, is provided to the children to ensure that they know: weight, area or volume are conserved when only shape is altered, leaving the quantity invariable. Consequently, children who fail to learn such concepts (classification, conservation and measurement) in their early days are likely to be puzzled by all forms of mathematics, for they fail to

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* teacher's inability to understand the concepts which are important to concept development was drawn from the author's own experiences in the course of his duties in the inservice courses of modern mathematics in various parts of Kenya.

understand what assumptions they may make in problems relating to length, area, weight or volume as they can only depend on examples given by their teachers and books, but are unable to deduce logically how to do problems of a slightly different kind.

At certain stages, Piaget has shown from his extensive investigations that children learn first through their actions and only gradually learn to internalize the concepts they form so that they can use them in imagination by referring to concrete aids. This process, as Piaget pointed out, takes place through the substitution of imagery, symbols and language for the concrete aids and occurs anew as new fields of mathematical and scientific knowledge are explored. For a child to understand classification, conservation or measurement concepts, he has to use whatever his environment can offer relevant to these topics; such as colour, shape, size, combination of these, lengths, areas and volumes. Children need models, containers, liquids, and other materials such as drawings so that they learn the concepts which they will use later in geometrical, algebraic and calculus reasoning.

For example, at the A-level, a student will approach the beginnings of calculus through the drawings to represent gradients, increasing area and volumes, made up from thin slices before he acquires the necessary concepts, symbols and abstractions.

Based on this study, the author has recommended that the three grades of pupils (three, four and five) need special help in learning how to develop modern mathematical concepts in classification, conservation and measurement. This may be achieved through the enrichment of the pupils' environments by strengthening communication, comprehension, application, analysis and evaluation abilities among the pupils. Additional extra-mural teaching or reciting the multiplication tables should be replaced with proper teacher-pupil ratio for learning the proper concepts in modern mathematics. The children should be helped to learn what the communicative demands of the situation are and how to meet these demands. For example, the teachers should learn that effective communication involves increasing the explicit features and decreasing the ambiguous features. Whereas reliance on perceptual judgement may be an asset to lower classes, it should be replaced with conceptualized judgement as the children mature in their mathematical logic. The findings of this study immediately reveal that children used more perceptual judgements than conceptualized judgements in their approach to the tasks in the study. Acquiring these skills may be a much more difficult and lengthy process than the acquisition of mechanized mode of thinking in modern mathematics performance. If this proves to be the correct observation, then teachers have a job to design regular and remedial classes which must be encouraged to place more emphasis on modern mathematics concept learning through communication and comprehension abilities. This approach may be important for later school success in the learning of modern higher mathematics.

The results of this study suggest that though mean-scores differ marginally among the schools, sexes, grades, socio-economic statuses and abilities, there is a need for specific research in the effects of varying environments and socio-economic strata towards the formation and learning of mathematical concepts among Kenya's primary school pupils. In addition, there is a need for research in the author's theory that pupils tested by AVET (Audio-Visual Experimental Technique) method in mathematical concept development would be better performers than the pupils tested by traditional method (paper and pen only). Perhaps one other area requiring research within the study would be to investigate the quality and quantity invested in the teachers of modern mathematics in Kenya.

The author has suggested that unless strict emphasis is maintained in the pforessional training of mathematics teachers, Kenya will always depend on foreign syllabus content and methods of teaching Kenyan pupils. By allowing this situation to exist in this country, the pupils may delay or even fail to form the correct mathematical concepts at their correct stages of cognitive development. This practice may tend to produce under-developed citizens who mature in unproductive attitudes, technology and responsibility. Correct formation and learning of modern mathematics among Kenyans should at least assist Kenyan children to develop the right thinking in the right directions for responsible citizenship, duty-mindedness and productive services. These properties are necessary for true national construction, progress and self-reliance.

REFERENCES

1. Beard, M.R., "An Investigation into Mathematical Concepts among Ghanaian Children." Teacher Education in New Countries: 9, 1, May, 1968, 9, 2, Nov., 1968, 144.

2. Best, J.W., Research in Education. Printice-Hall Inc., Englewood Cliffs, N.J. 7th Ed., 1963 ch.

3. Bruni, J.V., The Arithmetic Teacher: 21, 6, October, 1974, 474-79.

4. Carpenter, T.P., "Measurement Concepts of 1st and 2nd Grade Students." Journal for Research in Mathematics Education*. January, 1975, 3 - 12.

5. Carpenter, T.P., "The Development of the Concept of a Standard Unit of Measure in Children". Journal for Research in Mathematics Education: 7, 1, Jan. 76, 53.

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*Volumes unquoted

6. Copeland,R.W., How Children Learn Mathematics: Teaching Implications of Piaget's Research. Macmillan Publishing Co. Inc. 2nd Ed. 1974, pp. 51-55, 59-80, 247-303, 311 and 337.

7. Eshiwani,G.S., "The Teaching of Mathematics to Primary School Children." Part I, November 20, 1974, pp. 1-20.

8. Eshiwani, G.S., " Report to the National Committee on Educational Objectives and Policies". Republic of Kenya. April, 1976, pp. 22 - 27.

9. Evans,J.L., "Learning to Classify by Colour and by Function." A Study of Concept Discovery by Ganda Children, in "Otaala's: Piaget for Teachers, 1972 p. 9.

10. Furth,H.G., Piaget for Teachers. Prentice-Hall, Inc. 1970 p. 162.

11. Liedtake,L.D. "Activities, a Mathematics for Pre-school Children". The Arithmetic Teacher, 20, 7, Nov. 1973, 536 - 38.

12. Lovell, K., *The Growth of Basic Mathematics and Scientific Concepts in Children*. University of London Press, Ltd., 1971, pp. 11 - 24.

13. Lovell, K., *The Growth of Understanding in Mathematics: Kindergarten thro' Grade Three*. Early Childhood Education Series, 1971, pp. 7-23 102-108 and 177.

14. Martin, J.L., "A Test with Selected Topological Properties of Piaget's Hypothesis Concerning the Spatial Represe-ntation of the Young Children." *Journal for Research in Mathematics Education*: 7 1, Jan. 1976.

15. Modgil, S., Piagetian Research: *A Handbook of Recent Studies.* NFR, Publishing Co. Ltd., 1974, pp. 31, 56-65, 115 - 126.

16. Mussen, P.H., *Child Development and Personality.* Harper & Row, 4th Ed. 1974.

17. Otaala, B., *Piaget for Teachers:* Selected Readings, 1972 (Millie Almy: "The Usefulness of Piagetian Methods for Studying Primary School Children in Uganda."), p.4.

18. Otaala, B., "A Preliminary Investigation of conservation Abilities of Unschooled Iteso Adults." pp. 20-21.

19. Otaala, B., "Cognitive Development in Children: Development and Learning (Piaget)." pp. 6-7.

20. Piaget, J., The Child's Conception of Geometry. Routledge and Kogan Paul, 2nd. Ed., 1966, pp. 28 - 65, 90-104, 116-149, 301 and 354 - 398.

21. Povey, R.M., "Can Pre-school Children Form Concepts?" Education Research Journal: 17, 3, June 1975 180-191

22. Shihundu, J.A., "Essay on Teacher - Pupil Approach to the Teaching of Pre-geometry Mathematics among Young Children." Science Teacher 18, 5 & 6, June/July 1975, 38.

23. Siegel, I.E., Logical Thinking in Children, Research Based on Piagetian Theory. Holt, Rinehart & Winston, Inc., 1968, pp. ix and 295-96.

