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CLOSURE OF THE UNIVERSE BY WEAKLY INTERACTING MASSIVE NEUTRINOS

By

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IN THE DEPARTMENT
OF
PHYSICS

DECLARATION AND CERTIFICATION

This thesis is an original work submitted for the Degree of Doctor of Philosophy in Physics at the University of Nairobi and has not been, nor will it be presented to any other university for a similar or other degree award.

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<u>MR GEORGE MAUMBA OKEYO</u>

The undersigned certify that he has read and hereby recommend for acceptance by the University of Nairobi a thesis entitled: "Closure of the Universe by Weakly Interacting Massive Neutrinos" in fulfillment of the requirements for the award of the degree of Doctor of Philosophy (Physics) of the University of Nairobi.

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DEDICATION

To my Parents-Francis Nyaga Okeyo Maumba and Priskah Kerubo, my children Isaac Mecheo, Jacinta Nyamoita, Emmanuel Ondieki and spouse Catherine Mukolwe.

ABSTRACT

The role of neutrino mass in the context of grand unified theory has been investigated using the equilibrium condition between a critically expanding universe and weak neutrino interaction rates in the early universe. The critical expansion rate has been studied using the Friedman equation that was modified by expressing the scale factor as a function of time. This gave rise to the model equation used in the investigation to establish the critical nature of the expanding universe. The delicate equilibrium balance between expansion and weak neutrino reaction rates generated the Boltzmann transport equation; this was solved by Successive Approximation technique and a mass value of *1.57 GeV* was established for heavy or non-relativistic neutrino. For a light or relativistic neutrino, a mass value of *1.97 eV* was determined. The results were found to generate a cosmological mass gap for neutrino masses in the range between *1.97 eV* and *1.57 GeV* as opposed to the standard electroweak model of particle physics that allows existence of a massless neutrino. More importantly, the presence of a *GeV* neutrino mass predicts existence of a fourth family of leptons. When the calculated mass was used in the calculated See-Saw relation, a unification scale of 1.542×10^{13} *GeV* was achieved. This energy scale is interesting since the weak, electromagnetic and strong interactions would all have the same strength at around 10^{13} *GeV*-10^{16} *GeV* which suggests a very similar value for the grand unification scale. This appears promising as the anticipated discovery of tiny neutrino masses may help in probing the structure of the particle physics models that lie beyond the standard electroweak model.

Neutrino mass generation has also been investigated using the standard Higgs mechanism technique. The standard electroweak Lagrangian was modified by adding a mass term to it and a perturbation to the vacuum expectation value generated neutrino mass term couplings. This was found to be true with the aid of scalar fields, which, in this case was the Higgs boson. Also from the theory of neutrino oscillations, it was found that neutrinos of different masses travel with different velocities rather than with the same velocity. In particular, the dynamical effect i.e. the probability of neutrino oscillation was found to be very dependent on neutrino mass. This was studied by using the simple approach of mathematical theory of matrices rather than the complicated methods of renormalization. The results anticipate that the oscillation experiments (such as Fermilab's LBNE and the MINOS) may play an important role in putting the evidence for neutrino mass on a more solid and practical evidence.

LIST OF SYMBOLS

L	Lepton Number
GUT	Grand Unified Theories
B-L	Baryon-minus-Lepton Number
SU(2)	Special Unitary matrix
U(1)	Unitary matrix
GeV	Giga-electron-volt
CERN	European Organization for Nuclear Research
$W^{\pm}$	Charged weak vector bosons
Z^{o}	Neutral weak vector boson
Kpc	Kilo parsec
M(R)	Mass within radius R
He	Helium
D	Deuterium
Li	Lithium
Ω_B	Relative energy density of Baryons
ρ_γ	Photon energy density
ρ_ν	Neutrino energy density
eV	electron-volt
TOE	Theory of Everything
β	Beta symbol
KeV	Kilo-electron-volt
Q	Energy release in fission or fusion

$h.c$	Harmonic conjugate
$C_5H_{11}NO_2$	Chemical symbol for a Valine molecule
m_{ν_e}	Mass of neutrino
$e^+ e^-$	Positron-electron pair
T	Temperature
T_o	Present temperature
MeV	Mega-electron-volt
g_i	Particle spin states
$\boldsymbol{p}$	Particle momentum
G_F	Fermi coupling constant
T_ν	Neutrino temperature
n_ν	Neutrino number density
ρ_m	Cosmological energy density by matter
H_o	Hubble parameter
M_{cl}	Mass of cluster
r_{cl}	Cluster radius
σ	Velocity dispersion
$ITEP$	Institute of Theoretical and Experimental Physics
LSS	Large scale structure
$COBE$	Cosmic Background Explorer
$CMBR$	Cosmic Microwave Background Radiation
Z	Redshift

HDM	Hot Dark Matter
CDM	Cold Dark Matter
MDM	Mixed Dark Matter
ΛCDM	Cosmological Constant Cold Dark Matter
FRW	Friedman-Robertson-Walker metric
$\sum$	Summation symbol
Ψ	Wave or field function
$\psi_{L,\,R}$	Chiral field function
T_3	Isospinor third component
Y	Hypercharge
Φ	Higgs field
L_m^D	Dirac mass term Lagrangian
$\nu_{L,\,R}$	Neutrino fields
L_i	Weak isospinor doublet
g	Coupling constant
CC	Charged current interactions
NC	Neutral current Interactions
EM	Electromagnetic interactions
CKM	Cabibbo-Kobayashi-Maskawa mixing matrix
θ_W	Weinberg mixing angle
m_D	Dirac mass
m_M	Majorana mass
γ	Gamma matrices

W_μ^i	Weak gauge potential
X_μ	Electromagnetic gauge potential
K	Temperature Kelvin
S	Scale factor
k	Curvature parameter
ρ	Energy density
G	Newton's gravitational constant
t_{Pl}	Planck time
$R_{\mu\nu}$	Curvature tensor
$T_{\mu\nu}$	Perfect fluid energy tensor
$V_T(\phi)$	Effective potential
$d\tau$	Space-time interval
A	Action
V	Potential energy
E	Energy
it	Imaginary time
$\ddot{x}$	Rate of change of particle's velocity
P_b	Barrier penetration amplitude
SSB	Spontaneous Symmetry Breaking
m_{Pl}	Planck mass
f_i	Distribution function
Γ_i	Interaction rates

t_D	Decoupling time
T_D	Decoupling temperature
μ	Chemical potential
η	Baryon-to-Photon ratio
N_ν	Neutrino flavour
σ_o	Cross-section
$\lvert\psi_i >$	Eigenvector
MNS	Maki-Nakagawa-Sakata Matrix
l_{osc}	Oscillation length
θ_o	Oscillation mixing angle
CP	Charge-Parity symmetry
δ	Phase angle
KamLAND	Kamioka Liquid scintillator AntiNeutrino Detector
MSW	Mikheyev-Smirnov-Wolfenstein effect
P_ν	Neutrino momentum
E_ν	Neutrino energy
Δt	Uncertainty in time
ΔE	Uncertainty in the energy

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CHAPTER ONE

1.1. INTRODUCTION

There are many types of particles in nature. Particles such as the neutron, proton, electron and the neutrino on one hand and the photon, graviton, $W^{\pm}$ and the Z boson on the other hand can be classified into fermions and bosons, respectively. In high energy physics, however, a very large number of particles are encountered and it is useful to further subdivide these main groups according to the types of interaction in which they participate. All electrically charged particles, by virtue of their charge, can interact electromagnetically. Being aware of this, some particles respond only to the weak force; such particles are collectively known as leptons [Burcham and Jobes, 1995]. The electron e^{-}, electron-neutrino ν_e, muon μ^{-}, muon-neutrino ν_μ, the recently discovered tauon τ and tauon-neutrino [Kodama et al., 2001] are leptonic. All leptons have intrinsic angular momentum of one-half and are therefore fermions. Those particles which can participate in the strong interactions are known as hadrons. Unlike the leptons, which are all fermions, the hadron family contains both fermions and bosons. The hadrons with half-integer spin are also known as baryons and, amongst these, the neutron and proton are the most familiar. The mesons, originally named because they have masses intermediate between the light or zero-mass leptons and the heavier baryons, are bosons [Lewis, 1996].

In the general classification of particles, two particles considered most elementary are the electron and the neutrino. The fundamental properties of the electron are well established: it has a mass of 0.511 *MeV*, is negatively charged (-) and has an intrinsic angular momentum of one-half. The proper establishment of the electron's properties has enabled it to find wide applications in the fields of engineering, telecommunication, medicine and many other fields of pure and applied sciences. On the other hand, the properties of the neutrino are not yet fully understood/or established. It is not an easy task determining the properties of a neutral particle like the neutrino since it is not affected by the electromagnetic force that acts on the electron or any other charged particle. This follows from the fact (fully established in this investigation) that the neutrino does not carry any electric charge and is, therefore, affected only by a weak sub-atomic nuclear force of much shorter range than electromagnetism.

To properly describe an elementary particle, its spin, charge and mass must be fully established. Apart from the symmetry properties of intrinsic angular momentum (spin) and mass, other symmetry principles exist to cater for charge, baryon number, lepton number and many other quantum numbers associated with elementary particles [Mani and Mehta, 1988]. In particle reactions, leptons can be created or annihilated in particle–antiparticle pairs [Burcham and Jobes, 1995]. Evidence for lepton number conservation comes from neutrino reactions and from beta decay studies. The neutrino has its spin opposite to its direction of motion while the antineutrino has parallel spin and momentum i.e., the neutrino is left-handed and the antineutrino is right-handed in helicity. This situation is compatible with lepton number conservation only if the neutrino has no mass [Wick and Barry, 2000]. This is because a massive neutrino would change or decay into a less massive or massless neutrino and a lepton number would not be conserved [Hitoshi, 2002]. Particle-antiparticle conjugation will reverse the signs of all of a particle's additively conserved quantum numbers. The neutrino is immediately seen to be quite interesting then as it lacks charge, magnetic moment or any other measured quantum number that would necessarily reverse under such an operation. It is unique among the leptons and quarks in that the existence of a distinct antineutrino still remains an open question [Utpal, 2008].

All charged particles are Dirac particles, but the neutrino being a neutral particle, could be either a Dirac or a Majorana particle [Peccei, 1989]. A neutrino with a distinct antineutrino could be a Dirac neutrino and has a four–component field, whereas a two–component field will describe the possibility that the neutrino is its own antiparticle and hence a Majorana neutrino. The main difference between Dirac and Majorana neutrinos lies in their lepton number conservation or violation. If neutrinos are Dirac particles, lepton number will be conserved while for a Majorana neutrino, lepton number is violated by two units since they are their own antiparticles. Only the mass term in the Lagrangian can distinguish between a Dirac and a Majorana particle [Cheng and Ling-Fong, 1980]. There are still a lot of investigations going on to establish the true quantum nature of the neutrino.

Among the fundamental properties of the neutrino, the most controversial and interesting one is whether it has a non-zero mass or not [Wick and Barry, 2000]. The mass of a particle is known to contribute to its total energy because mass is just another form of energy. But also, mass may be converted into energy and vice versa and indeed a huge amount of energy may be produced from a relatively small mass. For instance, Fermi's calculation in 1931 aimed at producing results that agreed with experiment based on the fact that the end-point of a continuous distribution of the electrons emitted in beta decay as a function of temperature was sensitive to mass [Fermi, 1934]. He realized that it is possible to plot the spectrum in such a way that a non-zero mass would be revealed as a distortion at the endpoint tangent to the energy axis.

The standard Weinberg–Salam-Glashow model [Burcham and Jobes, 1995] which, henceforth, is coded as $SU(2)\times U(1)$, is the gauge group that describe particle interactions. It indicates that neutrinos are massless since they are described by the two component Weyl fields and because the symmetry breaking Higgs structure of the theory leads to the global symmetry corresponding to the lepton number L conservation. According to this model, there are no right-handed neutrinos that could combine with the left-handed neutrinos to form a Dirac mass term and lepton number conservation forbids Majorana masses for neutrinos [Cottingham and Greenwood, 1998]. The same applies in the grand unified theories (GUT_s) where the restriction coming from lepton number conservation is replaced by the baryon number minus lepton number symmetry $(B-L)$. The proton is seen to be no longer stable and therefore can decay. Symmetries giving rise to L and $(B-L)$ conservation are broken and neutrinos should be massive. If neutrinos are massive, then there must be some reason that their masses are smaller than those of other leptons and quarks [Hitoshi, 2002]. In the proposed extensions of the standard electroweak model, neutrinos can then be regarded as strange and at the same time special because they can have both Dirac and Majorana masses [Pati and Salam, 1974; Georgi and Glashow, 1974; Fritzsch and Minkowski, 1975; Buras, Ellis, Gaillard and Nanopoulos, 1978 and Ellis, 1980]. This is the basis of the seesaw mechanism and, in these investigations it is seen to arise naturally in models with both Dirac and Majorana masses.

The energy scales of the various neutrino reactions that occur on the surface of the earth are relatively low [Michael and Robert, 1992]. At these energies, the fundamental forces of nature are totally different from one another because the particles that transmit them have very different properties. However, above the energy scales of 10^2 GeV the electromagnetic and the weak forces become indistinguishable. They take on similar identities. Such phenomena have been observed in the large particle accelerators especially at CERN and FERMILAB. This observed similarity between the electromagnetic and weak forces suggests that the forces might be two components of a more fundamental force. The two forces are seen to become unified into the electroweak force above $\sim 10^2$ GeV [Guth and Weinberg, 1981]. On the other hand it is known from the kinetic theory of sub-atomic matter that temperature is just a measure of the amount of kinetic energy present; i.e. higher temperatures correspond to higher kinetic energies. Furthermore, since energy and mass are equivalent, then mass should be related to temperature so that for every mass there is a corresponding energy and temperature. The masses of the electroweak gauge particles correspond to the temperature of 10^2 GeV and the particles are found to have two types of energy associated with them: energy stored in their mass and energy determined by how fast they are moving-simply known as kinetic energy. As the temperature increases, so is their kinetic energy. The energy stored in the mass remains fixed however, because the masses of the particles are assumed to be constant.

