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The Flatness Problem as A Natural Cosmological Phenomenon

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Abstract

An interplay between elementary particle interactions at very high energies and the expansion of the Universe is studied in the context of the inflationary model to find a possible explanation for an observed flatness which remains a cosmological puzzle. As is well known from observation, the universe is at least fairly flat implying that the relative energy density is almost unity. This value of energy is extrapolated from the matter dominated and the radiation dominated phases of the false unbroken grand unified vacua separately, to the true vacuum. This present relative energy density of order unity must have been of order 10^{-61} at the Planck time of 10^{-43} seconds that leads to the gravity breaking from strong and weak interactions. A very fine tuning of one part in 10^{-61} is needed to keep the relative energy density close to unity over a very longtime to ensure that the universe is very old as observed. This in itself does not ensure that the curvature term in the Friedman field equations of motion is unimportant. These equations are solved separately, for the open, closed and flat universes corresponding to the value of the curvature term being minus one, zero and plus one. Analysis of these solutions indicates that the open and closed solutions converge to the flat universe solution. A power series expansion done for the relative energy density in the case of open and closed universes shows that even a first order correction is negligible and in this case the hyperbolic solutions become exponential which is the flat universe result. The flatness problem is then seen as a natural phenomenon in an inflationary

universe. Such a coarse-grained solution becomes the underlying reason for the curvature term to be identically zero rather than assuming the value of either minus one or plus one.

Introduction

The experimental justification of the hot big bang model rests on three principal pieces of evidence[1]:

i. The expansion of the universe based on the recession of the galaxies and Hubble's law $v = Hd$, where v is the recession speed, H is the proportionality constant and d is distance between two observers/galaxies.
ii. The discovery of the 2.726 ^{o} K cosmic microwave background radiations that fills the entire universe.
iii. The cosmic abundance of the very light elements especially Helium and Deuterium.

In spite of these striking characteristics, the model suffers from some serious general difficulties the solution of which demands an important addition or modification to the basic model[1]. These cosmological puzzles are broadly discussed in the literature[1,2,3,4,5,6,7]. Currently the inflationary models appear promising candidates that seem to offer the possibility of explaining the fundamental cosmological facts. They are based upon microphysical events which occurred early in the universe but well after the Planck epoch.

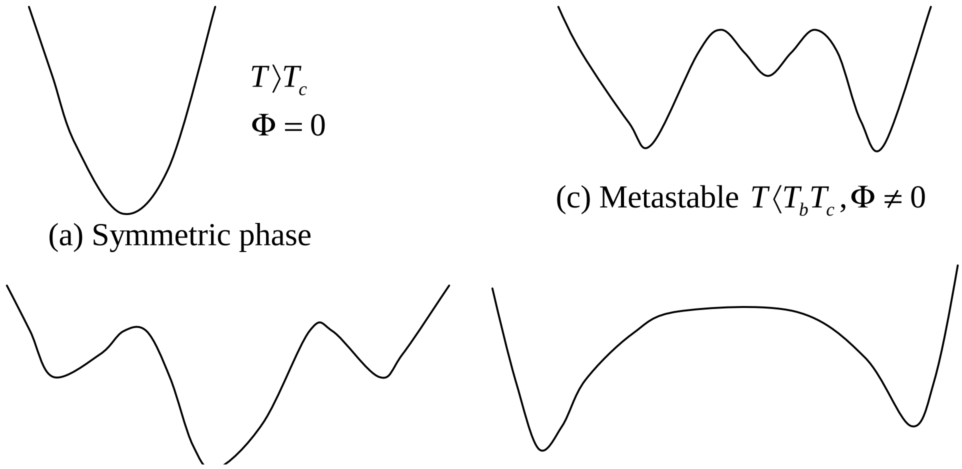
In this paper, the Friedman equation is solved explicitly for the three models (k=-1, k=0 and k=+1). By making the approximation based on the fact that the argument of the hyperbolic functions is very large, all the three models are seen to reduce to an exponential function ie both the k=+1 and k=-1 models reduce to the k=0 model in the approximation that the universe underwent a rapid expansion during its early stages of development. It is concluded that the flatness problem is probably a natural phenomenon in an inflationary universe.