24. Skemp,R.R., *The Psychology of Learning Mathematics*. Penguin Books, 1971, pp. 78-80.

25. Smart,J.R., " *The Arithmetic Teacher:* *13*, *4*, April, 1966, pp. 283-87

26. Taloumis, T., "The Relationship of Area Conservation to Area Measurement as Affected by Sequence of Representation of Piagetian Area Tasks to Boys & Girls in Grades 1-3." *Journal for Research in Mathematics Education:* *6*, *4*, Nov. *1975*, *232-41*:

27. Tuckman,B.W., *Conducting Educational Research*. Harcourt Brace Jovanorich Inc., 1972 Ed. (most of the chapters).

28. Unesco-Unicef., "The Development of Science & Mathematics Concepts in Young Children in African Countries." *Report of a Regional Seminar*, September, 1974, Nairobi, (Kenya) pp. 43-55.

29. Unesco-Unicef-Cedo., "The Development of Science and Mathematics Concepts in Children." *Report of a Regional Seminar*, May/June, 1972, Bangkok, (Thailand) pp. 64-65.

30. Wilson,B., "The International Meetings." Science Teacher: 18, 3&4, Feb./March 1975, pp. 21-23.

31. Zimmerman,B.J., "Enquiring & Retaining Conservation of Length thro' Modelling & Reversibility Cues." Merrill Palmer Quarterly, Behaviour & Development: 20, 3, July 1974, 145.

APPENDIX A.

MATHEMATICAL CONCEPTS DEVELOPMENT

(TEST - ITEMS)

STDS. 3, 4, & 5

SUBJECT'S NAMES ______________ ______________
(surname) (other names)

SUBJECT'S SCHOOL ______________________________

SUBJECT'S CLASS ______________________________

SUBJECT'S AGE ______________ ______________
(Years) (Months)

SUBJECT: BOY or GIRL ________

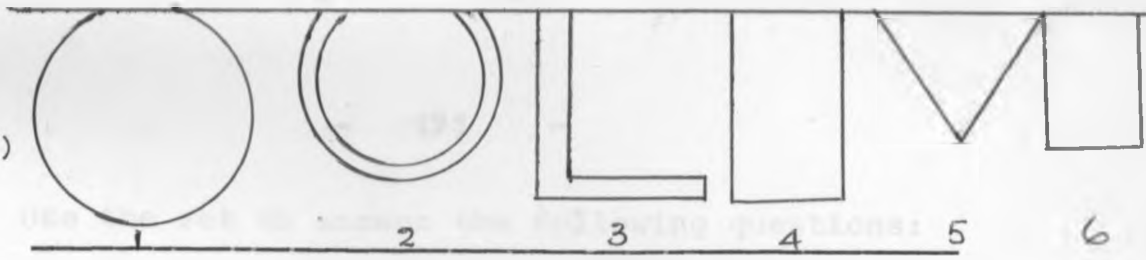
INSTRUCTION: There are 3 parts: A, B & C in this test. Please follow the instructions given in the parts carefully. Answer all the questions in this test on the test paper. Write clearly and neatly.

PART A:

There are materials of different colours, shapes and sizes in front of the class. Please look at these materials first before you answer the questions. The names of the materials are given by numbers.

There is a set of 6 members all on one string in front of your class. The 6 numbered 1, 2, 3, 4, 5 and 6. Here below is the drawing of the set.

Fig. (1)



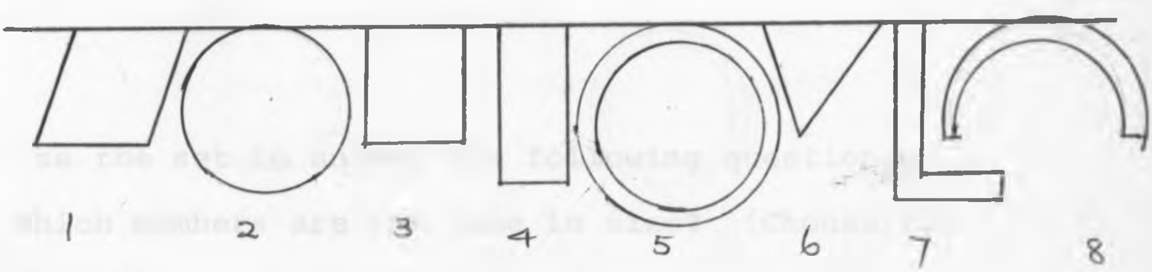
Use the set to answer the following questions:

What colours are the following members:

			POINTS
(1)	Member No. 1	(Yellow)	1
(2)	Member No. 3	(Black)	1
(3)	Member No. 5	(Red)	1
(4)	Member No. 6	(Grey)	1
(5)	Member No. 2	(Blue)	1
(6)	Member No. 4	(Green)	1

There is a set of 8 members all on one string infront of your class. The 8 members are numbered: 1, 2, 3, 4, 5, 6, 7 and 8. Here below is the drawing of the set:

Fig. (2)



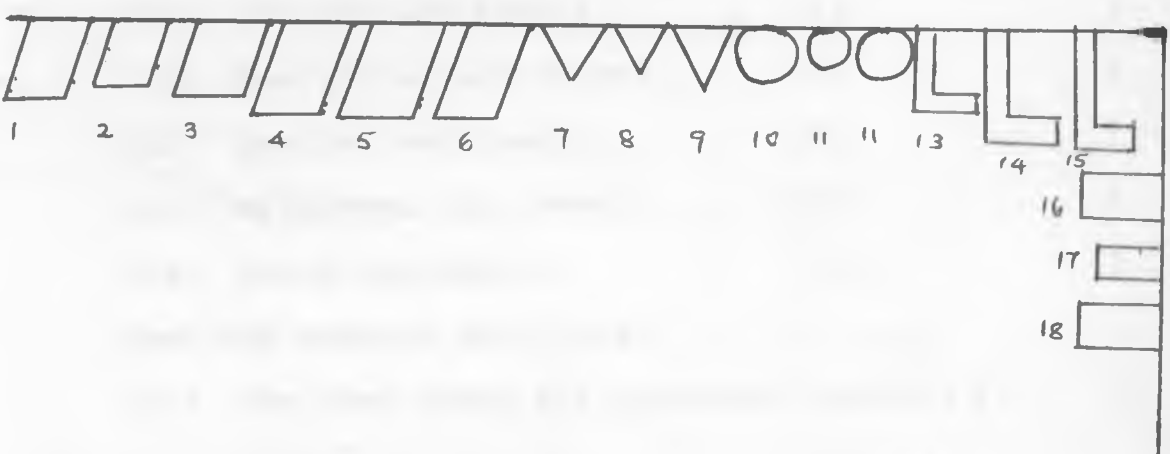
Use the set to answer the following questions:

What shapes are the following members:

			POINTS
(7)	Member No. 1	(rhombus/square)	1
(8)	Member No. 3	(square)	1
(9)	Member No. 4	(rectangle)	1
(10)	Member No. 2	(circle)	1
(11)	Member No. 5	(loop/ring)	1
(12)	Member No. 6	(triangle)	1
(13)	Member No. 7	(L-shap./rt.)	1
(14)	Member No. 8	(nest, semi-circle/ rain-bow)	1

There is a set of 18 members all on one string in front of your class. The 18 members are numbered: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, and 18. Here below is the drawing of the set:

Fig.(3)



Use the set to answer the following questions:

Which members are the same in size? (Choose two only from each group).