At very high temperatures, the kinetic energy of the particles dominates the energy contained within their mass. As a result, the particles will behave as if they had no mass at all. For the weak vector boson, $W's$ and Z particles, the kinetic energy begins to dominate at ~10^2 GeV. Thus these particles will behave as if they were massless above this temperature just like the electromagnetic vector boson, the photon. The unification of electromagnetism with the weak force has a consequence that there will only be three effective forces-the strong nuclear force, electroweak force and gravitational force-in operation above 10^2 GeV [Ellis and Gary, 1979]. However, the energy scales of 10^2 GeV represents the highest energy scale that can be probed directly by current particle physics accelerators. By moving above this energy scale, the realm of theoretical considerations is entered. John Ellis and other researchers actively involved in grand unification studies have proposed that the strong and the electroweak forces should become unified at around 10^{13} GeV to 10^{16} GeV into a single unified force under the 'grand unified theory' [Pati and Salam, 1974, Georgi and Glashow, 1974; Fritzsch and Minkowski, 1975; Buras, Ellis, Gaillard and Nanopoulos, 1978; Ellis, 1980]. This is the motivation behind the construction of the Large Hadron Collider (LHC) at CERN. However, to date, the precise form of the GUT theory of particle physics is still not settled because the higher temperatures at which it becomes possible to observe the phenomenon of grand unification are beyond the reach of the most powerful particle accelerators in the world today. But this hasn't stopped researchers, especially theoretical high energy physicists, from imagining what kind of a force should be in operation. The strongest motivation is that the imagined force must contain the strong, weak and electromagnetic forces as components. One testing ground in which the predictions of these grand unified theories may be investigated is expected to be found in the super hot conditions that were prevalent in the early universe-the big bang. At sufficiently early times the temperatures would have been above the temperature at which the GUT force takes over. If the unification idea is correct, then this force should have dominated the universe at these early times and it would have had a significant effect on the way in which the universe came to be what it is like today. This basically suggests that the grand unified theories can be probed by combining ideas from elementary particle interactions at very high energies and the big bang cosmology.

In this work, the interplay between elementary particle interactions at very high energies and the expansion of the early universe is studied in the context of the inflationary model to find a possible solution to the observed flatness of the universe that remains a cosmological puzzle. In the investigations, the puzzle has been attributed to existence of unseen matter in the universe that can be established from the validity of Newton's theory of gravity and Einstein's relativistic generalization, which cannot be tested directly at galactic or even cosmic distance scales [Georgi, 1991]. In particular, the velocity of stars beyond the luminous part of galaxies does not decrease with the distance as is expected from elementary considerations [Donald, 2000]. Therefore it is prudent to remain open to the possibility that the dark matter problem may be resolved by a suitable consideration or modification to the established conservation laws.

Particle dark matter is necessary since the inflationary model predicts that the universe is at the critical energy density state and baryons can contribute at most five percent of this energy density [Keith, 1984]**.** These dark matter particle candidates can be baryonic or nonbaryonic. Here, focus has been on the possibility that a major component of the dark matter could consist of subatomic non-baryonic weakly interacting massive particles that could be remnants of the big bang. The most promising subatomic elementary particle candidates for the dark matter problem include light or heavy neutrinos, axions, supersymmetric particles, monopoles and black holes with different masses and number densities [Graciela, 1987]. A nontrivial advantage for considering massive neutrinos as the subatomic weakly interacting massive dark matter candidate over the other proposed candidates is that neutrinos are known to exist. They too have a kinematical advantage over other dark matter candidates in that they may cluster on larger scales where the dark matter component is highly needed.
Because of the extreme symmetry of the homogenous and isotropic universe, the gravitational field acting on a mass located at a point depends on the masses within a sphere of the universe. But also, since the masses within a sphere act on the origin, the effects of

external masses will cancel themselves out since the masses are uniformly distributed. This will lead to an expanding universe. What is the expected fate of this expansion [Lidsey, 2000]? Will a phase transition with non-negligible viscosity slow down the expansion and finally reverse [Papini and Weiss, 1985]? It is known from the theory of heat that thermal gradients spontaneously decrease with time and sooner or later, the large astronomical temperatures would disappear. Matter would then reach an isothermal state, representing the maximum entropy with no free energy to sustain motions or life. The universe is then bound to die of isothermy [Hubert, 1987]. In order to see what role a neutrino may play in an expanding homogenous and isotropic universe, the standard big bang model predicts a contribution to the mass density of the universe, now of $\rho_{\nu} = 0.7\rho_{\gamma}$ where ρ_{γ} is the energy density of the photons for the redshifted cosmic micro-wave background radiation at temperature $T = 3K$ if neutrino is massless. For neutrino of non-zero mass, a contribution comparable to critical density results if the sum of the masses for three neutrino flavours is as little as 25 eV [Lubimov et al., 1980]. This implies that the neutrino masses within the existing experimental limits could close the universe and ultimately cause it to recollapse [Wick and Barry, 2000].

Before generation of baryon asymmetry in the early universe, matter and anti-matter were equally represented leading to zero chemical potential for all particles. Baryosynthesis, that is the occurrence of phenomena leading to a small excess of matter over antimatter, may have taken place at the grand unified theory period thereby resulting in non-zero chemical potentials. Without this event, essentially all the quarks would have annihilated later on, giving rise to no baryonic matter excess to display bound state energy spectra associated with nuclear and electromagnetic forces. In fact some heavy particles would have been preserved, as the annihilation processes would have gone out of equilibrium at low temperatures, but their number density would have been far too small for galactic and stellar formation to have been initiated later on. As time went on, the coupling constants of the four forces evolved differently. This is a clear indication that, if much of the history of the universe were to be described by equilibrium thermodynamics only, then the universe today would have been a very boring place. A number of crucial departures from equilibrium have taken place during the history of the universe. Good examples include photon decoupling, primordial nucleosynthesis, baryogenesis and perhaps even an inflationary phase transition [Scott and Michael, 1992]. The departure from equilibrium that has attracted attention in this study has involved neutrinos and the weak interactions in the early expanding universe. Since it is natural to consider the possibility of baryonic dark matter, the light emitting regions are seen, in terms of the cosmological energy density parameter, to account for $\Omega \approx 0.01$, the haloes of galaxies for $\Omega \approx 0.10$ at the largest scales(superclusters) the detected dark matter density reaches $\approx 0.2 \pm 0.1$ while inflation predicts that $\Omega = 1$. If indeed $\Omega = 1$, the remaining $(1-\Omega_{observed}) = 0.8 \pm 0.1$ which is not gravitationally detected should be in a smoothly distributed component that is not bound to any observed structure. The dark matter in the disk, the halos, in clusters of galaxies or in a smooth component in the universe, may all be different. The only hint of how much baryonic matter is in the universe comes from the big bang nucleosynthesis of light elements. From combined bounds on the abundance of ^4He, D^3, ^3He and ^7Li, present calculations infer the bound $0.0097 \le \Omega_B h^2 \le 0.016$ [Georgi, 1991]. It is then seen that Ω_B may be as large as 0.007 for $h = 0.7$. Thus primordial nucleosynthesis does not preclude the possibility that the dark matter consists of baryons.

In the conventional hot big bang model the neutrino has been found to exist in thermal equilibrium with all other particles at sufficiently early times when the various interaction rates exceeded the cosmological expansion rate. When this condition first fails at the temperature $(T = 1\,MeV)$, the neutrinos decouple or freeze out [Edward, 1986]. If the neutrino is non-relativistic at freeze-out, its number density will be reduced by a Boltzmann factor and this will imply that the cosmological energy density parameter will decrease as a power-law of the neutrino mass. This makes it necessary that the evolution of the universe depend crucially on the total matter density which is equal to the critical energy density. Out of this critical energy density, seventy percent comes from the vacuum energy also called dark energy or the cosmological constant, which is also being actively researched on. Baryonic and nonbaryonic matter account for about thirty percent; out of these, only five percent is baryonic matter and twenty-five percent is nonbaryonic. By numerical methods of Successive Approximation technique, the Boltzmann equation is set up and eventually solved in the context of the inflationary model in the early expanding universe. This is found to yield a neutrino mass value that is unique and interesting.

It has also been established that two neutrino states with slightly different masses and with the same quantum numbers can oscillate into each other. If they differ in any quantum numbers, that distinguishing symmetry will be broken in the process. However, in a matter-antimatter oscillation, charge and charge-parity symmetries are violated but in the neutrino oscillation, one of the neutrino species with definite lepton flavor will be transformed into another species violating lepton number. This is only possible if neutrinos have mass.

Precisely, if there is a process involving an electron, then it is automatic that an electron-neutrino will also been produced. When this neutrino propagates, it will be the physical state that will propagate and after some time, it will have a certain probability to be partly in a different flavor state, say, muon neutrino or tauon neutrino. When this neutrino is detected at a given distance, a neutrino with a different flavor may be found. If the two states have same masses, then both of them will propagate the same way and there will not be any oscillation. However, if the two states have different masses but the flavor states are the same as the physical states (no mixing), then the flavor states will have definite evolution and they will not change to other states. Thus neutrino oscillations can only occur if there is a mass difference between the different neutrino states and that the mass eigenstates are different from the flavour eigenstates, which is given by the neutrino mixing matrix. Consequently neutrino oscillations, through the Long Baseline Neutrino Experiments, will be an interesting observation that will provide solid experimental evidence to the result.

1.2. STATEMENT OF THE PROBLEM

A massive neutrino is increasingly becoming popular in high energy physics, especially in particle physics and cosmology. Recently, an increased number of investigations have reported that a neutrino has mass. In spite of these investigations, the exact absolute neutrino mass value is still vague and controversial. Consequently, further clarification of these proposals calls for more detailed study, especially on neutrino mass generation mechanism and oscillations to properly settle the controversy surrounding a massive neutrino. Investigations on neutrino weak interaction rates in the early universe and the expansion of the universe will help in gaining additional insight that will be crucial in determining the absolute mass of a neutrino and its potential role in a flat universe.

1.3. JUSTIFICATION OF THE STUDY

The discovery of a particle, the electron and radiation *(X-rays)*, in 1895 was driven by human curiosity but has ended up transforming the modern world. Consequently, in the context of this reasoning, it is hoped that other particles could be discovered with similar far-reaching and initially unanticipated benefits. In particular the discovery of new long-lived particles that can catalyze nuclear fusion or proton decay will have the potential to provide a limitless supply of energy and an improved crystallographic/medical imaging. Hence the justification for massive neutrino study is to properly establish and understand its fundamental properties that may shed more light on its unanticipated potential use in other areas of science and technology and to peer into sources that would be opaque to conventional/standard probes such as photons and protons or neutrons. For instance, at high energies, photons or gamma rays and protons are not viable probes since (i) photons can be absorbed on the infrared extragalactic background i.e., they are blocked by the cosmic microwave background radiation and (ii) protons with energies less than $10^{19}\,eV$ do not point back to their origin because galactic magnetic fields bend them significantly. They can therefore be degraded/absorbed by the cosmic radiation field because of photo-pair production. With high energy neutrino detector, a new "window" can be opened on the universe. New window, in this context, mean new discoveries. This necessitates a need for a new and weakly interacting particle that cannot lose information so easily. The success of such discovery depends on comprehensive knowledge on the physical properties, especially the mass of a neutrino. At the moment there is scarcity of such knowledge. Therefore, the study aims at shedding additional light on basic neutrino mass generation mechanism and the establishment of the absolute neutrino mass. Such information is necessary for identification of a neutrino as a possible dark matter candidate. Results of the weak interaction rates in a critically expanding universe will give a fundamental parameter which is essential for the description of the neutrino.

1.4. SIGNIFICANCE OF THE STUDY

The fundamental scientific motivation for high energy neutrino study is that neutrinos can come from cosmological distances and they can escape from optically thick sources. This implies that neutrino-based probes will allow us to observe what we cannot with other conventional detectors. In particular, we will be able to observe sources at cosmological distances. This may include observation of neutrinos from both diffuse and nearly point-like sources, whose measurements of their flux, energy spectrum, angular distribution and timing will be a fundamental observation of the universe. All these cannot be achieved if the fundamental properties of the neutrino are not well established.

Neutrinos also play a very important role in some branches of subatomic physics as well as astrophysics and cosmology. As established from this study, the smallness of the neutrino mass is very likely related to existence of a new, yet unexplored mass scales in High Energy Physics. These scales are so high that their direct experimental study may require supersensitive and high precision machines beyond the present Large Hadron Colliders. Neutrinos can therefore provide us with very valuable, though indirect, information about these mass scales and the new physics related to these scales. This is in line with our quest for the fundamental theory of nature (TOE).

Neutrinos play a very significant role in astrophysics and cosmology. They carry away up to 99% of the energy released in supernova explosions and therefore dominate the supernova energetic. This ensures a better distribution of energy within the universe.

Neutrinos are copiously produced in thermonuclear reactions which occur in the stellar interiors and in particular our sun [Tayler, 1970]. Solar neutrinos carry information about the core of the sun which is inaccessible to direct optical observations. The detection of solar neutrinos has, actually, confirmed the hypothesis that the sun is powered by thermonuclear reactions [Utpal, 2008]. At the same time, the sun and supernova gives us a possibility of studying neutrino properties over extremely long baselines and probe the neutrino mass differences as small as $10^{-5}eV$ or even smaller, beyond the reach of the terrestrial neutrino experiments.

The big bang nucleosynthesis model depends sensitively on neutrino interactions and number of light neutrino species [Edward, 1986]. Neutrinos of mass of a few eV could constitute the hot dark matter candidate that may be important for galaxy formation. Neutrinos may also have played an important role in baryogenesis i.e. the observed excess of baryons over antibaryons in the universe may be related to decays of heavy Majorana neutrinos [Utpal, 2008].

A beam of neutrinos can also be used to study the earth's interior core which is not possible with other conventional detectors. Although measurements of the seismic waves produced by earthquakes can be used to reconstruct a profile of the Earth's interior, they only provide indirect information. Since neutrinos are electrically neutral and only interact weakly with other particles, they can pass through thousands of kilometers of matter without being absorbed [Wick and Barry, 2000]. However, they can change flavor or oscillate as they pass through matter with, for instance, electron neutrino oscillating into muon neutrino and so on.

Since the amount of oscillation depends on the electron density in the matter, and since the electron density is directly related to the overall matter density, it should be possible to determine the matter density of the earth by making accurate measurements of the oscillations. The experimental challenges would be to build a neutrino factory with a vertical decay tunnel so that the neutrino beams can pass down through the centre of the earth e.g., a beam of neutrinos can be sent from an accelerator tens of thousands of kilometers through the earth to a detector on the other side of the globe. Existing neutrino beams are only a few degrees below the horizontal, whereas the neutrino beam that may be required would have to travel directly downwards. It is possible that such an experiment may begin by 2035

1.5. OBJECTIVES OF THE STUDY

The aim of the study was to find out if neutrinos can control the presently observed expansion of the universe. The specific objectives of the study were to investigate:

- Neutrino mass generation through spontaneous symmetry breaking as a phase transition.
- Why the standard Higgs mechanism does not endow a neutrino with mass in the context of the standard electroweak model.
- The possible neutrino mass that can provide for a critically expanding universe.
- Whether the determined mass can aid in the unification of the fundamental interactions.

CHAPTER TWO

LITERATURE REVIEW AND NEUTRINO MODELS

2.1. INTRODUCTION

In this chapter a brief review of the scientific literature reporting on neutrino mass is presented. Since various investigations concurrently approach the problem from the view point of the standard electroweak as well as the hot big bang model, then the chapter is intended to capture some major results pertaining to neutrino mass investigations in the context of the two broader models. In particular, neutrino kinematics in the standard electroweak model is presented, demonstrating the way in which the various neutrino quantum numbers are related to the Lagrangian. This is followed with a brief description of the neutral and charged current neutrino interactions from the perspective of gauge theories. The problem of mass and the chiral nature of neutrinos, spontaneous symmetry breaking and mass generation are also discussed. In the context of cosmology, the qualitative and quantitative aspects of the inflationary model are discussed. This critical nature of the universe is also established by directly analyzing the dynamical Friedman equation for the various curvature parameter values. This is followed with a presentation on the flat model that has been formulated and adopted for the work. The Boltzmann transport equation for neutrino interactions in a homogenous and isotropic expanding universe is also presented. Further, a model on neutrino oscillation is presented, explicitly establishing that the oscillation probability is very much dependent on neutrino mass.