The organization of the paper is as follows: In section II, we briefly review the inflationary scenario in the simplest way while in section III we formulate the flatness problem from the cosmological principle of homogeneity and isotropy. In section IV we give the justification for neglecting the energy density of relativistic particle radiation during the rapid expansion and in section V we give our conclusions and further research work.

Inflationary scenario

Grand Unified Models of particle physics predict that the state of thermal equilibrium of the quantum field will undergo a phase transition at a critical temperature T_c of order of the grand unification scale, $\approx 10^{15} GeV$. Hence if the quantum fields in the universe began in an arbitrary hot state at the big bang singularity, as assumed in the standard cosmological models, then such a phase transition would have occurred in the early universe as the fields cooled to below T_c as a result of the expansion of the universe. If the grand unified field-theory models turn out to be correct and if the universe behaved according to the standard cosmological models at these very early epoch, then this phase transition will have occurred and it is of considerable importance that we explore its consequences for cosmology[8].

The most dramatic and appealing consequence that has been predicted so far is the possible existence of an era of "inflation". In the original scenario[9], it was proposed that the zero temperature potential energy of a Higgs field Φ could have a local minimum with energy density (say) ρ_o above that of the absolute minimum. If Φ settled into the local minimum over a sufficiently large region of space as the field cooled, then it could remain trapped there in a metastable state until it tunneled through(by nucleation and expansion of bubbles) to the true minimum. While in the false vacuum, the kinetic and spatial derivative energies of the field would be negligible compared with its potential energy ρ_o. Hence the stress-energy tensor $T_{\alpha\beta}$ of the field would be dominated by the term ρ_o. Thus the effect of the field Φ on the dynamics of the universe via Einstein's equation would be just like the effect of having a large positive cosmological constant. In the Robertson-Walker models, this produces the de-Sitter solution which yields an expansion of the universe on an exponential time scale. Thus if Φ remained in the false vacuum for many e-folding times, an enormous inflation of the universe would have occurred which could be of relevance for shedding light on such issues as why there are relatively few (if any) magnetic monopoles in our universe, why the universe appears to be nearly cosmologically flat etc[4]. However, this model suffers from the serious problem that it does not appear to allow an exit from the inflationary era in such a way as to evolve to the presently observed universe. The new inflationary scenario[3,9,10] was proposed primarily to overcome this problem of obtaining a graceful exit from the inflationary era. The basic idea of new inflation is as follows: One considers the Higgs field Φ whose one-loop effective potential has the form sketched in figure 1. We assume that initially the field is in thermal equilibrium at high temperature and thus is in the symmetric minimum of V_T at Φ=0 as in curve (a). As the field cools due to expansion of the universe, it will fall into the metastable state indicated in curve (c) of figure 1 by the local minimum of V_T at Φ=0 for intermediate temperatures. At low temperatures, the dip in V_T goes away and the dynamics of the field is governed by the classical (ball rolling down a hill) evolution of Φ in the zero-temperature potential of curve (d) with initial conditions $\Phi \approx 0$, $\dot{\Phi} \approx 0$, where the magnitude of the differences from zero are estimated from quantum fluctuations. If $V(\Phi)$ is very flat near Φ=0, then it will take a long time for Φ to reach Φ_c. During this time, the stress-energy tensor of Φ will be dominated by the term ρ_o , where ρ_o is the height above the true minimum of the flat part of $V(\Phi)$. The universe has an extra energy density ρ_o at its disposal which must have dynamical effects via the Einstein's equation[4].



(a) Symmetric phase

(c) Metastable $T \langle T_b T_c, \Phi \neq 0$

(b) Lower values of $T \langle T_b, \Phi \neq 0$

(d) $T = 0, \Phi \neq 0$

Figure 1: shows a sketch of the behavior of the one-loop effective potential $V_T^{(1)}$ in Coleman-Weinberg models. At high temperatures, $V_T^{(1)}$ has the form shown in curve (a), with a single minimum at $\Phi = 0$. At lower values of T , $V_T^{(1)}$ develops side minima as shown in curve (b). At still lower temperatures $V_T^{(1)}$ has the form shown in curve (c). Finally curve (d) represents the effective potential at $T = 0$.