POINTS

(15) Group 1: Nos. 1,2, and 3 (1 and 3) 1

(16) Group 2: Nos. 7,8 and 9 (7 and 9) 1

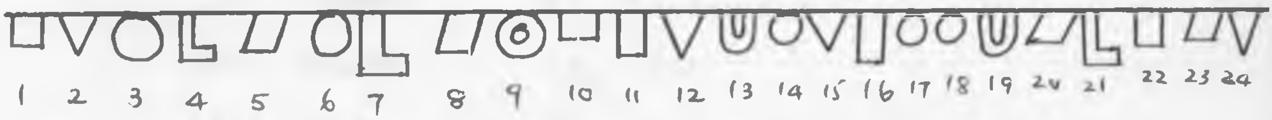
(17) Group 3: Nos. 10, 11 and 12 (10 and 12) 1

(18) Group 4: Nos. 4,5 and 6 (4 and 6) 1

(19) Group 5: Nos.16,17 and 18 (16 and 18) 1

There is a set of 24 members all on one string in front of your class. The 24 members are numbered: 1,2,3,4,5, 6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23, and 24. Below is the drawing of the set.

Fig.(4)



Use the set to answer the following questions:

POINTS

How many:

(20) Circles are there? (5) 1

(21) Rectangles are there? (3) 1

(22) Squares are there? (3) 1

(23) Triangles are there? (4) 1

(24) Loops are there? (1) 1

Name two members which are:

(25) The same shape but different colour (1) 1

(22)

(26) The same size but different shapes(None)

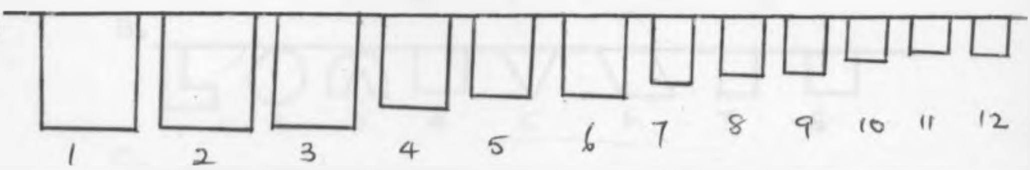
(27) The same colour but different size (9) 1

(28) Large in sizes but different in colour 1

(1) (5)

There is a set of 12 members all on one string in front of your class. The 12 members are of different sizes and are numbered: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Here below is the drawing of the set.

Fig.(5)

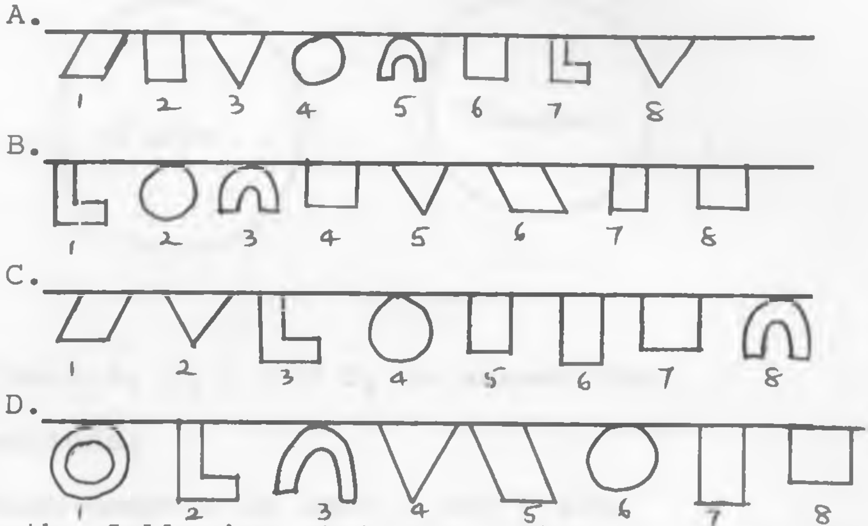


Use the set to answer the following questions:

			POINTS
Which members are:			
(29)	Red	(1, 5, and 9)	1
(30)	Green	(7, 12)	1
(31)	Black	(3, 11)	1
(32)	Blue	(2, 4 and 10)	1
(33)	Yellow	(6)	1
(34)	If all squares are removed from the set, how many squares will remain?	(zero/O)	1
Are there:			
(35)	More red members or more green members	(more red)	1
(36)	Less blue members or less black	(less black)	1
(37)	Equal green members or equal black members	the(same/equal)	1

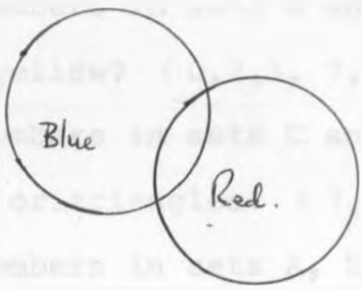
There are 4 sets A, B, C and D of 8 members each. The members in each set are numbered: 1, 2, 3, 4, 5, 6, 7 and 8. Here below are the drawings of the sets:

Fig. (6)

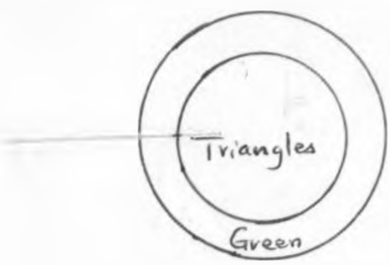


Match the following statements with the following diagrams:

(38) All circles are not triangles (2) 1



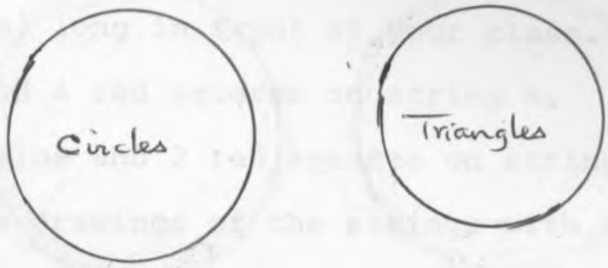
(39) Some rectangles are blue some are red



(3) 1

POINTS

(40) All triangles are green 1 1



Use the sets A, B, C and D, to answer the following questions:

(41) Which members in sets A and B are blue and green (Ø/none) 1

(42) Which members in sets C and D are squares and circles? (Ø/none) 1

(43) Which members in sets A and B are red or yellow? (0,2,3, 7, 6, 5 and 8) 1

(44) Which members in sets C and D are squares or triangles? (7, 2, 4) 1

(45) Which members in sets A, B, C and D are blue and red or are green? (Ø, 1-8) 1

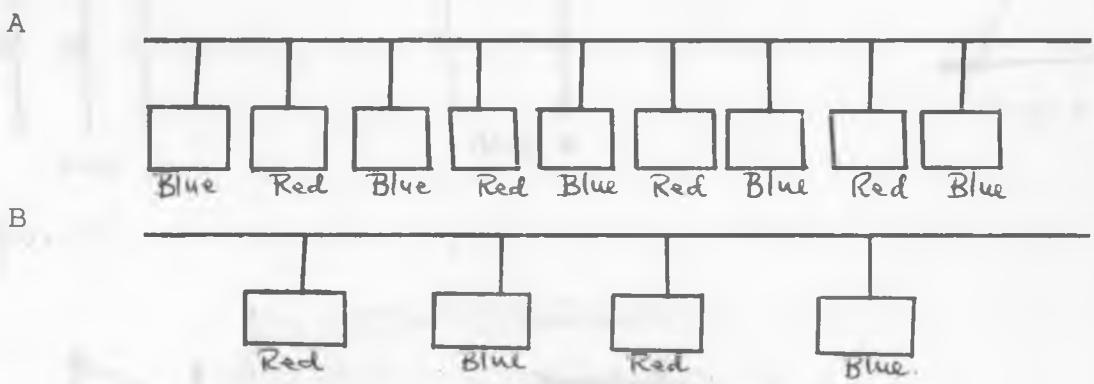
PART B

Answer all questions in this part

There are two strings A and B each measuring 1 metre (100cm) long in front of your class. There are 5 blue and 4 red squares on string A.