2.2. NEUTRINOS IN THE $SU(2)\times U(1)$ MODEL

The neutrino was first postulated in 1931 by Wolfgang Pauli (an interesting and more detailed discussion can be found in Laurie Brown's article; Brown, 1978) to preserve the conservation of energy and angular momentum in beta decay-the decay of an atomic nucleus (not known to contain or involve the neutron at that time) into a proton, an electron and, now, an antineutrino. He theorized that an undetected particle was carrying away the observed difference between the energy and momentum of the initial and final particles. He originally named his proposed light particle a neutron. When James Chadwick discovered a much more massive nuclear particle in 1932 [Chadwick, 1932], he also named it a neutron. This left the two particles with the same name. Enrico Fermi, who developed the theory of beta decay, coined the term neutrino in 1934 as a way of resolving the confusion. It is the Italian equivalent of “little neutral one”. In the July 20, 1956 issue of Science, Clyde Cowan and his research collaborators published a confirmation that they had detected the neutrino [Cowan et al., 1956], a result that was rewarded almost forty years later with the 1995 Nobel Prize. In this experiment, now known as the Cowan-Reines neutrino experiment, neutrinos created in a nuclear reactor by beta decay are shot into protons producing neutrons and positrons. The positron quickly finds an electron and they annihilate each other. The two resulting gamma rays are detectable. The neutron can be detected by its capture on an appropriate nucleus, releasing a gamma ray. The coincidence of both events-positron annihilation and neutron capture- gives a unique signature of an antineutrino interaction. In 1962 Leon M Lederman and other researchers [Lederman et. al., 1962] showed that more than one type of neutrino exists by first detecting interactions of the muon neutrino, which earned them the 1988 Nobel Prize. When the third type of lepton, the tau, was discovered in 1975 [Martin, 1975] at the Stanford Linear Accelerator Centre, it too was expected to have an associated neutrino (the tau neutrino). First evidence for this third neutrino type came from the conservation of missing energy and momentum in tau decays analogous to the beta decay leading to the discovery of the neutrino. The first detection of tau neutrino interactions was announced in summer of 2000 by the DONUT collaboration at Fermilab, making it the latest particle of the standard electroweak model to have been directly observed [Kodama et al., 2001].

To date the $SU(2)\times U(1)$ Model of particle physics assumes that the neutrino should be massless [Hitoshi, 2002]. However, in order for the anticipated phenomenon of neutrino oscillation to occur, the neutrino should have a nonzero mass. This idea was originally conceived by Bruno Pontecorvo in the 1950s [Bruno, 1958]. He pointed out that neutrinos created or detected with a well defined flavour are able to oscillate between the three available flavours while propagating through space. This occurs because the neutrino flavour eigenstates are not the same as the neutrino mass eigenstates and, hence, allows for a neutrino that was produced as an electron-neutrino at a given location to have a calculable probability to be detected as either a muon- or tauon- neutrino after it has travelled to another location. This quantum mechanical effect was first hinted by the discrepancy between the number of electron neutrinos detected from the sun’s core failing to match the expected number, dubbed “the solar neutrino problem” [Bahcall et al., 1998]. In 1998, research results at the Super-Kamiokande neutrino detector determined that neutrinos do indeed flavor oscillate and, therefore, have mass. While this shows that neutrinos have mass, the absolute neutrino mass scale is still not known. This is because neutrino oscillations are sensitive only to the difference in the squares of the masses. In particular, the best estimate of the difference in the square of the mass eigenstates was published by KamLAND in 2005 with a value $\Delta m_{21}{}^2 = 0.000079\, eV^2$ [Araki et al., 2005]. In 2006, the MINOS experiment measured oscillations from an intense muon neutrino beam determining the difference in the squares of the masses between neutrino mass eigenstates. The reported result is $\Delta m_{32}{}^2 = 0.0027\, eV^2$ [Fermilab's MINOS collaboration, 2006].

In the concerted efforts to determine the absolute neutrino mass scale, most experimentally-oriented researchers have selected tritium which has an energy release of 18.6-keV at the end point. In particular, an early tritium measurement by Hamilton and other researchers [Hamilton et al., 1965]* found an upper limit of $m_{\bar{\nu}_e} \leq 250 keV$. In 1972 Bergkvist, by combining electrostatic and magnetic spectrometric methods [Bergkvist, 1972] was able to reduce the limit substantially to $m_{\bar{\nu}_e} \leq 60 eV$. Then in 1980 Lubimov and other researchers [Lubimov et. al, 1980] at the ITEP, using a high-precision toroidal spectrometer and tritium in the form of a valine molecule $(C_5H_{11}NO_2)$ reported a nonzero mass in the range $14 eV \leq m_{\bar{\nu}_e} \leq 46 eV$; a result that set off a flurry of new high precision experiments.

Stimulated by Lubimov's result, several groups attempted improved versions of this experiment. A group of researchers headed by Robertson [Robertson et al., 1991], at Los Alamos used a much simpler source, that is, gaseous tritium molecules, tackling at the same time the serious safety issues associated with handling a kilocurie of this gas. After a series of measurements with a carefully constructed magnetic spectrometer that filled an entire room, they found $m_{\bar{\nu}_e} \leq 9.3 eV$. An experiment at Mainz using a frozen tritium source reported [Weinheimer, 1993] nearly a similar limit, $m_{\bar{\nu}_e} \leq 7.2 eV$ and a Livermore group using gaseous tritium and a toroidal magnetic spectrometer achieved comparable statistics [Stoeffi and Decman, 1995]. In 2009 lensing data of galaxy cluster were analyzed and predicted [Nieuwenhuizen, 2009] a neutrino mass of about 1.5 *eV*. This lies below the Mainz-Troitsk upper bound of 2.2 *eV* for the electron anti-neutrino as reported earlier [Weinheimer, 2002]. All these are set to be tested by 2015 in the KATRIN experiment that searches for a mass between 0.2 *eV* and 2 *eV*.

In spite of the interesting mass values calculated or measured so far, none of these can be considered to be consistent. It is also clear that in spite of the relative role that the neutrino seems to play in the standard electroweak model ($SU(2)\times U(1)$), the model still assumes that neutrinos are massless and the chiral fields

$$\Psi_L = \frac{1}{2}(1-\gamma_5)\Psi \qquad (2.2.1)$$

and

$$\Psi_R = \frac{1}{2}(1+\gamma^5)\Psi \qquad (2.2.2)$$

have different $SU(2)$ properties, with Ψ_L being part of $SU(2)$ doublets and Ψ_R being a singlet [Abers and Lee 1973]. From equations (2.2.1) and (2.2.2), Ψ is the neutrino wave function, γ_5 is the Pauli-Dirac matrix and γ^5 is its transpose. Also the electromagnetic gauge group's properties, which is coded as $U(1)$, is fixed by the electromagnetic charge identification [Hollik, 1999]

$$Q = T_3 + Y \qquad (2.2.3)$$

with T_3 being the third component of the $SU(2)$ generator and Y being the $U(1)$ generator. Since the right handed neutrino field ν_{Ri} has no $U(1)$ interactions, the fields are sterile under $SU(2)\times U(1)$ and the electroweak interactions act only on the left handed neutrino ν_{Li}. Though this might be a good idea, the statement of neutrino sterility may not be totally true since a $\nu_R - \nu_L$ Higgs coupling can exist. The Higgs doublet Φ whose vacuum expectation value is responsible for the breakdown of $SU(2)\times U(1)\rightarrow U(1)_{em}$ will then carry weak hypercharge of $-\frac{1}{2}$ so that its quantum numbers becomes

$$\Phi \sim \left(2,\ -\frac{1}{2}\right) \qquad (2.2.4)$$

Thus an invariant coupling of Φ with a

$$\bar{\nu}_{Li} \sim \left(2,\ +\frac{1}{2}\right) \qquad (2.2.5)$$

and

$$\nu_{Ri} \sim (1,\ 0) \qquad (2.2.6)$$

is allowed by $SU(2)\times U(1)$ [Peccei, 1988]. When the electroweak model breaks into its components-

$$SU(2)\times U(1)\rightarrow U(1)_{em} \tag{2.2.7}$$

and the scalar field Φ acquires a nonzero vacuum expectation value $\langle\Phi\rangle$, the Yukawa couplings generates a Dirac mass term for the neutrino. However, since in the standard model no observational evidence exists at present for a Dirac mass matrix with associated eigenvalue $(m_D)_{\nu i}$, then the mass term should vanish; that is

$$\begin{aligned} L_{mass}{}^{Dirac} &= -\bar{\nu}_{Li}(m_D)_{ij}\nu_{Rj} - \bar{\nu}_{Ri}(m_D{}^*)_{ij}\nu_{Lj} \\ &= 0 \end{aligned} \tag{2.2.8}$$

2.2.1 CHARGED AND NEUTRAL CURRENT INTERACTIONS OF NEUTRINOS

In $SU(2)\times U(1)$, the right handed neutrinos are ignored and only their left handed counterparts considered. These neutrinos with the corresponding left handed charged leptons form an $SU(2)$ doublet [Lahanas, 1987]

$$L_i = \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_L \tag{2.2.1.1}$$

Their interactions arise from replacing in the fermion kinetic energy terms, the ordinary derivatives with the covariant derivatives so that the Lagrangian can take the form

$$L = i\bar{L}_i\gamma^{\mu}D_{\mu}L_i\,, \tag{2.2.1.2}$$

where $D_\mu L_i$ expands out to

$$D_\mu L_i = \left[\partial_\mu - ig\frac{\tau_a}{2}W_a{}^{\mu} + i\frac{g^{'}}{2}Y^{\mu}\right]L_i\,, \tag{2.2.1.3}$$

with $W_a{}^{\mu}(a=1,2,3)$ and Y^{μ} being the $SU(2)$ and $U(1)$ gauge fields, g and $g^{'}$ are their respective coupling constants. It follows from equations (2.2.1.2) and (2.2.2.3) that the neutrino interaction Lagrangian can be written

$$L_{\text{int}} = gW_a{}^{\mu}J_{a\mu} + g^{'}Y^{\mu}J_{\mu}\,, \tag{2.2.1.4}$$

with the $SU(2)$ and $U(1)$ currents containing neutrinos being given by

$$J_a{}^{\mu} = \bar{L}_i\gamma^{\mu}\frac{\tau_a}{2}L_i$$

and

$$J^{\mu} = -\frac{1}{2}\bar{L}_i\gamma^{\mu}L_i \tag{2.2.1.5}$$

To describe the physical interactions of neutrinos in the $SU(2)\times U(1)$ model, two effects resulting from the

$$SU(2)\times U(1)\rightarrow U(1)_{em} \tag{2.2.1.6}$$

breaking down are taken into account:

- The mass and charge eigenstates for the gauge fields are linear combinations of the $W_a{}^{\mu}$ and Y^{μ} fields, with the physical excitations being defined by [Peccei, 1988]

$$W_{\pm}{}^{\mu}=\frac{1}{\sqrt{2}}(W_1{}^{\mu}\mp iW_2{}^{\mu}) \tag{2.2.1.7}$$

  and

$$\begin{pmatrix} Z^{\mu} \\ A^{\mu} \end{pmatrix}=\begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_3{}^{\mu} \\ Y^{\mu} \end{pmatrix}, \tag{2.2.1.8}$$

where θ_W, the Weinberg angle, relates the coupling constants g and $g^{'}$ to the electric charge e via the unification condition

$$e=g\sin\theta_W=g^{'}\cos\theta_W \tag{2.2.1.9}$$

and A^{μ} is the photon field [Peccei, 1988]. From equation (2.2.1.4), it is then found that

$$L_{\mathrm{int}}=\frac{e}{2\sqrt{2}\sin\theta_W}\left[J_{+}{}^{\mu}W_{-\mu}+J_{-}{}^{\mu}W_{+\mu}\right]+\frac{e}{2\cos\theta_W\sin\theta_W}\left[J_{NC}{}^{\mu}Z_{\mu}\right]+eJ_{em}{}^{\mu}A_{\mu}, \tag{2.2.1.10}$$

where the charged currents (CC) $J_{\pm}{}^{\mu}$, the neutral current (NC) $J_{NC}{}^{\mu}$ and the electromagnetic current (EM) $J_{em}{}^{\mu}$ are defined by

$$\begin{aligned} J_{\pm}{}^{\mu}&=2\left[J_1{}^{\mu}\mp iJ_2{}^{\mu}\right] \\ J_{NC}{}^{\mu}&=2\left[J_3{}^{\mu}-\sin^2\theta_W J_{em}{}^{\mu}\right] \\ J_{em}{}^{\mu}&=J_3{}^{\mu}+J^{\mu} \end{aligned} \tag{2.2.1.11}$$

- As a result of the $SU(2)\times U(1)$ breakdown, quarks and charged leptons acquire mass. However since the quark mass eigenstates are not the same as the weak interaction eigenstates, there exists flavour mixing in the charged current interactions. For the leptonic currents, however, there is no flavor mixing in the limit that neutrino masses are neglected.