Formulation of the Flatness Problem

The evolution of the universe is described by the Robertson-Walker metric through the Friedmann equation[4]

$$\frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G}{3c^2}\rho \tag{1}$$

where S(t) is the scale factor, k is the curvature parameter and ρ is the energy density. In the k=0 model we have at any epoch (in the radiation dominated era)

$$S \propto t^{1/2} \tag{2}$$

so that $$\frac{kc^2}{S^2} = (\Omega - 1)\frac{\dot{S}^2}{S^2} = \frac{\Omega - 1}{4t^2} \tag{3}$$

where $\Omega = \dfrac{\rho}{\rho_c}$ is the cosmological relative energy density parameter. For the present epoch, we have $$\frac{kc^2}{S_o^2} = (\Omega_o - 1)\frac{\dot{S}_o^2}{S_o^2} \tag{4}$$

Given $S(t) \propto T^{-1}$ we have for $k = \pm 1$, $(\Omega - 1) = (\Omega_o - 1) 4 H_o{}^2 t^2 \dfrac{T^2}{T_o{}^2}$ (5)

In the early cosmological chronology[2], we have the following

Planck time $(\Omega - 1) = 3.108 \times 10^{-61} h^2 (\Omega_o - 1)$

GUTs time $(\Omega - 1) = 4.626 \times 10^{-53} h^2 (\Omega_o - 1)$

Electroweak symmetry breaking phase

$$(\Omega - 1) = 4.626 \times 10^{-27} h^2 (\Omega_o - 1) \qquad (6)$$

Quark confinement time

$$(\Omega - 1) = 5.140 \times 10^{-22} h^2 (\Omega_o - 1)$$

Neutrino decoupling time

$$(\Omega - 1) = 4.626 \times 10^{-17} h^2 (\Omega_o - 1)$$

It is interesting to note that the coefficient on the right-hand side of equation (5) keeps on gaining i.e. at the Planck time the approximate value of 10^{-61} leads gravity breaking the strong and electroweak interactions; the value 10^{-53} leads the strong force breaking from its counterpart electroweak interaction and 10^{-27} leads to the electroweak symmetry breaking. The above scenario is not good for cosmology, meaning a phase transition is bound to take place whenever the coefficient on the right hand side of the above equation attains a particular value. However if we can rewrite equation (1) as

$$-kc^2 = \dot{S}^2 - \frac{8\pi G}{3c^2} \rho S^2 \qquad (7)$$

then we can interpret $(-kc^2)$ as the total energy of the universe with the kinetic energy term represented by $\dot{S}^2$ and the gravitational potential energy by the term containing ρ. If the total energy is positive (k=-1) then the kinetic energy is great enough (the initial velocity is greater than the escape velocity) and the universe will continue to expand forever i.e. the universe is open. If the total energy is negative (k=+1) the universe will recollapse i.e. the universe is closed. In the k=0 model, the universe is at the escape velocity and it will expand indefinitely. Since our universe has been expanding[6] at almost exactly the critical rate to avoid recollapse, we expect that some mechanism in the early history of the universe could have driven the universe to its present state.

While Guth's[9] original model was flawed, the new version based on slow-roll-over transition proposed by Linde[3], Albrecht and Steinhardt[10] and A A Starobinsky[11] appears promising. It is associated with the evolution of a weakly-coupled scalar field which for some reason was initially displaced from the minimum of its potential[1].

Imagine the universe being cooled through the critical temperature T_c. As the temperature T drops below T_c, the state of lowest energy shifts in the false vacuum (metastable state) at $\Phi = 0$ until at some stage the $\Phi - \mathit{field}$ tunnels across the $V(\Phi)\rangle 0$ barrier and falls down the V(0) slope to its true vacuum as shown in figure 2 below.

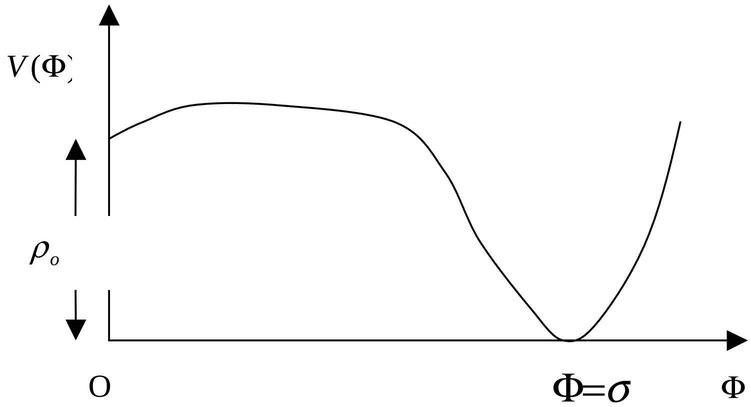


Figure 2: Effective potential for a field Φ of the type corresponding to inflationary models with a slow-roll-over.