There are 2 blue and 2 red squares on string B. Here below are the drawings of the strings with the squares.

Fig. (7)



Use the 2 strings and the squares to answer the following questions:

(46) Which string has more squares on it: A or B?

(A) 1

(47) Which string has less squares B or A?

(B) 1

(48) Which string is shorter: (A or B?)

(none/equal) 1

(49) Which string is longer: (B or A)

(none/equal) 1

(50) What is the length in meters of strings: A and B? (2 metres/200 cm.) 1

There are 3 ropes: A, B, and C in front of your class. The ropes are equal in length, each rope measuring 30 cm long. Here below are the drawings of the ropes in different positions.

Fig.(8)

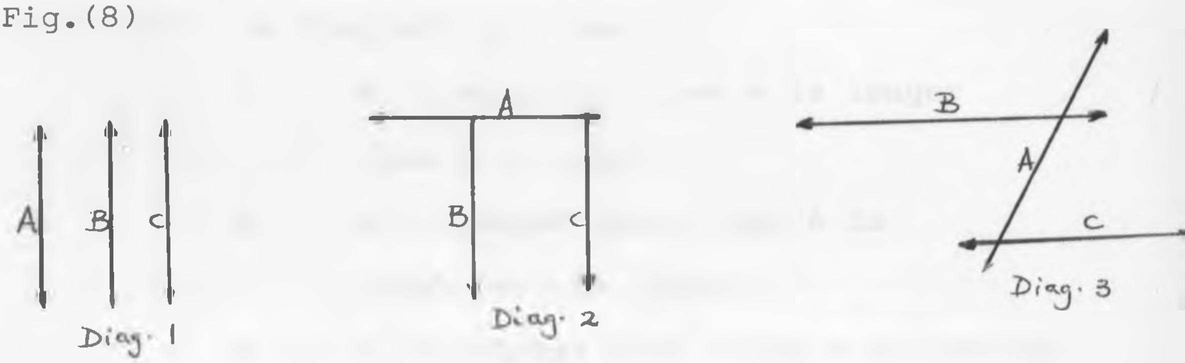


Fig.(9)

Use the diagrams to answer the following questions: (choose one answer only)

POINTS

(51) In diagram 1 and 3:

A B is longer than C in length

B B is shorter than C in length

C B is equal to C in length

D Don't know. (C) 1

POINTS

(52) In diagrams 2 and 4:

A C is equal to B in length

B C is shorter than B in length

C C is longer than B in length

D Don't know. (A) 1

(53) In diagrams 1, 3 and 5:

A B is equal to A and A is longer than C in length

B B is longer than A and A is equal to C in length

C B is shorter than A and A is shorter than C in length.

D B, A and C are equal in length

E Don't know. (D) 1

(54) In diagrams 2, 3 and 4:

A C is shorter than B and B is longer than A in length

B C is equal to B and B is shorter than A in length

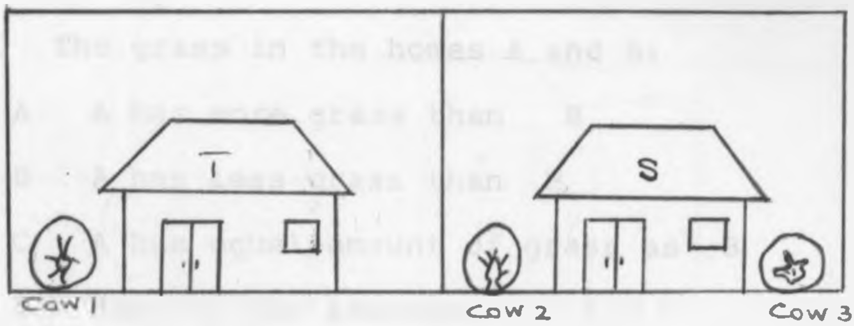
C C, B and A are equal in length

D C is longer than B and B is longer than A in length

(C) 1

There is a drawing of homes A and B of equal size in front of your class. There are 2 houses T and S of equal size in the homes. The fields are fully covered with green grass. There is one cow in home A and 2 cows in home B. The cows ate all the grass in the circles. Here below is the drawing of the 2 homes.

Fig.(10)



Use the drawings of the two homes to answer the following questions. (choose one correct answer only)

POINTS

(55) The homes:

A Home A is bigger than home B in size

B Home A is smaller than home B in size

C Home A is equal to home B in size

D None of the answers A, B and C

E Don't know. (C) 1

(56) The cows:

A Cow 1 ate less grass than cow 3

B Cow 1 ate the same amount of grass as cow 3.

C Cow 1 ate the same amount of grass as cow 3

D None of the answers A, B and C

E Don't know (C) 1

(57) The grass in the homes A and B:

A A has more grass than B

B A has less grass than B

C A has equal amount of grass as B

D None of the answers A, B and C

E Don't know (A) 1

(58) How many cows should be added to home A to make the amount of grass in home A and B equal.

A No cow

B One cow

C Two cows

D Three cows

E Any number of cows (B) 1

(59) How many cows should be taken away from home B to make the amount of grass in homes A and B equal?

A Ten cows

B No cow

C One cow

D Three cows

E Many cows (C) 1

There is a drawing of 3 fields numbered 1, 2, and 3 in front of your class. The fields are made up of a rectangle (field no. 1) and 2 equal small rectangles (fields nos. 2 and 3). Here below is the drawing of the 3 fields.

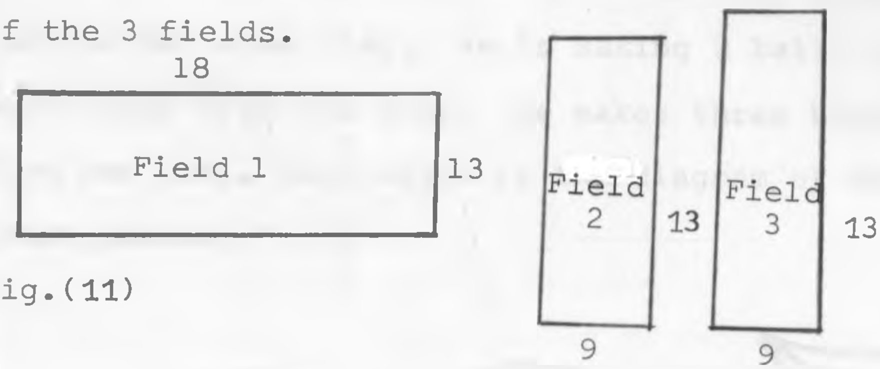


Fig.(11)

Use the drawing of the 3 fields to answer the following questions.

(60) The fields:

A No. 1 has more squares than no. 3

B No. 1 has less squares than no. 3

C No 1 has equal number of squares as no.3

D None of the answers A, B and C

E Don't know (A) 1

(61) The fields:

A No. 1 = no. 2

B No. 2 = no. 3

C No. 1 = no. 3

D None of the answers A, B and C

E Don't know (B) 2

(62) The fields:

A No. 2 + no. 3 = No. 1

B No. 2 - no. 3 = no. 1

C No. 1 + no. 2 = No. 3

D None of the answers A, B and C

E Don't know (A) 1

Look at what is going on in front of your class. Someone has some clay. He is making 2 balls of equal size from the clay. He makes three bananas from one ball. Here below is the diagram of what has taken place.