The Yukawa interactions of quarks and leptons with the Higgs doublet when

$$SU(2)\times U(1)\rightarrow U(1)_{em} \tag{2.2.1.12}$$

breaks, will produce mass matrices for fermions of the form

$$L_{mass} = -\bar{u}_{iL} M_{ij}{}^{u} u_{jR} - \bar{d}_{iL} M_{ij}{}^{d} d_{jR} - \bar{l}_{iL} M_{ij}{}^{l} l_{jR} + h.c \tag{2.2.1.13}$$

In equation (2.2.1.13), neutrino mass term is not included since, no ν_{iR} fields are considered. The matrices $M_{ij}{}^{f}\,(f = u,\, d,\, l)$ are, in general, not diagonal. However, they can be diagonalized by a basis change on the quark and lepton fields so that the fields can take the form

$$\Psi_L{}^f \rightarrow U_L{}^f \Psi_L{}^f; \qquad \Psi_R{}^f \rightarrow U_R{}^f \Psi_R{}^f \tag{2.2.1.14}$$

This basis change produces interfamily mixing in the charged current and for three families, the charged current $J_{+}{}^{\mu}$, before the basis change, is found to be [Peccei, 1988]

$$J_{+}{}^{\mu} = 2\left(\bar{\nu}_e\ \bar{\nu}_u\ \bar{\nu}_\tau\right)_L \gamma^{\mu} I \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L + 2\left(\bar{u}\ \ \bar{c}\ \ \bar{t}\right)_L \gamma^{\mu} I \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{2.2.1.15}$$

After the basis change the current then becomes

$$J_{+}{}^{\mu} = 2\left(\bar{\nu}_e\ \bar{\nu}_u\ \bar{\nu}_\tau\right)_L \gamma^{\mu} U_L{}^{l} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L + 2\left(\bar{u}\ \ \bar{c}\ \ \bar{t}\right)_L \gamma^{\mu} \left(U_L{}^{u}\right)^{*} U_L{}^{d} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{2.2.1.16}$$

For the quark sector the matrix entering in (2.2.1.16) is the 3×3 unitary Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [Cabibbo, 1963; Kobayashi and Maskawa, 1973]

$$V = (U_L{}^{u})^{*} U_L{}^{d} \tag{2.2.1.17}$$

while for the leptonic sector, the mixing matrix $U_L{}^{l}$ is eliminated by a unitary redefinition of the neutrino fields

$$\nu_L \rightarrow U_L{}^{l} \nu_L \tag{2.2.1.18}$$

since $m_{\nu i} = 0$. The form of the charged current then becomes

$$J_+^{\mu} = 2\left(\overline{\nu}_e\ \overline{\nu}_\mu\ \overline{\nu}_\tau\right)_L \gamma^\mu I \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L + 2\left(\overline{u}\ \overline{c}\ \overline{t}\right)_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$$J_-^{\mu} = \left(J_+^{\mu}\right)^* \qquad (2.2.1.19)$$

Equation (2.2.1.19) shows that neutrino CC interactions do not cause family change transitions as long as it is assumed that $m_{\nu i} = 0$. The basis change equation (2.2.1.14) does not affect the electromagnetic and the neutral currents since what is involved in both currents is

$$\left(U_L^{\ f}\right)^* U_L^{\ f} = \left(U_R^{\ f}\right)^* U_R^{\ f} = 1 \qquad (2.2.1.20)$$

Thus, these currents are diagonal in flavour and by using equation (2.2.1.11), the neutral and electromagnetic fermion currents takes the simple form

$$\left(J_{NC}^{\ \mu}\right)^f = \overline{f}\gamma^\mu\left[Q_L^{\ f}(1-\gamma_5) + Q_R^{\ f}(1+\gamma_5)\right]f \qquad (2.2.1.21)$$

$$\left(J_{em}^{\ \mu}\right)^f = e^f\,\overline{f}\gamma^\mu f \qquad (2.2.1.22)$$

where the chiral charges $Q_{L,R}^{\ f}$ are now defined by

$$Q_L^{\ f} = T_3^{\ f} - e^f \sin^2\theta_W$$

$$Q_R^{\ f} = -e^f \sin^2\theta_W \qquad (2.2.1.23)$$

with $T_3^{\ f}$ and e^f being, respectively, the eigenvalues of the $SU(2)$ generator T_3 and of the electric charge Q for the left handed components of the fermions in question. These corresponds to

$$T_3^{\ f} = \begin{cases} \frac{1}{2} & \nu_i,\ u_i \\ & \qquad ; \\ -\frac{1}{2} & l_i,\ d_i \end{cases} \qquad (2.2.1.24)$$

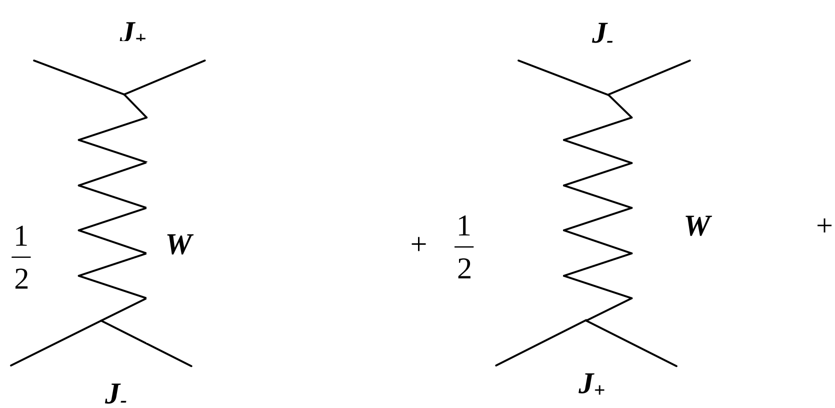
$$e^f = \begin{cases} \frac{2}{3} & u_i \\ 0 & \nu_i \\ -\frac{1}{3} & d_i \\ -1 & l_i \end{cases} \tag{2.2.1.25}$$

For three generations, the neutral current interaction will then become

$$\begin{aligned}\left(J_{NC}{}^{\mu}\right)_{neutrinos} &= \frac{1}{2}\left[\bar{\nu}_e\gamma^{\mu}(1-\gamma_5)\nu_e + \bar{\nu}_{\mu}\gamma^{\mu}(1-\gamma_5)\nu_{\mu} + \bar{\nu}_{\tau}\gamma^{\mu}(1-\gamma_5)\nu_{\tau}\right] \\ &= \bar{\nu}_{eL}\gamma^{\mu}\nu_{eL} + \bar{\nu}_{\mu L}\gamma^{\mu}\nu_{\mu L} + \bar{\nu}_{\tau L}\gamma^{\mu}\nu_{\tau L} \end{aligned} \tag{2.2.1.26}$$

Because the CC, NC and EM interactions of leptons are family diagonal, as in (2.2.1.26), then the standard electroweak model in the absence of neutrino mass terms conserves separately the lepton number for each family.

Except for experiments at the large $p\bar{p}$ colliders at CERN and FERMILAB [The ALEPH Collaboration, 1993] where real $W's$ and $Z's$ have been produced, it is possible that other experimental investigations of the standard model may occur in circumstances where the momentum transfer q^2 is far much less than $M_W{}^2$ and $M_Z{}^2$. In this case, the weak part of the interaction Lagrangian of equation (2.2.1.10) can be replaced by an effective current-current Lagrangian as in figure 2.2.1.1 below



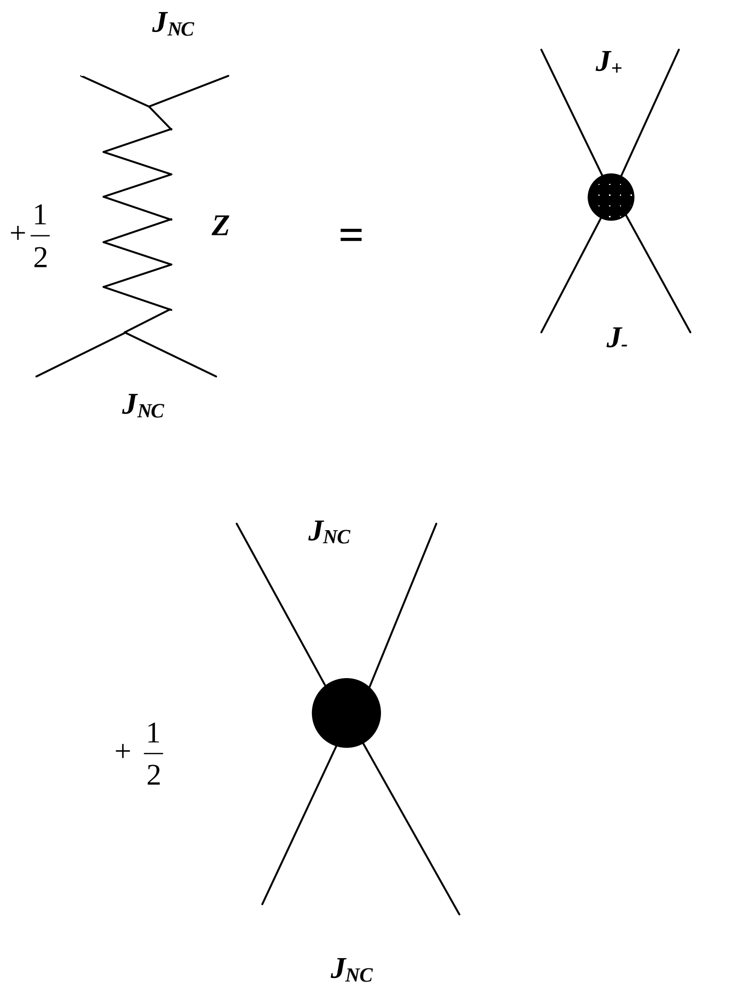


Figure 2.2.1.1: Effective interactions in the low energy scale ($q^2 << M_W{}^2, M_Z{}^2$)

so that the effective Lagrangian takes the form [Peccei, 1988]

$$L_{eff} = \frac{i}{2!}\int L_{\text{int}} L_{\text{int}} \approx \left(\frac{e}{2\sqrt{2}\sin\theta_W}\right)^2 \frac{1}{M_W{}^2} J_+{}^{\mu} J_{-\mu} + \frac{1}{2}\left(\frac{e}{2\cos\theta_W \sin\theta_W}\right)^2 \frac{1}{M_Z{}^2} J_{NC}{}^{\mu} J_{NC_\mu}$$

(2.2.1.27)

For the charged current interactions, comparison with the Fermi theory identifies the Fermi coupling constant G_F as [Lahanas, 1987]

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W M_W{}^2} \tag{2.2.1.28}$$

which provides the formula for the charged gauge boson's mass (W) once $\sin^2\theta_W$ is determined experimentally. Using equation (2.2.1.28) and defining a parameter ρ by

$$\rho = \frac{M_W{}^2}{\cos^2\theta_W M_Z{}^2}, \qquad (2.2.1.29)$$

the effective weak Lagrangian can then be written as

$$L_{eff}{}^{weak} = \frac{G_f}{\sqrt{2}}\left\{J_+{}^{\mu}J_{-\mu} + \rho J_{NC}{}^{\mu}J_{NC_\mu}\right\} \qquad (2.2.1.30)$$

2.2.2. A NEUTRINO MASS IN $SU(2)\times U(1)$ MODEL

Since neutrinos are neutral particles, it is possible that they can have two distinct types of mass term [Zralek, 1997]; either Dirac (a particle-antiparticle mass term) or Majorana (a particle-particle mass term). Only the first kind of mass term is available for the charged leptons and the quarks, since Majorana masses for these particles would violate charge conservation.

On the basis of the transformation properties of the fields ν_L and ν_R under $SU(2)\times U(1)$, neutrinos can get a mass from the Higgs Yukawa couplings. This means that the neutrino mass term that can arise after $SU(2)\times U(1)$ symmetry breakdown, can be written as [Utpal, 2008]

$$L_{mass} = -m_D\left[\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L\right] - \frac{1}{2}m_M^L\left[\nu_L^T C\nu_L + \bar{\nu}_L C\bar{\nu}_L^T\right] - \frac{1}{2}m_M^R\left[\nu_R^T C\nu_R + \bar{\nu}_R C\bar{\nu}_R^T\right]$$

(2.2.2.1)

The equation shows that it is possible to have a Dirac mass m_D plus a Majorana mass for the left handed m_M^L and right handed m_M^R neutrino fields. The Dirac mass m_D arise from the Yukawa coupling of neutrinos with the standard Higgs doublet

$$\Phi = \begin{pmatrix}\phi^0 \\ \phi^+\end{pmatrix} \qquad (2.2.2.2)$$

so that the Yukawa Lagrangian can take the form

$$L_{Yukawa} = -\Gamma^{\nu}(\bar{\nu}\quad \bar{l})_L\Phi\nu_R - \Gamma^{\nu*}\bar{\nu}_R\Phi^*\begin{pmatrix}\nu \\ l\end{pmatrix}_L, \qquad (2.2.2.3)$$

In this case, l will be the lepton associated with the neutrino field ν in question [Peccei, 1988] and

$$m_D = \Gamma^{\nu}\langle\phi^0\rangle \qquad (2.2.2.4)$$

This mass term is lepton number conserving since the Lagrangian L_{Yukawa} is found to be invariant under the transformation

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L \to \begin{pmatrix} \nu \\ l \end{pmatrix}_L^{'} = e^{i\alpha} \begin{pmatrix} \nu \\ l \end{pmatrix}_L$$

$$\nu_R \to \nu_R^{'} = e^{i\alpha}\nu_R \tag{2.2.2.5}$$

The presence of a Dirac mass matrix for neutrinos implies lepton mixing and, since the vacuum expectation value is roughly of the order $\langle\phi^0\rangle \cong 246\,GeV$ [Weinberg S 1976], then to get neutrino masses in the eV range requires Yukawa couplings which are extraordinary small; that is, $\Gamma^{\nu_e} \sim 10^{-10} - 10^{-11}$. In contrast to the Dirac mass m_D which is connected to the scale of the $SU(2)\times U(1)$ breaking of $\langle\phi^0\rangle \cong 246 GeV$, the majorana mass m_M^R is allowed by $SU(2)\times U(1)$ since the right handed neutrinos are singlets. In addition, m_M^R is an independent scale altogether [Linde, 1977] and the presence of m_M^R breaks lepton number with the combinations $\nu_R^T C \nu_R$ and $\overline{\nu}_R C \overline{\nu}_R^T$ having $L = +2$ and $L = -2$ respectively. Although m_M^R is an explicit renormalizable mass term, it can also arise from a Yukawa coupling with some $SU(2)\times U(1)$ singlet Higgs field σ which acquires a non zero vacuum expectation value. This means that [Peccei, 1988]

$$m_M^R = \begin{cases} m_M^R & \text{explicit mass} \\ \dfrac{g_\nu}{\sqrt{2}}\langle\varphi\rangle & \text{spontaneous mass} \end{cases}, \tag{2.2.2.6}$$

where g_ν is a neutrino coupling constant. If φ carries a lepton number, then the Yukawa coupling of σ with ν_R fields can be lepton number conserving and lepton number is only broken spontaneously by the φ vacuum expectation value $\langle\varphi\rangle$. In this latter case, unless lepton number is gauged, a Goldstone boson will appear in this theory. This is usually known as a Majoron [Graciela, 1987].