If we denote by ρ_o the difference between the energies of the two vacua, then until the tunneling has taken place, the universe has an extra energy density ρ_o at its disposal which must have dynamical effects in the Einstein's equation for the standard cosmology described by the Friedmann equation

$$\frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G}{3c^2}(\rho_o + \rho_r) \tag{8}$$

where $\rho_r \propto S(t)^{-4}$ is the energy density of the relativistic particle radiation[12]. Since it falls as the universe expands while ρ_o stays constant, the vacuum energy density dominates. Therefore we can assume ρ_r and solve the equation of motion (8) for the k=0, k=+1 and k=-1 models.

<u>The k=0 model</u>

Equation (8) reduces to $\frac{\dot{S}^2}{S^2} = \frac{8\pi G}{3c^2}\rho_o$ so that $\frac{\dot{S}}{s} = \sqrt{\frac{8\pi G}{3c^2}\rho_o} = H$ (9a)

where H is Hubble's constant. We simply integrate equation (9a) to get

$$S \propto \exp Ht \tag{9b}$$

The k=+1 model

Equation (8) reduces to $\frac{\dot{S}^2 + c^2}{S^2} = \frac{8\pi G}{3c^2}\rho_o$ which can be rewritten as $\dot{S}^2 = \frac{8\pi G}{3c^2}\rho_o S^2 - c^2$ so that $\dot{S} = \frac{dS}{dt} = \frac{1}{\alpha}\sqrt{S^2 - \beta^2} = \frac{\beta}{\alpha}\sqrt{\frac{S^2}{\beta^2} - 1} = c\sqrt{u^2 - 1}$ (10a)

where $\alpha = \sqrt{\frac{3c^2}{8\pi G\rho_o}}, \beta = c\alpha$ and $u = \frac{S}{\beta}$. Therefore equation (10a) becomes $\frac{du}{\sqrt{u^2 - 1}} = \frac{cdt}{\beta}$, which can be integrated to obtain $S = \beta \cosh\frac{1}{\alpha}t = c\alpha \cosh\frac{1}{\alpha}t$ (10b)

from which we arrive at $S = \sqrt{\frac{3c^4}{8\pi G\rho_o}} \cosh\sqrt{\frac{8\pi G\rho_o}{3c^2}}t$ and $S \propto \cosh\sqrt{\frac{8\pi G\rho_o}{3c^2}}t$ (10c)

The k=-1 model

Equation (8) reduces to $\frac{\dot{S}^2 - c^2}{S^2} = \frac{8\pi G}{3c^2}\rho_o$ from which we have $\dot{S}^2 = \frac{8\pi G}{3c^2}\rho_o S^2 + c^2 = \frac{1}{\alpha^2}(S^2 + \beta^2)$ and $\dot{S} = \frac{dS}{dt} = \frac{1}{\alpha}\sqrt{(S^2 + \beta^2)} = c\sqrt{\frac{S^2}{\beta^2} + 1}$ (11a)

Making use of our previous substitutions, we have
$\frac{du}{\sqrt{u^2 + 1}} = \frac{1}{\alpha}dt$ which can be integrated to obtain $S = \sqrt{\frac{3c^4}{8\pi G\rho_o}} \sinh\sqrt{\frac{8\pi G\rho_o}{3c^2}}t$ (11b)

so that $S \propto \sinh\sqrt{\frac{8\pi G\rho_o}{3c^2}}t$ (11c)

In the approximation that $\sqrt{\frac{8\pi G\rho_o}{3c^2}}t$ is very large[13], both equations (10c) and (11c) reduce to (9b) which is the flat model. This rapid expansion in an exponential fashion continues until the tunneling takes place and Φ attains its true vacuum value. Note that starting with the curvature term $\frac{kc^2}{S^2}$ in comparison to the expansion term $\frac{\dot{S}^2}{S^2}$ prior to inflation, leads the former being reduced by approximately $S^2 \approx 10^{58}$ while the latter almost stays constant. This large factor manifests as the fine-tuning in the flatness problem.