Fig.(12) 

Use the diagrams and what you have seen done in front of your class to answer this question:

(63) The bananas:

A The ball has the same amount of clay as three bananas.

B The 3 bananas have the same amount of clay as the 2 balls.

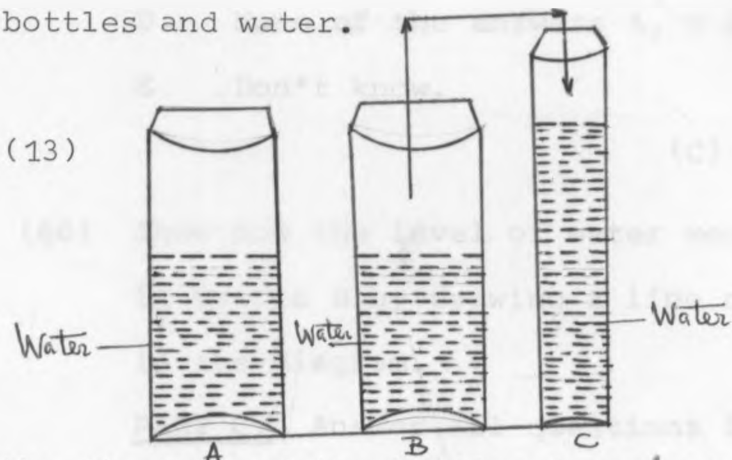
C The ball has less clay than the 3 bananas.

D None of the answers A, B and C

E Don't know (A) 1

Look at the 3 bottles A, B and C in front of your class. Bottles A and B have equal amount of water. Bottles C is empty. Water in bottle B is poured into bottle C which is narrower and taller than bottles A and B. Here below are the diagrams of the bottles and water.

Fig.(13)



Use the diagrams of the bottles to answer the following questions:

A Bottle A has more water than bottle C

B Bottle A has less water than bottle C

C Bottle A has equal amount of water as bottle C

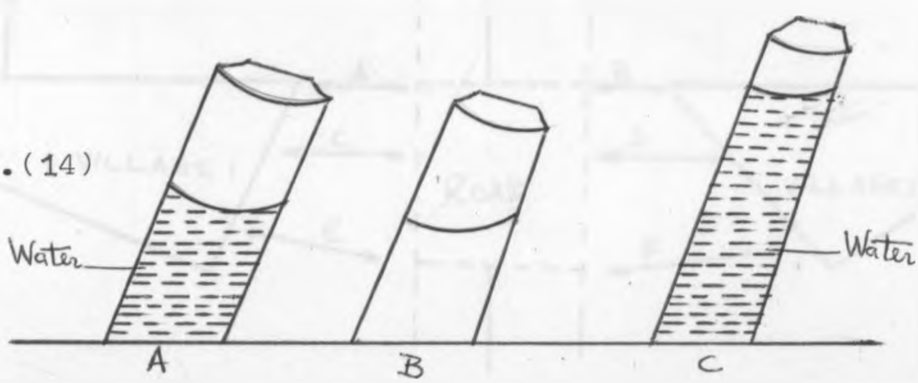
D None of the above answers A, B and C.

E Don't know

(C) 1

Bottles A, B and C are turned (tilted) as in the following diagrams:

Fig.(14)



(65) The bottles:

A Bottle A has more water than bottle C

B Bottle A has less water than bottle C

C Bottle A has equal amount of water as bottle C

D None of the answers A, B and C

E Don't know.

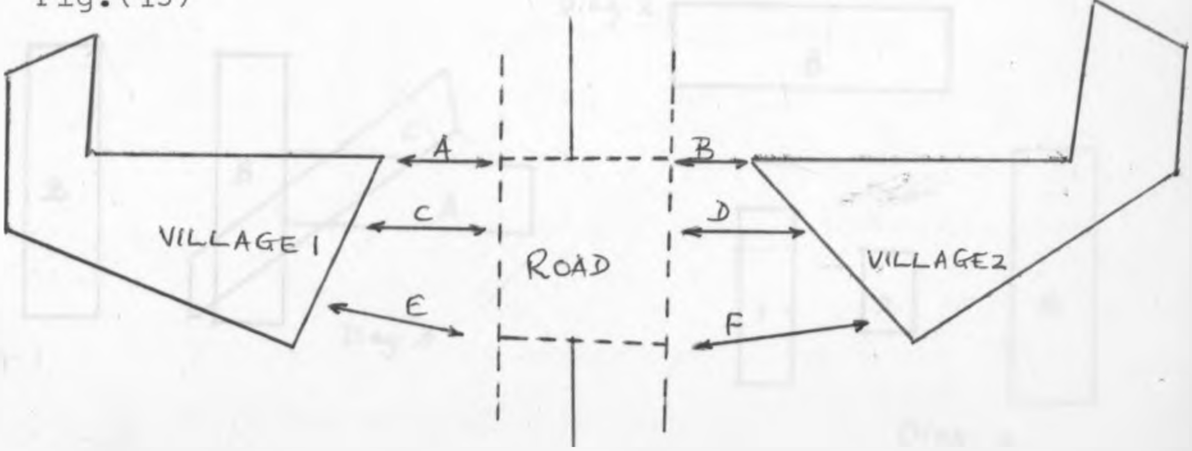
(C) 1

(66) Show how the level of water would look like in bottle B by drawing a line on bottle B in the diagram. 1

PART C: Answer all questions in this part.

There is a diagram of 2 villages in front of your class. The villages are separated equally by a road R. There are six paths A, B, C, D, E and F which cross the black road into the villages. Here below are the drawings of the villages, the road and the paths:

Fig. (15)



Use the drawings to answer the following questions:

Which paths are equal in length to the following paths:

(67) A ________________ (B) ________________ 1

(68) D ________________ (C) ________________ 1

(69) F ________________ (E) ________________ 1

(70) By how many times do you think path A is path B?