On the other hand the Majorana mass m_M^L cannot be an explicit mass term as it violates $SU(2)\times U(1)$ but can arise after the breaking of $SU(2)\times U(1)$. If m_M^L is taken to be the result of the renormalisable interactions, then it is necessary to introduce an $SU(2)$ triplet Higgs field $\vec{\Delta}$ in the theory. From the Yukawa interaction that [Schetchter and Valle, 1980]

$$L_\Delta = -\frac{g}{\sqrt{2}}\left[(\nu,\ l)_L^T C\vec{\tau}.\vec{\Delta} \begin{pmatrix} \nu \\ l \end{pmatrix}_L \right] \tag{2.2.2.7}$$

and the assumption that the zero charge component of $\vec{\Delta}$ acquires a non zero vacuum expectation value $\langle \Delta^0 \rangle$, then the Majorana mass term can take the form

$$m_M^L = \sqrt{2} g \langle \Delta^o \rangle \qquad (2.2.2.8)$$

The presence of a triplet Higgs field expectation value, however, changes the prediction for ρ so that [Utpal, 2008]

$$\rho \approx 1 - 2\left(\frac{\langle \Delta^0 \rangle}{\langle \phi^0 \rangle}\right)^2 \qquad (2.2.2.9)$$

The mass scale m_M^L could also arise from L-violating non renormalisable interactions from which

$$L = \frac{1}{\Lambda}\left[\Phi^T C \vec{\tau} \Phi\right] \bullet \left[(\nu, l)_L^T C \vec{\tau} \begin{pmatrix} \nu \\ l \end{pmatrix}_L\right] + h.c \qquad (2.2.2.10)$$

For $\Phi \to \langle \Phi \rangle$, equation (2.2.2.10) yields a Majorana mass

$$m_M^L = \frac{2 \langle \phi^0 \rangle^2}{\Lambda} \qquad (2.2.2.11)$$

By making use of the conjugate spinor ν^C, the general neutrino mass term equation (2.2.2.1) can then be written in a more symmetrical form [Peccei, 1988]

$$\begin{aligned} \overline{(\nu^C)}_L (\nu^C)_R &= \nu_R^T C C \overline{\nu}_L^T \\ &= -\nu_R^T \overline{\nu}_L^T \\ &= \overline{\nu}_L \nu_R \end{aligned} \qquad (2.2.2.12)$$

where C and C^{-T} are charge and anti-charge conjugates respectively Thus,

$$\overline{\nu}_L \nu_R = \frac{1}{2}\left[\overline{\nu}_L \nu_R + \overline{(\nu^C)}_L (\nu^C)_R\right] \qquad (2.2.2.13)$$

and using the definitions [Peccei, 1988]

$$\nu_R^T C \nu_R = \overline{(\nu^C)}_L \nu_R \qquad (2.2.2.14)$$

and

$$\overline{\nu}_L C (\overline{\nu}_L)^T = \overline{\nu}_L (\nu^C)_R \qquad (2.2.2.15)$$

equation (2.2.2.1) can reduce to the form

$$L_{mass} = -\frac{1}{2}\left[\overline{\nu}_L \quad \overline{(\nu^C)}_L\right]\begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix}\begin{pmatrix} (\nu^C)_R \\ \nu_R \end{pmatrix} + h.c \qquad (2.2.2.16)$$

where the matrix

$$K = \begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix} \qquad (2.2.2.17)$$

has two eigenvalues m_1 and m_2, and its eigenstates are Majorana neutrino states. This implies that, it is possible to build a hierarchy between these two eigenvalues if there is a hierarchy in the matrix K. Since m_M^L is connected to the vacuum expectation value of a triplet Higgs field $\vec{\Delta}(\ \langle\Delta^0\rangle \leq 246\,GeV)$ or is inversely proportional to a large scale Λ, a hierarchy via m_M^R can be build by taking $m_M^R \equiv M \ \ >> m_D >> m_M^L \approx 0$ so that equation (2.2.2.17) reduces to

$$K \approx \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \qquad (2.2.2.18)$$

The matrix K is diagonalised via a unitary transformation [Gupta, 2004]

$$U^* K U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad (2.2.2.19)$$

If $M >> m_D$, then

$$U \approx \begin{pmatrix} 1 & \frac{m_D}{M} \\ -\frac{m_D}{M} & 1 \end{pmatrix} \qquad (2.2.2.20)$$

The diagonalisation of the matrix K corresponds to a basis change for the left handed and right handed neutrino fields

$$\begin{pmatrix} (\nu^C)_R \\ \nu_R \end{pmatrix} = U\begin{pmatrix} \nu_{1R} \\ \nu_{2R} \end{pmatrix}$$

$$\left(\overline{\nu}_L \quad \overline{(\nu^C)}_L\right) = \left(\overline{(\nu_1^C)}_L \quad \overline{(\nu_2^C)}_L\right)U^* \qquad (2.2.2.21)$$

from which it is found that

$$\nu_R \approx \nu_{2R} - \frac{m_D}{M}\nu_{1R} \qquad (2.2.2.22)$$

and

$$\nu_L \approx (\nu_1^C)_L + \frac{m_D}{M}(\nu_2^C)_L \qquad (2.2.2.23)$$

Equations (2.2.2.22) and (2.2.2.23) show that ν_R is mostly made up of the heavy neutrino field ν_2, while ν_L is mostly the light neutrino field ν_1 respectively. In terms of ν_1 and ν_2, the mass Lagrangian (2.2.2.16) then becomes [Utpal, 2008]

$$L_{mass} = -\frac{1}{2}m_1\left[\overline{(\nu_1^C)}_L \nu_{1R} + \bar{\nu}_{1R}(\nu_1^C)_L\right] - \frac{1}{2}m_2\left[\overline{(\nu_2^C)}_L \nu_{2R} + \bar{\nu}_{2R}(\nu_2^C)_L\right] \qquad (2.2.2.24)$$

which are Majorana mass terms for these fields. The Majorana fields η_1 and η_2 are then defined by [Peccei, 1988]

$$\begin{aligned}\eta_1 &= \left(\nu_1^C\right)_L + \nu_{1R} \\ &= (\eta_1)^C\end{aligned} \qquad (2.2.2.25)$$

$$\begin{aligned}\eta_2 &= \left(\nu_2^C\right)_L + \nu_{2R} \\ &= (\eta_2)^C\end{aligned} \qquad (2.2.2.26)$$

so that mass Lagrangian (2.2.2.24) reduces to

$$L_{mass} = -\frac{1}{2}m_1\left(\bar{\eta}_1\eta_1\right) - \frac{1}{2}m_2\left(\bar{\eta}_2\eta_2\right) \qquad (2.2.2.27)$$

Consequently, combination of equations (2.2.2.22) and (2.2.2.25) leads to

$$\nu_R \approx \eta_{2R} - \frac{m_D}{M}\eta_{1R} \qquad (2.2.2.28)$$

and

$$\nu_L \approx \eta_{1L} + \frac{m_D}{M}\eta_{2L} \qquad (2.2.2.29)$$

Equations (2.2.2.28) and (2.2.2.29) show that ν_R is mostly η_2 and ν_L is mostly η_1. From equations (2.2.2.27) it is explicit that if m_2 is sufficiently heavy, then it would not be easy to have any evidence for η_2 yet. However, the see saw relation allows a light neutrino η_1 to exist which is essentially the same as the massless left handed neutrino of the standard electroweak model.

2.2.3. SPONTANEOUS SYMMETRY BREAKING AND MASS GENERATION

In the context of the electroweak model, all particles should be massless but in realty matter particles are massive. Particles especially the weak gauge bosons are massive since the weak interaction is of short range. Therefore, unifying *SU(2)* and *U(1)* can only be possible if a mechanism can be found that can give the weak gauge bosons and other particles the required masses but not the photon. One such mechanism is spontaneous symmetry breaking (SSB) which is a very crucial ingredient in the electroweak standard model of Glashow, Weinberg and Salam [Weinberg, 1967; Salam, 1968; Glashow and Georgi, 1972]. In this scheme, the fundamental idea is that the weak interactions should be mediated by gauge bosons $\left(W^{\pm}, Z^{0}\right)$ which are, to begin with, massless. The Lagrangian for the theory also contains terms for massless leptons. A scalar field (the Higgs field) is then introduced with a non-vanishing vacuum-expectation value. The resulting spontaneous breakdown of symmetry gives masses to the charged leptons and gauge bosons, but not to the electromagnetic gauge boson. This is done by considering the free Lagrangian of the form

$$L = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\,\psi \tag{2.2.3.1}$$

which becomes

$$L = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi \tag{2.2.3.2}$$

for $m = 0$. The following projection operators [Feyman and Gell-Mann, 1958]

$$\Psi_{L} = \left(\frac{1-\gamma_{5}}{2}\right)\Psi \text{ and } \Psi_{R} = \left(\frac{1+\gamma_{5}}{2}\right)\Psi \tag{2.2.3.3}$$

are applied so that the Lagrangian (2.2.3.2) takes the form

$$L = i\overline{\psi}\gamma^{\mu}\left\{\left(\frac{1+\gamma_{5}}{2}\right)+\left(\frac{1-\gamma_{5}}{2}\right)\right\}\partial_{\mu}\psi \tag{2.2.3.4}$$

This finally reduces to

$$L = i\,\overline{\psi}\,\gamma\,\partial\psi$$

$$= i\,\overline{\psi_{R}}\,\gamma\,\partial\psi_{R} + i\,\overline{\psi}_{L}\,\gamma\,\partial\psi_{L} \tag{2.2.3.5}$$

Since transformations are between particles whose space-time properties are the same, then the only possibility is a mixing of e_L and ν_e from which the isospinor doublet has to be written as

$$L_{IS} = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \qquad (2.2.3.6)$$

To preserve gauge invariance, the doublet is assigned a non-Abelian weak isospin charge

$$I_W = \frac{1}{2} \qquad (2.2.3.7)$$

so that the neutrino takes

$$I_W{}^3 = \frac{1}{2} \qquad (2.2.3.8)$$

as its third component and the electron

$$I_W{}^3 = -\frac{1}{2} \qquad (2.2.3.9)$$

The other particle $R = e_R$, which is an isosinglet, has

$$I_W = 0 \qquad (2.2.3.10)$$

This will lead the Lagrangian (2.2.3.5) to take the form

$$L = i\bar{R}\gamma\,\partial R + i\bar{L}\gamma\,\partial L \qquad (2.2.3.11)$$

Gauge invariance demands that the Lagrangian L remains invariant under the operators

$$L \to e^{-\frac{i}{2}\tau.\alpha} L \qquad (2.2.3.12)$$

and

$$R \to R \qquad (2.2.3.13)$$

which are rotations in the weak isospin space. This generates the group $SU(2)$ whose transformation matrix is suggested to be of the form

$$SU(2)\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix} \to \begin{pmatrix} e^{-\frac{i}{2}\vec{\tau}.\vec{\alpha}} & 0 & 0 \\ 0 & e^{-\frac{i}{2}\vec{\tau}.\vec{\alpha}} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix} \qquad (2.2.3.14)$$

The relation between the electric charge Q and the third component I is given by the Gell-Mann-Nishijima relation [Lewis, 1985]

$$Q = I_W{}^3 - \frac{1}{2};\ \text{for L}$$
$$Q = I_W{}^3 - 1;\ \text{for R} \tag{2.2.3.15}$$

and since the electromagnetic ($U(1)$) symmetry leads to a conserved charge of which e_R possesses one value and ν_e and e_L the other value, then it is not the electric charge Q as they have different values of Q. This charge is the "Weak Hypercharge" Y_W defined by a quasi-Gell-Mann-Nishijima relation [Stancu, 1996] as

$$Q = I_W{}^3 + \frac{Y_W}{2} \tag{2.2.3.16}$$

From equation (2.2.3.15) the following quantum numbers can easily be calculated;

$$L \rightarrow Y_W = 1$$
$$R \rightarrow Y_W = 2 \tag{2.2.3.17}$$

It is evident that the left-handed field should couple with half the strength of the right-handed fields to the hypercharge gauge field so that the $U(1)$ transformation matrix can now take the form

$$U(1)\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix} \rightarrow \begin{bmatrix} e^{i\frac{\beta}{2}} & 0 & 0 \\ 0 & e^{i\frac{\beta}{2}} & 0 \\ 0 & 0 & e^{i\beta} \end{bmatrix} \begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix} \tag{2.2.3.18}$$

To establish the gauge invariance of the Lagrangian (2.2.3.11) the product of the matrix (2.2.3.18) and (2.2.3.14), that is, $(SU(2) \times U(1))$ should be allowed operate on the Lagrangian (2.2.3.11) and observe the results. This is done by taking the matrix product of $(SU(2) \times U(1))$ so that

$$SU(2) \times U(1) \rightarrow \begin{pmatrix} e^{-\frac{i}{2}\tau\alpha} & 0 & 0 \\ 0 & e^{-\frac{i}{2}\tau\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} e^{\frac{i}{2}\beta} & 0 & 0 \\ 0 & e^{\frac{i}{2}\beta} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \tag{2.2.3.19}$$

The above operation yields

$$SU(2)\times U(1)\rightarrow\begin{pmatrix} e^{-\frac{i}{2}(\tau\alpha-\beta)} & 0 & 0 \\ 0 & e^{-\frac{i}{2}(\tau\alpha-\beta)} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \tag{2.2.3.20}$$

On using the substitutions

$$\frac{1}{2}(\vec{\tau}.\vec{\alpha}-\beta)=\phi\,,\beta=\theta\,, \tag{2.2.3.21}$$

on the matrix equation (2.2.3.20) leads to

$$SU(2)\times U(1)\rightarrow\begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \tag{2.2.3.22}$$

When the matrix (2.2.3.22) is allowed to operate on $\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix}$, it yields

$$SU(2)\times U(1)\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix}\equiv\begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix}\begin{pmatrix} \nu_e \\ e_L \\ e_R \end{pmatrix}\rightarrow\begin{pmatrix} \nu_e' \\ e_L' \\ e_R' \end{pmatrix}=\begin{pmatrix} e^{-i\phi}\nu_e \\ e^{-i\phi}e_L \\ e^{i\theta}e_R \end{pmatrix} \tag{2.2.3.23}$$

On using (2.2.3.23) on the Lagrangian (2.2.3.11) yields

$$\begin{aligned} L &= i\overline{R}\gamma\partial R+i\overline{L}\gamma\partial L \\ &= i\overline{e_R}\gamma\partial e_R+i\overline{e_L}\gamma\partial e_L+i\overline{\nu_e}\gamma\partial\nu_e \end{aligned} \tag{2.2.3.24}$$

or

$$\begin{aligned} L^{'} &= (i\overline{R}\gamma\partial R+i\overline{L}\gamma\partial L)^{'} \\ &= ie^{-i\theta}\overline{e_R}\gamma\partial\left(e^{i\theta}e_R\right)+ie^{i\phi}\overline{e_L}\gamma\partial\left(e^{-i\phi}e_L\right)+ie^{i\phi}\overline{\nu_e}\gamma\partial\left(e^{-i\phi}\nu_e\right) \end{aligned} \tag{2.2.3.25}$$

This simplifies to

$$L^{'}=L\equiv i\overline{e_R}\gamma\partial e_R+i\overline{e_L}\gamma\partial e_L+i\overline{\nu_e}\gamma\partial\nu_e \tag{2.2.3.26}$$

Thus, it is explicitly clear that, the Lagrangian is invariant under $SU(2)\times U(1)$. To gauge $SU(2)$, three gauge potentials $W_\mu{}^i$ are introduced so that, by acting on the isospinor L, the ordinary derivative is replaced by the covariant derivative [Lahanas, 1987]

$$D_\mu L = \partial_\mu L - \frac{i}{2} g\vec{\tau}\cdot\overrightarrow{W}_\mu L\,, \tag{2.2.3.27}$$

where g is the $SU(2)$ coupling constant. Gauging $U(1)$ symmetry introduces a gauge potential X_μ and a coupling constant $g^{'}$ so that, since L has half the hypercharge of R, the covariant derivatives then become

$$\begin{aligned} D_\mu L &= \partial_\mu L + \frac{i}{2} g^{'} X_\mu L \\ D_\mu R &= \partial_\mu R + i g^{'} X_\mu R \end{aligned} \tag{2.2.3.28}$$