Fine-Tuning in the Flatness Problem

Let the total energy density ρ_t be $(\rho_o + \rho_r)$. Then since $\rho_r \propto T^4$ and $T \propto S^{-1}$, we have $\rho \propto S^{-4}$. We can expand the energy density as

$$\rho_t(x) = a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + ... + a_n x^n$$

where $x = S^{-4}$. after finding the coefficients, we have

$$\rho_t(x) = \rho_o + S^{-4} - 4S^{-9} + 20S^{-14} - 120S^{-19} + ...$$

Notice that even to first-order, $\rho_r \propto S^{-4}$ is negligible.

Conclusion

The nature and age of the universe constrain the value of the relative energy density Ω to be not too different from unity. Had the energy density ρ been substantially different from the critical energy density ρ_c, the heavy elements, the stars and the galaxies could not have formed. For ρ far much less than ρ_c, the universe would have expanded too rapidly for stars or galaxies to form while for ρ far much greater than ρ_c, the universe would have recollapsed before star (or galaxy) formation could have been achieved. The fine-tuning required in maintaining Ω nearly unity over the required period is so extremely delicate that the inflationary mechanism probably seems to have played a significant role. The conventional result that Ω is nearly unity today means that in the first moments of the big bang it was precisely unity. The natural inference is that the value is and has always been exactly unity. An important implication of this result is that the value $\Omega = 1$ is a natural phenomenon in an inflationary scenario. Alternatively, our result would also imply that there is a large amount of unseen matter in the universe that is contributing indirectly to the energy density so that the value $\Omega = 1$. This possibility provides a window for further research work.

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References

[1] [1] Hughes, I S, 1991, Elementary Particles, Cambridge University Press, Cambridge UK, Chap 15.

[2] Coughlan, G.D., and Dott, J.E, 1991, Ideas of Particle Physics, Cambridge University Press, Cambridge, UK, Chap.42.

[3] Linde, A.D., 1982, "A new inflationary Universe scenario: A possible solution of the Horizon, flatness, homogeneity, isotropy and primordial monopole problems", Phys Lett 108, No.6, 389.
[4] Jayant, V.N, 1993, introduction to Cosmology, Cambridge University Press, Cambridge, U K, Chap. 6.
[5] Maumba, G.O, Cosmological puzzles, University of Nairobi Preprint NRB-TP-1-06.
[6] Hawking, S.W, and Moss, I.G, 1982, "SupercooledPhase transitions in the very early universe", Phys.Lett 110B, No.1, 35.
[7] Wikipedia, 2006, Big Bang, The free encyclopedia, p1.
[8] Gene, F.M, et al, 1985, "Does a phase transition in the early universe produce the conditions needed for inflation?" Phys.Rev D31, No.4, 273.
[9] Guth, A.H, 1981, "Inflationary Universe: A possible solution to the horizon and flatness problems", Phys.Rev D 23, No. 2, 347.
[10] Andreas, A., and Paul, J.S, 1982, "Cosmology for Grand Unified Theories with radiatively induced symmetry breaking", Phys. Rev Lett 48, No. 17, 1220.
[11] Starobinsky, A.A, 1982, "Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations", Phys. Lett 117B, No. 3,4 175.
[12] Robert, J.S, and Michael S.T, 1985, "Decaying particles do not heat up the universe" Phys.Rev D 31, No. 4, 681.
[13] Thomas and Finney, 1993, Calculus and Analytic Geometry, Narosa Publishing House, New Delhi, India, Chap. 9.

List of symbols and abbreviations

c-speed of light
G-gravitational constant
GeV-giga electronvolt
h-accounts for the uncertainty in Hubble's constant
H-Hubble's proportionality constant
k-the curvature parameter
S(t)-cosmic scale factor
T_c-critical temperature
$T_{\alpha\beta}$-Energy-Momentum tensor
v-speed of recession
V_T-temperature dependent effective potential
ρ-total energy density
ρ_c-critical energy density
ρ_o-vacuum energy density
Φ-Higgs scalar field
Ω-relative cosmological energy density parameter
Ω_o-present relative cosmological energy density parameter.