A One time

B Two times

C Three times

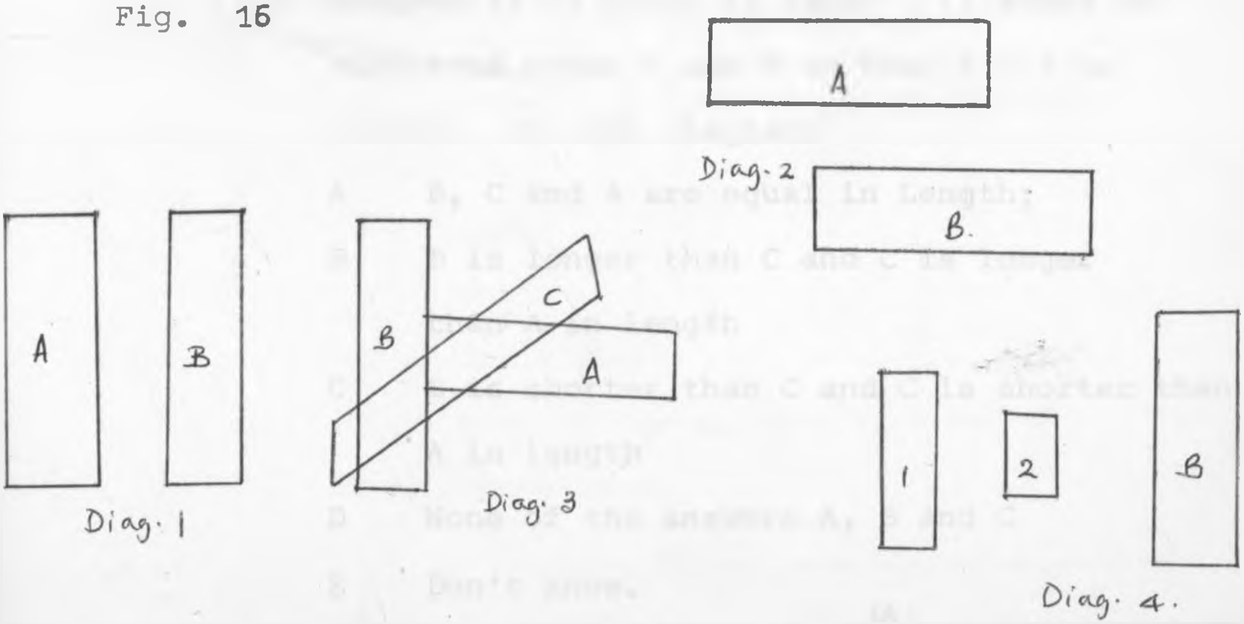
D None of the answers A, B and C

E Don't know

________________ (A) ________________ 1

There is a diagram of strips of paper in 4 different diagrams in front of your class. The strips of paper A and B in diagram 1 are equal in length. The strips of paper A and B are moved in different places of diagrams 2, 3 and 4. Here below are the diagrams of the strips of paper A and B.

Fig. 16



Use the diagrams to answer the following questions:

(71) Diagram 1: The strips of paper A and B are equal in length. About how long do you think is B:

A About 20 cm.

B About 30 cm.

C About 40 cm.

D None of the answers A, B and C

E Don't know.

(B) 1

(72) Diagram 2:

The lengths of A and B are:

A A is longer than B in length

B A is shorter than B in length

C A is equal to B in length

D None of the answers A, B and C

E Don't know.

(C) 1

(73) Diagram 3: A strip of paper C is added to strips of paper A and B so that C = A in length. In this diagram:

A B, C and A are equal in Length;

B B is longer than C and C is longer than A in length

C B is shorter than C and C is shorter than A in length

D None of the answers A, B and C

E Don't know.

(A)

(74) Diagram 4: The strip of paper A is separated into strips 1 and 2.

In this diagram:

A B is equal to strip 1 take away strip 2.

B B is equal to strip 1 add strip 2

C B is shorter than 1 add strip 2

D B is longer than strip a add strip 2

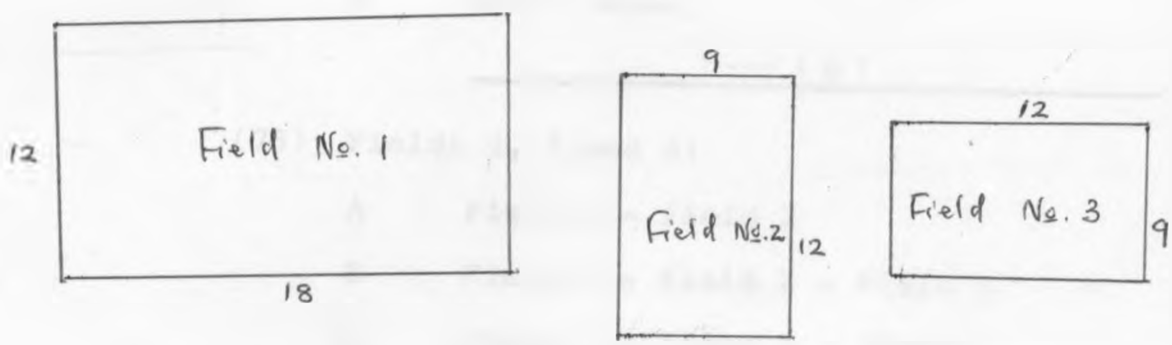
E Don't know.

(B)

There is a diagram of 3 fields in front of your class.

Here below is the drawing of the 3 fields.

Fig. 17



Use the diagrams to answer the following questions:

The fields:

(75) About how many times is field 1 bigger than field 2.

A Once

B Twice

C Three times

D None of the answers A, B and C

E Don't know.

(B)

1

(76) Fields 2 and 3

A Field 2 is bigger than field 3

B Field 2 is smaller than field 3

C Field 2 is equal to field 3

D None of the answers A, B and C

D Don't know.

(C) 1

(77) How many square units are in field 1?

A 117

B 216

C 107

D None of the answers A, B and C

E Don't know.

(B) 1

(78) Fields 1, 2 and 3:

A Field 1 = field 2

B Field 1 = field 2 - field 3

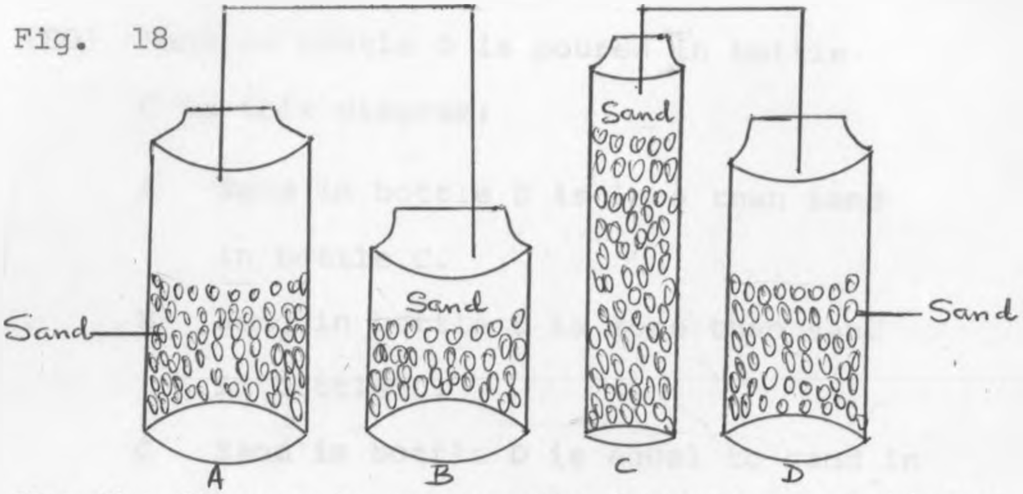
C Field 1 = field 2 + field 3

D Don't know.

(C) 1

There are 4 bottles A, B, C and D in front of your class. B and C are different in size. Bottles A and D are equal in size and have sand half-way filled. Bottle B is wider and shorter than bottles A and D. Bottle C is narrower and taller than bottles A, B and D. Here below is the drawing of the bottles of sand:

Fig. 18



Use the diagram to answer the following questions:

Sand in bottle A is poured in bottle B.

(79) In this diagram:

A Sand in bottle D is more than sand in bottle B.

B Sand in bottle D is less than sand in bottle B

C Sand in bottle D is equal to sand in bottle B

D None of the answers A, B and C

E Don't know.

(C) 1

(80) Sand in bottle D is poured in bottle C in this diagram:

A Sand in bottle D is less than sand in bottle C.

B Sand in bottle D is more than sand in bottle C.

C Sand in bottle D is equal to sand in bottle C.

D None of the answers A, B, and C.

E Don't know.

(C) 1

Total 80

APPENDIX B

PILOT STUDY

AN INVESTIGATION INTO MATHEMATICAL CONCEPT DEVELOPMENT IN CLASSIFICATION, CONSERVATION AND MEASUREMENT AMONG KENYA PRIMARY SCHOOL CHILDREN.