Substitution of equations (2.2.3.27) and (2.2.3.28) into (2.2.3.11) and the gauge field kinetic terms included leads to the Lagrangian

$$\begin{aligned} L_1 &= i\overline{R}\gamma^\mu(\partial_\mu + ig^{'}X_\mu)R + i\overline{L}\gamma^\mu(\partial_\mu + \frac{i}{2}g^{'}X_\mu - \frac{i}{2}g\vec{\tau}\cdot\overrightarrow{W}_\mu)L \\ &- \frac{1}{4}(\partial_\mu W_\nu - \partial_\nu W_\mu + gW_\mu\times W_\nu)^2 - \frac{1}{4}(\partial_\mu X_\nu - \partial_\nu X_\mu)^2 \end{aligned} \tag{2.2.3.29}$$

Next an isospinor scalar field (the Higgs field)

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} \tag{2.2.3.30}$$

that carries the quantum numbers

$$I_W = \frac{1}{2},\ Y_W = 1 \tag{2.2.3.31}$$

is introduced into this Lagrangian, with both ϕ^+ and ϕ^0 being complex fields (the particle and antiparticle are distinct). Thence,

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \\ \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \end{pmatrix}, \qquad (2.2.3.32)$$

where ϕ_1, $\phi_{2,}$ ϕ_3, ϕ_4 are real and the covariant derivative of ϕ defined as

$$D_\mu \phi = \left(\partial_\mu - \frac{i}{2} g \vec{\tau} \cdot \overrightarrow{W}_\mu - \frac{i}{2} g^{'} X_\mu \right) \phi \qquad (2.2.3.33)$$

Since the Higgs field ϕ interacts with the leptons with strength G_e, then the overall Lagrangian containing ϕ becomes

$$L_2 = (D_\mu \phi)^* (D_\mu \phi) - \frac{m^2}{2} \phi^* \phi - \frac{\lambda}{4} (\phi^* \phi)^2 - G_e (\overline{L}\, \phi\, R + \overline{R}\, \phi^* L) \qquad (2.2.3.34)$$

The second and last terms can be written out fully as

$$-G_e (\overline{\nu}_e e_R \phi^+ + \overline{e}_L e_R \phi^0 + \overline{e}_R \nu_e \overline{\phi} + \overline{e}_R e_L \overline{\phi}_0)$$

and (2.2.3.35)

$$\phi^* \phi = (\phi^+)^* \phi^+ + (\phi^0)^* \phi^0 = \frac{1}{2} (\phi^2{}_1 + \phi_2{}^2 + \phi_3{}^2 + \phi_4{}^2)$$

In the case $m^2 > 0,$ the situation describes a scalar field with mass m, and the lowest energy state corresponds to $\phi = 0$. However, if $m^2 < 0$, the lowest energy state is not at $\phi = 0$ but at

$$(\phi^* \phi)_0 = -\frac{m^2}{\lambda} \qquad (2.2.3.36)$$

The symmetry is then broken by choosing the isospinor frame in which

$$(\phi_1)_0 = \sqrt{2}\eta \qquad (2.2.3.37)$$

and

$$(\phi_2)_0 = (\phi_3)_0 = (\phi_4)_0 = 0 \qquad (2.2.3.38)$$

so that

$$(\phi)_0 = \begin{pmatrix} 0 \\ \eta \end{pmatrix}, \qquad (2.2.3.39)$$

for real η. The fact that the symmetry is local means that a different isospin rotation can be performed at each point in space so that $\phi(x)$ reduces to the form

$$\phi(x) = \begin{pmatrix} 0 \\ \eta + \frac{\sigma(x)}{\sqrt{2}} \end{pmatrix} \qquad (2.2.3.40)$$

at every point. Thus equation (2.2.3.33) reduces to

$$D_\mu \phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \partial_\mu \sigma \end{pmatrix} - \left[\frac{i}{2} g \begin{pmatrix} W_\mu{}^3 & W_\mu{}^1 - iW_\mu{}^2 \\ W_\mu{}^1 + iW_\mu{}^2 & -W_\mu{}^3 \end{pmatrix} + i \frac{g^{'}}{2} X_\mu \right] \begin{pmatrix} 0 \\ \eta + \frac{\sigma}{\sqrt{2}} \end{pmatrix} \tag{2.2.3.41}$$

$$= -\frac{i}{2} \begin{pmatrix} g\eta(W_\mu{}^1 - iW_\mu{}^2) + \frac{g\sigma}{\sqrt{2}}(W_\mu{}^1 - iW_\mu{}^2) \\ i\sqrt{2}\partial_\mu \sigma + \eta(-gW_\mu{}^3 + g^{'} X_\mu) + \frac{\sigma}{\sqrt{2}}(-gW_\mu{}^3 + g^{'} X_\mu) \end{pmatrix} \tag{2.2.3.42}$$

The Hermitian conjugate of (2.2.3.42) is

$$\left(D_\mu \phi\right)^* = \frac{i}{2} \begin{pmatrix} g\eta\xi + \frac{g\sigma}{\sqrt{2}}\xi & -i\sqrt{2}\partial_\mu \sigma + \eta\varsigma + \frac{\sigma}{\sqrt{2}}\varsigma \end{pmatrix} \tag{2.2.3.43}$$

where

$$\xi = (W_\mu{}^1 + iW_\mu{}^2) \tag{2.2.3.44}$$

and

$$\varsigma = (-gW_\mu{}^3 + g^{'} X_\mu) \tag{2.2.3.45}$$

Hence, the product of the equations (2.2.3.42) and (2.2.3.43) yields

$$(D_\mu \phi)^* (D_\mu \phi) = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g^2\eta^2}{4}\left[(W_\mu{}^1)^2 + (W_\mu{}^2)^2\right] + \frac{\eta^2}{4}\left(gW_\mu{}^3 - g^{'} X_\mu\right)^2 \tag{2.2.3.46}$$

It is then suggested that the various terms be defined as;

$$Z_\mu = \cos\theta_W W_\mu{}^3 - \sin\theta_W X_\mu$$
$$= \frac{gW_\mu{}^3 - g^{'} X_\mu}{\left(g^2 + g^{'2}\right)^{1/2}} \tag{2.2.3.47}$$

$$A_\mu = \sin\theta_W W_\mu{}^3 + \cos\theta_W X_\mu$$
$$= \frac{g^{'} W_\mu{}^3 + gX_\mu}{\left(g^2 + g^{'2}\right)^{1/2}} \tag{2.2.3.48}$$

where θ_W (the Weinberg angle) is defined by

$$\cos\theta_W = \frac{g}{\left(g^2 + g^{'2}\right)^{1/2}} \tag{2.2.3.49}$$

and

$$\tan\theta_W = \frac{g^{'}}{g} \tag{2.2.3.50}$$

Then, from equation (2.2.3.46), $W_\mu^{1,2}$ *and* Z_μ pick up masses with

$$M_{W1}^{2} = M_{W2}^{2} = \frac{g^2\eta^2}{2} \qquad (2.2.3.51)$$

$$M_Z^{2} = \frac{g^2\eta^2}{2\cos^2\theta_W} = \frac{M_W^{2}}{\cos^2\theta_W} \qquad (2.2.3.52)$$

and A_μ remains massless. A_μ can physically be identified with the usual electromagnetic field vector potential. To this end, it is important to know why this field remains massless. This can be argued out by considering the relation

$$Q = I_3 + \frac{Y}{2} \qquad (2.2.3.53)$$

that relates the electric charge Q, the third component of weak isospin I_3 and the hypercharge *Y*. With $Y = 1$ for the doublet, the $I_3 = +\frac{1}{2}$ component has charge +1 (in units of e) and the $I_3 = -\frac{1}{2}$ component is electrically neutral. Fluctuations around the vacuum will correspond to the emission or absorption of a Higgs boson, meaning that any quantum numbers carried by the Higgs boson can be spontaneously created or annihilated. Conservation of charge will dictate that a non-zero vacuum expectation value for the neutral component is the most appropriate. This follows from the fact that a non-zero vacuum expectation value for the charged spinor Ψ^+ would imply non-conservation of electric charge. Now, if the choice Ψ^0 with $I_3 = -\frac{1}{2}$ and $Y = 1$ breaks both the $SU(2)$ and $U(1)_Y$ symmetry, then the gauge boson associated with that symmetry will remain massless. Such symmetry exists, for, if the vacuum is operated on with the electric charge operator Q, then

$$\begin{aligned} Q\Psi_0 &= \left(I_3 + \frac{Y}{2}\right)\Psi_0 \\ &= 0 \end{aligned} \qquad (2.2.3.54)$$

so that the vacuum is invariant under a transformation $\Psi_0 \to \Psi_0^{'} = e^{[i\alpha(x)Q]}\Psi_0 = \Psi_0$ for any value of $\alpha(x)$. This is also a $U(1)$ transformation whose generator is a linear combination $(Q = I_3 + \frac{Y}{2})$ of the generators of the overall $SU(2)\times U(1)_Y$ transformations, and it is just the $U(1)$ transformation of electromagnetism $U(1)_{em}$. It is a subgroup of $SU(2)\times U(1)_Y$ and of the generators I and Y of $SU(2)\times U(1)$ only the combination

$$Q = I_3 + \frac{1}{2} \qquad (2.2.3.55)$$

which is the generator of $U(1)_{em}$, satisfies

$$\begin{aligned} Q\Psi_0 &= (I_3 + \frac{Y}{2})\Psi_0 \\ &= 0 \end{aligned} \qquad (2.2.3.56)$$

and consequently the associated gauge boson should remain massless. The other generators break the symmetry and the associated bosons become massive. From equation (2.2.3.1), if

$$m_D \neq 0 \qquad (2.2.3.57)$$

then a Dirac mass term can be written as

$$-m_D \overline{\psi}_D \psi_D = -m_D \overline{\psi}_R \psi_L - m_D \overline{\psi}_L \psi_R \qquad (2.2.3.58)$$

where the second term is the hermitian conjugate of the first term. In the context of the electroweak model, it is convenient to work with the states ψ_L and ψ_R as independent states; *CPT* invariance will then include the states $\psi^c{}_R$ and $\psi^c{}_L$. However, in the grand unified theories, the states ψ_L and $\psi^c{}_L$ are considered independent states, so that they can be put into a single representation of the grand unification group

$$\Psi_L \equiv \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}_L \qquad (2.2.3.59)$$

CPT invariance will then include the *CP* conjugate states in the theory, which are the right-handed particles

$$\Psi_R \equiv \begin{pmatrix} \psi^c \\ \psi \end{pmatrix}_R \qquad (2.2.3.60)$$

The states Ψ_L and Ψ_R can then belong to the representations $\Re(G)$ and $\overline{\Re}(G)$, respectively of any unifying group G that commutes with the Lorentz group. One can then write the Dirac masses as

$$\begin{aligned} L_D &= -m_D \overline{\psi}_R \psi_L - m_D \overline{\psi}_L \psi_R \\ &= m_D \psi^T{}_R C^{-1} \psi^c{}_L + m_D \psi_L{}^T C^{-1} \psi_R{}^c \end{aligned} \qquad (2.2.3.61)$$

Grand unified theories also regard $SU(3) \times SU(2)_L \times U(1)$ as a low-energy relic of a more unified theory, based on a simple group G which incorporates QCD and $SU(2) \times U(1)$ (Glashow-Weinberg-Salam) as subgroups:

$$\begin{array}{c} G \to SU(3) \times SU(2)_L \times U(1) \to SU(3) \times U(1) \\ \sim 10^{14}\,GeV \qquad\qquad\qquad \sim 10^{2}\,GeV \end{array} \tag{2.2.3.62}$$

Indicated in equation (2.2.3.62) are the expected scales at which the primordial symmetry G is successively broken down; the scale 10^2 GeV is that of the gauge bosons, $W^{\pm}$ and Z^0 and the accepted structure of the known fermions is as shown in figure 2.2.3.1 below:

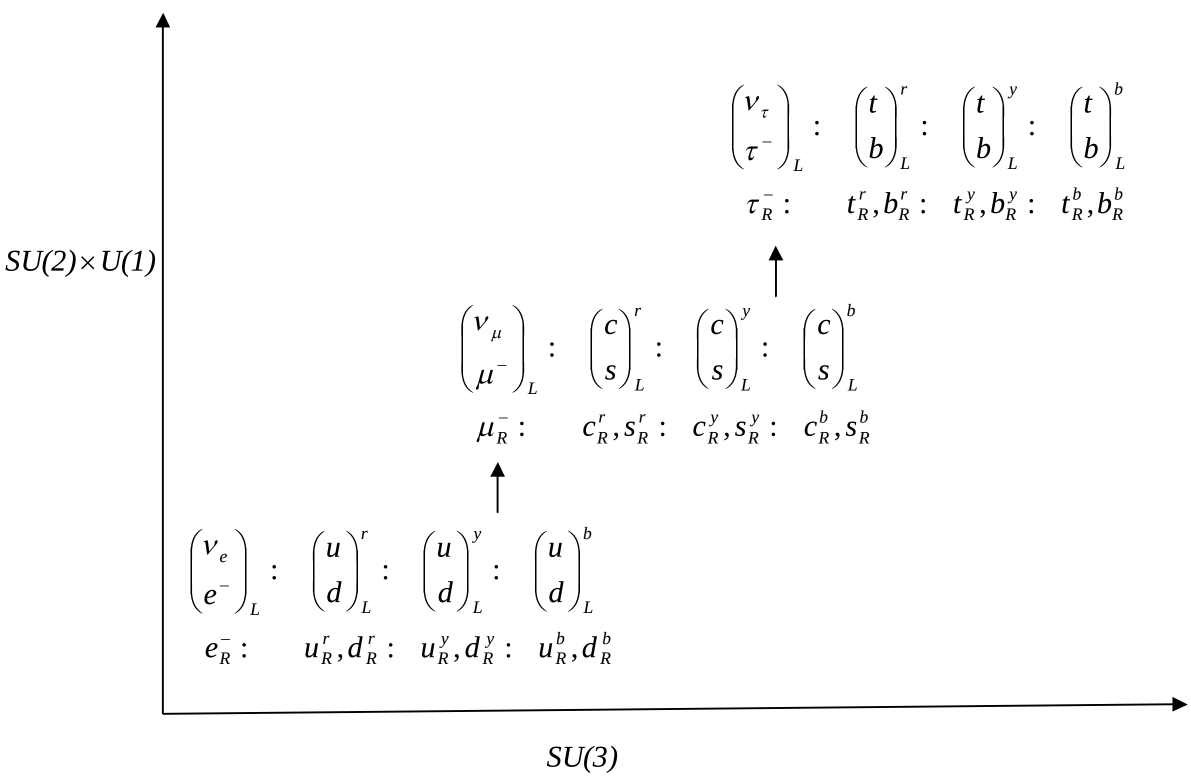


Figure 2.2.3.1: The multiplet structure of fermionic particles that participate in the strong, weak and electromagnetic interactions.