INTRODUCTION

The primary purpose of the pilot study was to select materials and instruments which would be appropriate and meaningful to the main proposed study. Further to the primary purpose, the study was to determine the possibility of predicting the results of the main study.

The pilot study took place at Likoni Primary School of the Municipality of Mombasa, Coast Province. The sample in the study consisted of 72 pupils, selected at random. The pupils came from standards 3, 4 and 5. There was an equal number of boys (36) and girls (36). Each class had boys (12) and girls (12). Each class also provided for pupils' above average (8) average (8) and below average (8) in ability, distributed equally for boys (4) and girls (4), as recorded in pupils' annual examinations in mathematics, 1975. The age range was between 9 and 17 years.

There were three main tasks in the pilot study: classification, conservation and measurement. Classification task included 45 items on: colour, shape, size, class-inclusion, hierarchical and logical connectives ("AND", "OR").

Conservation task included 21 items on: length, area and volume. Measurement task included 14 items on: distance, length, area and volume. There was a questionnaire (see Appendix C) on socio-economic statuses of the pupils' home environments to assist the experimenter in finding out the degree of environmental influence on the pupil's concept formation in mathematical development.

The entire experiment included 72 pupils, 80 items, 6 research assistants and the experimenter (author).

METHOD AND PROCEDURE

All the 80 items were administered in one large classroom, for effective supervision. Each item was expected to last between 1-2 minutes. The pupils sat so as to occupy reasonable room for independent and comfortable working. All the pupils sat opposite the experimenter and the research assistants. To control for any possible leakage of the test-items among the research assistants and the pupils, the experimenter kept the items and other relevant equipment in his custody. The research assistants assisted with seating plans, registration of the pupils, supply of working tools and the handling of the experimental test-items in the presence of the experimenter, at the time of experimentation.

The experimenter, employed his own method called "Audio-Visual Experimental Technique", in future to be referred to as: (AVET). In this technique (AVET), the materials in the experiments were displayed one set at a time, in front of the class. The pupils were required (actual did) to observe the displayed materials while at the same time, one of the research assistants read loudly one question at a time to the pupils.

The pupils used the test-items, the questions (see Appendix A). The pupils combined the audio-visual and the conceptualized thinking to comprehend the items and provide the information per se. The pupils wrote all the answers in the space provided for in the test-items booklet. The writing by pupils was in pencil.

The test lasted up to 3 hours excluding a brief break of 30 minutes between the parts A and B. The research assistants again assisted with marking the test-items. The rest of the work on grading and analysing was handled by the experimenter himself. The scoring system was (1,0) for right answers and wrong answer respectively.

DATA ANALYSIS

The author carried out scale and item analysis on the classification, conservation and measurement tasks. The table below shows the results which included Reliability Co-efficient, using Kuder-Richardson Formula 20.

1. Scale Analysis:

Table (15)

Mean Scores by Task (N = 675)

Task	Items	Means	S.D.	Rel .Coeff.
Classification	45	17.00	7.00	0.90
Conservation	21	7.60	4.0	0.70
Measurement	14	6.00	3.0	0.70
Total	80	31.00	12.0	0.90

2. Item Analysis:

The author carried out an item analysis on separate tasks (classification 45 items, conservation 21 items and measurement 14 items) together with the standard deviation of each and the difficulty-index of each item. The results showed that the following were difficult items:

Classification: 4, 7, 11, 13, 14, 25, 26, 27, 28, 37 - 45

Conservation: 48, 49, 50, 51, 57.

Measurement: 71.

HYPOTHESES

The author set out to test the following null hypotheses:

1 ... There is no difference in performance between boy and girl pupils in classification, conservation and measurement concepts.

2 ... There is no difference in performance between pupils of grades: 3, 4, and 5 in classification, conservation and measurement concepts.

3 ... There is no difference in performance between pupils from different parental or gurdian educational levels in classification, conservation and measurement concepts.

4 ... There is no difference in performance between pupils of above-average, average and below average abilities in classification, conservation and measurement concepts.

RESULTS

The results of the pilot study clearly indicated that the pupils who took part in the study did well in classification task, scoring a mean of 17 points and a standard deviation of about 7.0 Items on multiple classification, hierarchical classification and logical connectives proved difficult to a majority of the pupils. This failure in classification task indicated that the pupils were not exposed to these tasks in their first two years of primary school education. It was interesting to note that "grey" colour was confused for white by almost all the pupils, yet this particular colour is commonest in use at the Coast.

Conservation task showed that pupils who took part in the test were unable to conserve objects when such objects underwent certain transformations. The experimenter may wish to carry-out further experiments into this part to find out why pupils of standard 3: upwards cannot conserve. This should form one of the experimenter's main tasks in the main study.

Measurement tasks on the other hand proved equally difficult. It looked as though there will be need for pupils to learn and understand the difference between measurement and a measure. The pupils' concept of measurement is still under-developed. Again this area will receive attention on the main study.

Based on the experience and findings of the pilot study, the experimenter firmly concluded that there was need to do research in mathematical concept development among the Kenyan Primary school pupils.

APPENDIX C

MATHEMATICAL CONCEPT DEVELOPMENT AMONG KENYAN PRIMARY SCHOOL CHILDREN

(QUESTIONNAIRE OF SUBJECT'S SOCIO-ECONOMIC STATUS:)

STDS. 3, 4 & 5

This is not a test. Research Assistants please make sure that the subjects (pupils) give the correct information. The subjects should complete this questionnaire as directed by the author. All queries affecting this exercise should be referred to the author only.

Subject's names ____________________ ____________________
(Surname first) (Other names)

Subject's School ____

Subject's Class ____

Subject: Boy or Girl ____________________________________

INSTRUCTION: Subjects are to choose one letter from the given letters A, B, C, D, E, F, etc., which offers nearly the most correct answer from several other answers. For example:-

the question: "Whom do you like best in your family?"

A. Both father and mother

B. Father only

C. Mother only

D. None

(check one only)

If the answer is: "Both father and mother", We write A. in the column called: "Answer-space", opposite (check one only)	AN-SPACE A
1. Which is your home district? A. Mombasa B. Lamu C. Kisii D. Meru E. Kiambu F. Kakamega G. Siaya H. Machakos I. Narok J. Iringa K. Kwale	

AN-SPACE

L. Embu

M. Busia

N. Turkana

O. Isiolo

P. Mandera

Q. Other (specify)

R. Don't know

(check one only

2. Which is your tribe?

A. Arab

B. Miji-kenda

C. Luo

D. Kikuyu

E. Kamba

F. Kalenjin

G. Luyia

H. Other (specify)

I. Don't know

(check one only)

3. How old are you now?

A. Below 7 years

B. 7 years

C. 8 years

D. 9 years

E. 10 years

F. 11 years

G. Over 11 years

H. Don't know
(check one only)

AN-SPACE

4. Which is your religion?

A. Muslim

B. Christian

C. Hindu

D. Other (specify)

E. None

(check one only)

5. Are your parents:

A. Both living?

B. Both dead?

C. Father only living?

D. Mother only living?

E. Separated?

F. Don't know?

(check one only).

6. What is your father's work?

A. Farmer

B. Teacher

C. Doctor

D. Politician

E. Driver

F. Government servant

G. Trader

H. Other (specify)

I. None

J. Don't know

(check one only).

AN-SPACE

7. What is your mother's or your guardian's work?

   A. Farmer

   B. Teacher

   C. Doctor

   D. Politician

   E. Driver

   F. Government servant

   G. Trader

   H. Other (specify)

   I. None

   J. Don't know

   (Check one only).

8. How many children are there in your family including yourself?

   A. One only

   B. Two only

   C. Three only

   D. Four only

   E. Five only

   F. Six only

   G. Seven only

   H. More than seven.

   (check one only).

AN-SPACE

9. Where does your family's money come from?

A. Family's farm produce

B. Family's business

C. Parent's salary

D. All A, B and C.

E. None

G. Don't know.

(check one only).

10. Which type of house do you live in?

A. Permanent

B. Semi-permanent

C. Temporary

D. Other (specify)

E. None

F. Don't know

(check one only).

11. Which meals do you have in a day?

A. Breakfast, lunch and supper

B. Breakfast and lunch only

C. Lunch and supper only

D. Breakfast only

E. Lunch only

F. Supper only

G. Breakfast and supper

H. None

(check one only).

AN-SPACE

12. How do you travel to your school?

A. By car

B. By public bus

C. By bicycle

D. On foot

E. Other (specify)

(check one only)

13. How much money do you carry each day?

A. Cents ten only

B. Cents fifty only

C. One shilling only

D. More than one shilling

E. Other (specify)

F. None

(check one only).

14. How many servants work at your home?

A. One only

B. Two only

C. Four only

D. Other (specify)

E. None

F. Don't know

(check one only).

AN-SPACE

15. Which of the following things are at your home?
    A. Radio only.
    B. Television only
    C. Record player only
    D. All A, B and C
    E. Other (specify)
    F. Don't know
    G. None
    (check one only).

16. Which type of light do you use at home for reading?
    A. Candle wax
    B. Oil lamp
    C. Torch
    D. Pressure lamp
    E. Electricity
    F. Other (specify)
    (check one only).

17. Who helps you with your studies at home?
    A. Father and mother
    B. Father only
    C. Mother only
    D. Brother and sister
    E. Brother only
    F. Sister only

AN-SPACE

G. House - worker

H. Baby - seater

I. Private teacher

J. Other (specify)

K. None

L. Don't know

(check one only)

18. What work do you do at home?

A. Look after animals

B. Sell goods in the shop

C. Help in the garden

D. Look after the baby

E. Play

F. Other (specify)

G. Nothing

H. Don't know

(check one only).

19. How many times did you go to the hospital for treatment during this year?

A. More than three times

B. Two times only

C. One time only

D. None

E. Don't know

(check one only).

AN-SPACE

20. Which disease troubles your home most?
    A. Malaria
    B. Chest disease
    C. Diarrhoea
    D. Wounds
    E. Eye disease
    F. Other (specify)
    G. None
    (check one only).

21. What class of education did your father reach?
    A. Below C.P.E.(Below Std. 7).
    B. From C.P.E. to O-level (std. 7-Form 4)
    C. Above O-level (Form 5 - University)
    (check one only)

22. Which class of education did your mother or guardian reach?
    *A. Below C.P.E.(Below Std.7)
    B. From C.P.E. to O-level (Std. 7 - Form 4)
    C. Above "O" level (Form 5 - University).
    (check one only)

* A..Lower, B...Middle, C...Upper Classes.

APPENDIX D

MATHEMATICAL CONCEPT DEVELOPMENT AMONG KENYAN PRIMARY SCHOOL PUPILS

CODING SHEETS

VARIABLE	CODE	COLUMNS
1. Student Identity	A = Mombasa Primary School	1 - 5
	B = Alibhai Panju P. "	
	C = Fahari Primary School	
	D = Buxton " "	
	E = Mvita " "	
	F = Mtongwe " "	
	G = Makande " "	
	H = Spaki " "	
	J = Makupa " "	
	K = Tudor " "	
	3 = Standard 3	
	4 = Standard 4	
	5 = Standard 5	
	Student No.=001-999	6 blank
2. Sex	1 = Boy student	7
	2 = Girl student	
3. Age	Years = 00 - 20	8 - 9

VARIABLE	CODE	COLUMNS
4. Socio-Ec. Status	1 = Low class	10
	2 = Middle Class	
	3 = Upper Class	
5. Ability	1 = Below Average	11
	2 = Average	
	3 = Above -Average	
		12 blank

VARIABLE				COLUMNS
6 - 50 classification			1 = correct answer 0 = wrong answer	
6 = item 1	var. 5		"	13
7 = "	2	6	"	14
8 = "	3	7	"	15
9 = "	4	8	"	16
10 = "	5	9	"	17
11 = "	6	10	"	18
12 = "	7	11	"	19
13 = "	8	12	"	20
14 = "	9	13	"	21
15 = "	10	14	"	22
16 = "	11	15	"	23
17 = "	12	16	"	24
18 = "	13	17	"	25
19 = "	14	18	"	26
20 = "	15	19	"	27
21 = "	16	20	"	28
22 = "	17	21	"	29
23 = "	18	22	"	30
24 = "	19	23	"	31
25 = "	20	24	"	32
26 = "	21	25	"	33
27 = "	22	26	"	34
28 = "	23	27	"	35
29 = "	24	28	"	36
30 = "	25	29	"	37
31 = "	26	30	"	38
32 = "	27	31	"	39
33 = "	28	32	"	40
34 = "	29	33	"	41
35 = "	30	34	"	42
36 = "	31	35	"	43
37 = "	32	36	"	44
38 = "	33	37	"	45
39 = "	34	38	"	46
40 = "	35	39	"	47

VARIABLE			CODE	COLUMN(S)
41 = Item	36	40	"	48
42 = "	37	41	"	49
43 = "	38	42	"	50
44 = "	39	43	"	51
45 = "	40	44	"	52
46 = "	41	45	"	53
47 = "	42	46	"	54
48 = "	43	47	"	55
49 = "	44	48	"	56
50 = "	45	49	"	57
51 =			00 - 45 Actual marks scored	58 blank. 59 - 60 61 blank
52 - 72 Conservation			1 = correct answer 0 = wrong answer	62
52 = Item	1	50	"	63
53 = "	2	51	"	64
54 = "	3	52	"	65
55 = "	4	53	"	66
56 = "	5	54	"	67
57 = "	6	55	"	68
58 = "	7	56	"	69
59 = "	8	57	"	70
60 = "	9	58	"	71
61 = "	10	59	"	72
62 = "	11	60	"	73
63 = "	12	61	"	74
64 = "	13	62	"	75
65 = "	14	63	"	76

VARIABLE			CODE	COLUMN(S)
66 = "	15	64	"	77
67 = "	16	65	"	78,79 blank
68 = "	17	66(p2)	"	7
69 = "	18	67	"	8
70 = "	19	6	"	9
71 = "	20		"	10
72 = "	21		"	12 blank
73 =			00 - 21 Actual marks scored.	13 - 14
74-78 Measurement			1 = correct answer 0 = wrong answer	15 blank
74 = Item	1	71	"	16
75 = "	2	72	"	17
76 = "	3	73	"	18
77 = "	4	74	"	19
78 = "	5	75	"	20
79 = "	6	76	"	21
80 = "	7	77	"	22
81 = "	8	78	"	23
82 = "	9	79	"	24
83 = "	10	80	"	25
84 = "	11	81	"	26
85 = "	12	82	"	27
86 = "	13	83	"	28
87 = "	14	84	"	29
				30 blank
88			00 - 14 = Actual marks scored.	31 - 32
				33 blank
89 = GRAND TOTAL			00 - 80 = ACTUAL GRAND TOTAL MARK SCORED	34 - 35
				36 - 79 blank.