They seem to occur in generations or families containing 15 helicity states each of which has the group theoretic $SU(3) \times SU(2)_L$ content of:

$$(3,2) \quad + \quad 2(\bar{3},1) \quad + \quad (1,2) \quad + \quad (1,1) \tag{2.2.3.63}$$

Here, only particle content of the first generation has been given since the electroweak interaction does not admit the right-handed neutrinos.

Currently, grand unified theories are attempting to unify all the interactions within each generation with the intention of making quarks and leptons share representations of *G*, which may provide direct quark $\rightarrow$ lepton transitions. However, the major obstacle to the unification of the different known interactions is the disparity between the different coupling constants

$$g_3 \neq g_2 \neq g_1 \tag{2.2.3.64}$$

measured at low energies. Nevertheless, the property of asymptotic freedom guarantees that the strong interaction gets weaker at higher momenta, whereas the renormalization group demand that the *SU(2)* and *U(1)* coupling constants should also change with momentum with g_1 rising and g_2 remaining intermediate between g_1 and g_3. These evolutions suggest that it is possible for g_3, g_2 and g_1 to become equal at some momentum scale at which grand unification can take place as sketched in the figure 2.2.3.2 below:

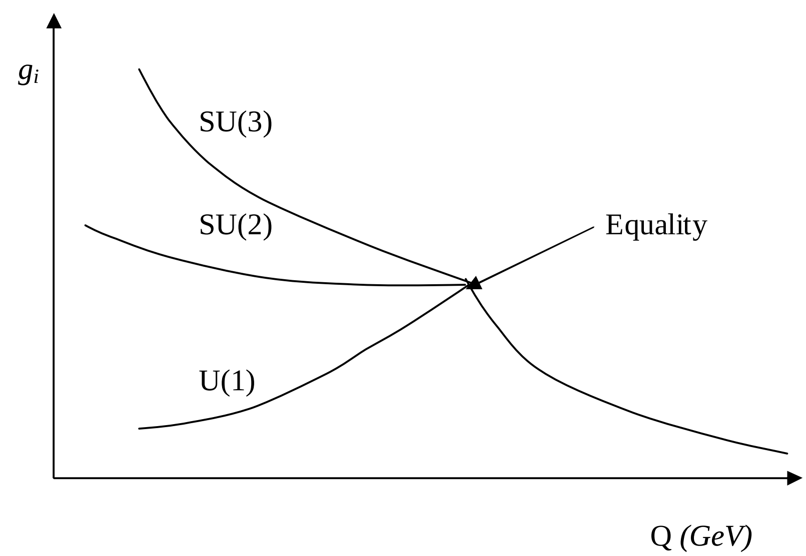


Figure 2.2.3.2: Schematic picture of the evolution of SU(3), SU(2) and U(1) coupling constants

The slow logarithmic variations of the different coupling constants suggest that this may take place at around 10^{14} *GeV* [Ellis, 1980]

2.3. A NEUTRINO IN THE BIG BANG MODEL

The $SU(2) \times U(1)$ model is a low-energy-scale gauge model i.e. it takes place at *10^2 GeV* energy scales. In addition, the model assumes that the neutrino is massless. Thence, to endow the neutrino with mass, higher energy scales should be probed. But also it is not easy to achieve these higher energies since the present particle accelerators are incapable of producing them. Consequently, the only natural particle accelerator available is the very early and hot universe. This means that neutrino properties beyond the $SU(2)\times U(1)$ energy scales can be studied from the view-point of a neutrino-dominated and evolving early universe. In particular, long before neutrinos had experimentally been detected, Alpher and his research collaborators [Alpher et al., 1953] had noted that they would have been in thermal equilibrium in the early universe through interactions with mesons at temperatures above $5\,MeV$. They noted that the subsequent annihilation of $e^{+}e^{-}$ pairs would heat the photons but not the decoupled neutrinos so that by conservation of entropy the temperature ratio $\frac{T_o}{T}$ would decrease from its initial high temperature value of unit, down to the numerical value

$$\frac{T_o}{T}=\left(\frac{4}{11}\right)^{\frac{1}{3}} \tag{2.3.1}$$

at $T_{dec} << 1\,MeV$; T_o is the expected neutrino temperature (to be calculated in this work) and $T = 2.726\,K$ is the observed photon temperature [Wick and Barry, 2000]. As long as they remain relativistic, neutrinos would retain a Fermi-Dirac distribution with phase-space density given as [Srivastava, 2008]

$$f_{\upsilon}=\frac{g_{\upsilon}}{(2\pi)^{3}}\left[\exp\left(\frac{p}{T_{\upsilon}}\right)+1\right]^{-1} \tag{2.3.2}$$

where g_{v} is neutrino degrees of freedom and p is neutrino momentum. In their calculation, Chiu and Morrison managed to show that the rate for the reaction $e^{+}e^{-}\leftrightarrow \upsilon_{\varepsilon}\overline{\upsilon}_{\varepsilon}$ in plasma should be

$$\lambda_{\varpi}\approx G_F^{2}T_{\upsilon}^{5} \tag{2.3.3}$$

for the universal Fermi interaction [Chiu and Morrison, 1960]. Here $G_F = 1.17\times 10^{-5}\,GeV^{-2}$ is Fermi coupling constant and T_{υ} is the temperature of the neutrino. Zeldovich later established that the expansion rate in the radiation- dominated era should be

$$\lambda_{e}=\sqrt{\frac{8\pi G_{N}\rho}{3}} \tag{2.3.4}$$

with

$$\rho=\frac{\pi^{2}}{30}g_{*}T^{4} \tag{2.3.5}$$

and g_* counts the relativistic degrees of freedom [Zeldovich, 1965]. However, Zeldovich and Chiu, in their analysis concluded that relic neutrinos although nearly as numerous as the blackbody photons, cannot make an important contribution to the cosmological energy density since they are probably massless [Zeldovich and Chiu, 1966]. Also, Gershtein and Zeldovich [Gershtein and Zeldovich, 1966] made the connection that if relic neutrinos are massive, then a bound on their mass will follow from a simple relation, that is,

$$\mathrm{m}_{\upsilon} n_{\upsilon} < \rho_m \tag{2.3.6}$$

where ρ_m is the cosmological energy density in form of matter, m_ν is the neutrino mass and n_υ is the neutrino number density. Using the general relativistic constraint

$$\frac{\rho}{\rho_{crit}} t_0^2 H_0^2 < \left(\frac{\pi}{2}\right)^2 \tag{2.3.7}$$

they derived the energy density contributed by matter as $\rho_m < 2 \times 10^{-25} gcm^{-3}$ and inferred that $m_\nu < 400\, eV$ for photon temperature of $3K$ (H_o is Hubble's constant at time t_o). Their calculation of the relic neutrino abundance was rather approximate in that they adopted $g_\nu = 4$ i.e. assumed massive neutrinos to be Dirac particles with fully populated right-handed (RH) states. Cowsik and McClleland [Cowsik and McClelland, 1972] using direct limits on the cosmological energy density parameter $\frac{\rho_{matter}}{\rho_{cirtical}}$ and h obtained a more restrictive bound of $m_\nu < 8eV$ assuming $m_\upsilon = m_{\upsilon e} = m_{\upsilon n}$. They also assumed that $T_\upsilon = T$ and that right-handed states are fully populated.

An upper bound of $m_\nu < 130eV$ was reported by Marx and Szalay [Marx and Szalay, 1976] who numerically integrated the cosmological Friedmann equation from ν_μ decoupling to the present epoch, subject to the condition $t_o > 4.5\, Gyears$. The modern version of the bound was arrived at by Bernstein and Feinberg [Bernstein and Feinberg, 1981]. Here the conservative limits $t_o > 10$ *gigayears* and $h_o > 0.4$ imply $\frac{\rho_m}{\rho} h^2 < 1$ ie $\rho_m < 10.54 keVcm^{-3}$.

When they combined these values with the relic neutrino number density, they arrived at a result

$$\frac{\rho_\nu}{\rho_{crit}}h^2 = \sum_i \left(\frac{m_{\nu i}}{93eV}\right)\left(\frac{g_{\nu i}}{2}\right) < 1 \qquad (2.3.8)$$

(Here, h is the value of the Hubble parameter in units of 100 km per mega parsec per second and m_{ν_i} is the neutrino mass species). The bound (2.3.8) together with the Hubble Key project determination of $h = 0.72 \pm 0.08$ implies that the sum of all neutrino masses cannot exceed about 15*eV*. It also assumes that neutrinos constitute all of the dark matter permitted by the dynamics of the universal Hubble expansion. However, Cowsik and McClleland [Cowsik and McClelland, 1972] had earlier suggested that neutrinos with a mass of a few *eV* could, also, naturally be the missing mass in cluster of galaxies. They arrived at this conclusion by requiring that the mass density within a cluster should be

$$M \approx \frac{1}{G_N^3 r_{cl}^3} M_{cl} \qquad (2.3.9)$$

which they obtained by modeling a cluster of mass M_{cl} as a square potential well of core radius r_{cl} filled with a Fermi-Dirac gas of neutrinos at zero temperature. Although the microscopic phase-space density in equation (2.3.2) is conserved for collisionless particles, the coarse-grained phase-space density in bound objects can decrease below its maximum value during structure formation. Modeling the bound system as an isothermal sphere with velocity dispersion σ and core radius

$$r_{cl}^2 = \frac{9\sigma^2}{4\pi G_N \rho(r_{Cl})} \qquad (2.3.10)$$

Tremaine and Gunn [Tremaine and Gunn 1979] obtained the relation

$$M_\nu > 120eV\left(\frac{\sigma}{100kms^{-1}}\right)^{-\frac{1}{4}}\left(\frac{r_{cl}}{kPc}\right)^{-\frac{1}{2}} \qquad (2.3.11)$$

They found this bound to be consistent with the cosmological upper bound in equation (2.3.8) down to the scale of galaxies. However, there is a conflict for smaller objects like, dwarf galaxies, which require a minimum mass of $\approx 100eV$ [Spergel et al., 1988; Lake, 1989]. The central phase space density of the observed dark matter cores in these structures decreases rapidly with increasing core radius, rather than being constant as would be expected for neutrino [Burkert, 1997]. Moreover since neutrinos would cluster more efficiently in larger potential wells, there should be a trend of increasing mass-to-light ratio with scale. This was indeed claimed to be the case initially but later it was recognized that the actual increase is far less than expected [Blumenthal et al., 1984]. Thus a massive neutrino was disfavored as the constituent of the missing mass in galaxies and clusters.

Nevertheless cosmological arguments have continued to be of major interest since the 1980s when the $ITEP$'s tritium beta decay experiments reported a $\approx 30eV$ mass for the electron neutrino [Lubimov et al., 1980]. In particular the attention of cosmologists has been focusing on how the large-scale-structure (LSS) of galaxies, clusters and super clusters would have formed if the universe is indeed dominated by such massive neutrinos. The basic picture is that structure grows through gravitational instability from primordial density perturbations [Padmanabhan, 1993]. These density perturbations were first detected by $COBE$ via the temperature fluctuations they induce in CMB [Smoot et al., 1992]. On small scales $(\leq\ 10\ \mathrm{mpc})$ structure formation is complicated by non-linear gravitational clustering as well as non-gravitational processes but on large scales, gravitational dynamics is linear and provides a robust probe of the nature of the dark matter. Density perturbations in a medium composed of relativistic collisionless particles are subject to a form of Landau damping which effectively erases perturbations on scales smaller than the free-streaming length [Bond et al., 1980] $\sim 41mpc\left(\frac{m_\nu}{30eV}\right)^{-1}$. This is essentially the distance traversed by a neutrino from the big bang until it becomes non-relativistic and corresponds to the scale of super clusters of galaxies. Thus huge neutrino condensations containing a mass $\sim 3\times10^5 M_\Theta\left(\frac{M_\nu}{30eV}\right)^{-2}$ would have begun growing at a red-shift $Z_{eq}\sim 7\times10^3\left(\frac{M_\nu}{30eV}\right)$, when the universe became matter-dominated and gravitational instability set in. This is well before the epoch of recombination at $Z_{rec}\sim10^3$ so that the baryons were still closely coupled to the photons, while the neutrinos were mildly relativistic

$$\frac{v}{c}\sim 0.1 \tag{2.3.12}$$

and, hence, hot. After the universe became neutral, baryonic matter could have accreted into these potential wells, forming a thin layer of gas in the central plane of the pancakes. Thus super clusters would be the first objects to condense out of the Hubble flow in a “hot dark matter” (HDM) cosmogony and smaller structures such as galaxies would form only later through the fragmentation of the pancakes [Zeldovich, 1970]. The gross features of such a “top down” model for structure formation are compatible with several observed features of large scale structure, in particular the distinctive voids and filaments seen in large galaxy surveys. Studies [Peebles, 1982; White et al., 1983; Bond and Szalay, 1983] have, however found that galaxies form too late through the break-up of the pancakes at a red-shift $Z \leq 1$ counter to observations of galaxies and quasars at $Z > 4$, i.e., galaxies should have formed last in a HDM universe, whereas our galaxy is in fact dynamically much older than the local group. Therefore, the HDM model was abandoned and considerable attention was turned to cold dark matter (CDM), i.e., particles which were non-relativistic at the epoch of matter-domination. Detailed studies of CDM universes gave excellent agreement with observation of galaxy clustering and a standard cold dark matter model for large scale structure formation was established, viz a critical density CDM universe with an initially scale-invariant spectrum of density perturbations [Peebles, 1982; Bond and Efstathiou, 1984;Davis et al., 1985; Ostriker, 1993]. Plausible particle candidates were provided notably the neutralino in supersymmetric models which has a relic abundance of order of the critical density [Jungman et al., 1996]. Nevertheless neutrinos were resuscitated some years later as a subdominant component of the dark matter when the CDM cosmogony itself ran into problems [Liddle and Lyth, 2000]. To appreciate the background to this, it is necessary to recapitulate the essential ingredients of a model for cosmic structure formation. A key assumption made concerns the nature of the primordial density perturbations, which grows through gravitational instability in the dark matter. Such fluctuations are assumed to have a power spectrum of the scale-free form:

$$p(k) = \langle |\delta_k|^2 \rangle \qquad (2.3.13)$$

where

$$\delta_k = \int \frac{\delta\rho}{\rho}(\bar{x}) e^{-\varepsilon \bar{k}.\bar{x}} d^3 x \qquad (2.3.13')$$

is the Fourier transform of spatial fluctuations in the density field of wavelength $\lambda = \frac{2\pi}{k}$.

Powerful support for their conjecture was provided by the inflationary model that had been developed earlier by Allan Guth and Alexander Linde [Guth, 1981; Linde, 1982] and later applied to the cosmological model by Edward Kolb, Michael Turner and Alexander Linde [Kolb and Turner, 1990; Linde, 1990]. According to this model, perturbations arise from quantum fluctuations of a scalar field ϕ, the vacuum energy of which drives a period of accelerated expansion in the early universe. Primordial perturbations have another unique observational signature in that they induce temperature fluctuations in the cosmic microwave background through gravitational red/blue shift corresponding to spatial scales larger than the Hubble radius on the last scattering surface. The anisotropy in the cosmic microwave background (CMB) measured by Cosmic Background Explorer $(COBE)$ allows determination of the fluctuation amplitude at the scale $H_0^{-1} \approx 3000h^{-1}mpc$ corresponding to the present Hubble radius. With this normalization, it became clear that a HDM model had too little power on small-scales for adequate galaxy formation [Brandenberger et al., 1987; Bertschinger and Watts, 1988]. However, it also became apparent that the standard CDM model when normalized to $COBE$ had too much power on small-scales [Smoot G F et al., 1992]. It was, thus, a logical step to invoke a suitable mixture of CDM and HDM models to try and match the theoretical power spectrum to the data on galaxy clustering and motions [Wright et al., 1992; Davis et al., 1992; Taylor and Rowan, 1992].

Shafi and Stecker motivated by theoretical considerations of supersymmetric grand unified theories had earlier discussed the possibility that the dark matter may have both a hot and a cold component [Shafi and Stecker, 1984]. In the post-$COBE$ era, studies on mixed dark matter (MDM) model were performed and a neutrino fraction of about 20% was found to give the best match with observations [Dalen and Schaefer, 1992; Klypin et al., 1993; Jing et al., 1994; Ma and Bertschinger, 1994; Pogosian and Starobinsky, 1995; Liddle et al., 1996]. The implied neutrino mass was ~5*eV*. This was an exciting time for neutrino cosmology as both laboratory data and astronomical observations supported the possibility that a substantial fraction of the cosmological mass density is in the form of massive neutrinos. However, in the absence of a standard model of inflation it might be argued that the inflationary spectrum may instead have $n > 1$, thus allowing a large HDM component. Yet another way of suppressing small-scale power in the CDM cosmogony is to decrease the matter content of the universe, since this postpones the epoch of matter-radiation equality and thus shifts the peak of power spectrum to larger scales. Further, the spatial geometry can be maintained flat if there is a cosmological constant with $\rho_\wedge = 1 - \rho_m \approx 0.7$. Evidence for such a cosmology (ΛCDM) has come subsequently from observations of the Hubble diagram of type Ia supernova which suggests that the expansion is in fact accelerating [Riess et al., 1998; Perlmutter et al., 1999]. When this is coupled with the observation that ρ_m does not exceed 0.3 even on the largest scales probed, it is yields a scale invariant power spectrum

$$f_v = \frac{\rho_v}{\rho_m} < 0.13 \tag{2.3.14}$$

which corresponds to an upper bound of $2.1eV$ on the sum of neutrino masses [Elgaroy et al., 2002]. To fully establish how a neutrino can lead to a flat universe, the relevant cosmological ingredients must be fully utilized. Essentially, since cosmology deals with the physical structure of the universe at large scales, it is suggested that the physical processes occurring in its early phases of evolution may have observable, though indirect, consequences in the present structure. However, a proper cosmological setting relies on the following guiding assumptions [Keith, 1984]:

- There does not exist privileged observers and on average, the universe is not expected to look any different from any other spatial position (The Copernican Principle).
- Physical laws do not depend on space-time (The relativity Principle).

These two principles taken together constitute the Cosmological Principle which can be stated as; the universe is isotropic in all its measurable properties at all times over all space. That is, the universe is spatially homogenous and isotropic. As Keith observes, there are two immediate consequences of the cosmological principle. The first is that the only true velocity fields allowed are either overall expansion or contraction. Other possibilities such as rotation, shear, combined expansion and contraction are all contained in the anisotropic Bianchi models. Furthermore, any expansion or contraction present must have no apparent centre i.e. the relative velocity between any two observers must depend only on their separation

$$v_{12} = Hr_{12} \tag{2.3.15}$$

where H is a universal spatial constant known as Hubble's constant. The second consequence of the cosmological principle is that, there must exist a measure of distance which is independent of direction such that

$$d = \frac{zc}{H} \tag{2.3.16}$$

where z is the redshift due to expansion of an emitted signal. More generally, the second consequence implies that there exists a metric g which does not depend on direction but is a symmetric tensor of the form

$$g = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{2.3.17}$$

that defines the line element ds^2. The metric must also be non-singular so that it has an inverse defined by

$$g^{-1} = g^{\mu\nu} \frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial x^{\nu}} \tag{2.3.18}$$

and

$$g^{\mu\nu} g_{\nu\lambda} = \delta^{\mu}{}_{\lambda} \tag{2.3.19}$$

If the cosmological principle is applied to the metric, then

$$g_{0i} = 0 \tag{2.3.20}$$

$$g_{ij} = 0 \qquad i \neq j \tag{2.3.21}$$

and

$$ds^2 = g_{00} dt^2 + g_{ii} dx^i dx^i \tag{2.3.22}$$

A set of coordinates can further be defined so that the homogenous and isotropic metric takes the form

$$ds^2 = -dt^2 + S^2(t) d\sigma^2 \tag{2.3.23}$$

where $d\sigma^2$ is the three-space metric of constant curvature and is time-independent. The different three-space geometries are those of positive, negative and zero curvature, so that the metric takes the form

$$d\sigma^2 = dr^2 + f^2(r)(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.3.24}$$

In this equation, the function $f(r)$ is defined by

$$f(r) = \begin{cases} \sin r & (k = +1) \\ \sinh r & (k = -1) \\ r & (k = 0) \end{cases} \tag{2.3.25}$$

and k is the curvature parameter of the space-time. Homogeneity and isotropy then guarantees that the form of *f* be independent of θ and ϕ so that the resulting metric, known as the Friedmann-Robertson-Walker metric (FRW), takes the form,

$$ds^2 = -dt^2 + S^2(t)\left[\frac{dr^2}{1-kr^2} + r\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{2.3.26}$$

To derive the equations describing the dynamical evolution of a cosmological model with *FRW* metric, general relativity is required. To begin with, the covariant derivative of a vector A^{ν} is defined by

$$D_\mu A^\nu = A^\nu_{,\mu} + \Gamma^\nu_{\mu\sigma} A^\sigma$$

$$= A^\nu_{;\mu} \qquad (2.3.27)$$

$$D_\mu A_\nu = A_{\nu,\mu} - \Gamma^\sigma_{\mu\nu} A_\sigma$$

$$= A_{\nu;\mu} \qquad (2.3.28)$$

where

$$A^\nu_{,\mu} = \partial_\mu A^\nu$$

or

$$A_{\nu,\mu} = \partial_\mu A_\nu \qquad (2.3.29)$$

is the ordinary derivative and the affine connection (or Christoffel) symbol is given in terms of the metric by

$$\Gamma^\mu_{\nu\sigma} = \frac{1}{2}\left[g_{\rho\sigma,\nu} + g_{\nu\rho,\sigma} - g_{\nu\sigma,\rho}\right] g^{\rho\mu} \qquad (2.3.30)$$

In terms of the affine connection, the Riemann curvature tensor can be written as [Srivastava, 2008]

$$S^\sigma_{\mu\rho\nu} = \Gamma^\sigma_{\nu\mu,\rho} - \Gamma^\sigma_{\rho\mu,\nu} + \Gamma^\lambda_{\nu\mu}\Gamma^\sigma_{\rho\lambda} - \Gamma^\lambda_{\rho\mu}\Gamma^\sigma_{\lambda\nu} \qquad (2.3.31)$$

A space-time will be defined to be flat if

$$S^\sigma_{\mu\rho\nu} = 0 \qquad (2.3.32)$$

Contraction on σ and ρ gives the Ricci tensor $S_{\mu\nu}$ and further contraction yields the curvature scalar S_c which is defined by [Gupt, 2004]

$$S_{\mu\nu} = S^\sigma_{\mu\sigma\nu} = \Gamma^\sigma_{\mu\nu,\sigma} - \Gamma^\sigma_{\sigma\mu,\nu} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho} - \Gamma^\sigma_{\rho\mu}\Gamma^\rho_{\sigma\nu} \qquad (2.3.33)$$

$$S_c = S^\mu_\mu = S_{\mu\nu} g^{\mu\nu}$$

To derive the field equations, the set of the Christoffel symbols are worked out to yield [Keith, 1984]

$$\Gamma^0_{ij} = \frac{\dot{S}}{S} g_{ij} \qquad (2.3.34)$$

$$\Gamma^i_{0j} = \Gamma^i_{j0} = \left(\frac{\dot{S}}{S}\right)\delta^i_j \qquad (2.3.35)$$

$$\Gamma^1_{11} = kr\left(1 - kr^2\right)^{-1} \qquad (2.3.36)$$

$$\Gamma^1_{22} = -\left(1-kr^2\right)r \tag{2.3.37}$$

$$\Gamma^1_{33} = -\left(1-kr^2\right)r\sin^2\theta \tag{2.3.38}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \tag{2.3.39}$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta \tag{2.3.40}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta \tag{2.3.41}$$

The calculated Christoffel symbols are then used to establish that the only non-vanishing Ricci coefficients are

$$S^0_0 = \frac{3\ddot{S}}{S} \tag{2.3.42}$$

$$S^1_1 = S^2_2 = S^3_3 = \left[2\left(\frac{\dot{S}}{S}\right)^2 + \left(\frac{\ddot{S}}{S}\right) + \left(\frac{2k}{S^2}\right)\right] \tag{2.3.43}$$

and the curvature scalar is

$$S_c = S^\mu_\mu = 6\left[\left(\frac{\dot{S}}{S}\right)^2 + \left(\frac{\ddot{S}}{S}\right) + \left(\frac{2k}{S^2}\right)\right] \tag{2.3.44}$$

Concentrating on the 0-0 term in the field equations yields the standard Friedman equation [Keith, 1984]

$$\left(\frac{\dot{S}}{S}\right)^2 = \frac{8\pi G_N\rho}{3} - \frac{k}{S^2} + \frac{\Lambda}{3} \tag{2.3.45}$$

More interestingly, when the equation (2.3.45) is written (with the cosmological constant equated to zero) in the form

$$-k = \dot{S}^2 - \frac{8\pi G}{3}\rho S^2, \qquad (2.3.46)$$

then the left hand side can be interpreted as the total energy $(-k)$ of the universe with the kinetic energy term represented by $\dot{S}^2$ and the potential energy by the term containing ρ in the right hand side. If the left-hand side (total energy) is positive ($k = -1$), then the kinetic energy term is great enough (the initial velocity is greater than the escape velocity) and the universe will expand forever (the universe is open). If the total energy is negative ($k = +1$), the universe will recollapse (the universe is closed). In the $k = 0$ model, the universe is at the escape velocity and it will expand indefinitely. It is not yet clear which situation describes the present universe. However, since the universe has been expanding at almost exactly the critical rate to avoid recollapse, it is expected that some mechanism in the early universe could have driven it to its present state. Hence, from the inflationary model, if the energy difference between two vacua is denoted by ρ_v, then until the tunneling has taken place, the universe has an extra energy density ρ_v at its disposal which must have dynamical effects in Einstein's equation for the standard cosmology described by the Friedmann equation of motion for cosmic dynamics:

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi G}{3}\rho_v \qquad (2.3.47)$$

2.4. INSTANTONS

In the standard cosmology, the universe is suggested to have started from a big bang and continued to expand. When extrapolated backwards, the universe is found to be very dense in and the average energy per particle is much higher. The extrapolation predicts a singularity at $t = 0$ and as this epoch is approached ($S(t) = 0$), the Hubble constant H increases rapidly becoming infinite at $S(t) = 0$. The epoch is normally referred to as a singularity. Quantitatively, $S(t) = 0$ implies a breakdown of the concept of space-time geometry and has been recognized as an inevitable feature of Einstein's general theory of relativity [Narlikar, 1993]. Qualitatively, it is a feature that prevents one from investigating what happened at $S(t) = 0$ or prior to it. This abrupt termination of the past signifies an incompleteness of the general theory of relativity hence a need for a better and consistent solution [Maumba et al., 2011]. In the context of the particle physics of the big bang cosmology, little known field solutions from the non-linear gauge field theories are suggested as a possible solution. These extended solutions known as solitons may represent stable configurations with well-defined energies that are nowhere singular. On this account, if gauge theories are to be taken seriously, then so must these solutions, for they promise to give rise to some physics that may even solve the problem of quark confinement. A number of soliton solutions exist in the literature [Lewis, 1985]. Of particular interest are those localized in time as well as in space, known as instantons. The basic idea is to write down the instanton solution for the equation of motion exhibiting its topological nature and then examine the physical consequences that may follow from it. To begin with, Euclidean space is considered to have co-ordinates (x_1, x_2, x_3, x_4) with

$$x_4 = ix_0 \tag{2.4.1}$$

and

$$x_0 = ct \tag{2.4.2}$$

Its Euclidean field tensor $F^a_{\mu\nu}$, defined in the same way as the Minkowski tensor [Brandenberger, 1985], then takes the form

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\mu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu \tag{2.4.3}$$

with

$$A_\mu = \frac{1}{2}\sigma^a A^a_\mu$$

$$F_{\mu\nu} = \frac{1}{2}\sigma^a F^a_{\mu\nu} \tag{2.4.4}$$

This means that the tensor $F_{\mu\nu}$ can be written as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\lfloor A_\mu \ , \ A_\nu \rfloor \tag{2.4.5}$$

Defining

$$\partial_{[\mu} A_{\nu]} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.4.6}$$

implies that $F_{\mu\nu}$ can be written as

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]} - ig\lfloor A_\mu \ , \ A_\nu \rfloor \tag{2.4.7}$$

The dual of $F_{\mu\nu}$ (denoted $\tilde{F}_{\mu\nu}$) is correspondingly defined by

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} \tag{2.4.8}$$

With the antisymmetric tensor $\varepsilon_{1234} = 1$, this yields

$$\tilde{F}_{\mu\nu} \approx F_{\mu\nu} \tag{2.4.9}$$

In the Minkowski space, it is defined that when $\varepsilon^{0123} = 1$, then $\varepsilon_{0123} = -1$ so that

$$\tilde{F}_{\mu\nu} \approx -F_{\mu\nu} \tag{2.4.10}$$

Under the gauge transformations that

$$A_{\mu}^{'} = SA_{\mu}S^{-1} - \frac{i}{g}(\partial_{\mu}S)S^{-1} \tag{2.4.11}$$

$$F_{\mu\nu}^{'} = SF_{\mu\nu}S^{-1} \tag{2.4.12}$$

and the vector

$$\begin{aligned} K_{\mu} &= \frac{1}{4}\varepsilon_{\mu\nu\kappa\lambda}\left(A_{\nu}^{a}\partial_{\kappa}A_{\lambda}^{a} + \frac{g}{3}\varepsilon_{abc}A_{\nu}^{a}A_{\kappa}^{b}A_{\lambda}^{c}\right) \\ &= \varepsilon_{\mu\nu\kappa\lambda}Tr\left(\frac{1}{2}A_{\nu}\partial_{\kappa}A_{\lambda} - \frac{ig}{3}A_{\nu}A_{\kappa}A_{\lambda}\right) \end{aligned} , \tag{2.4.13}$$

then,

$$\begin{aligned} \partial_{\mu}K_{\mu} &= \frac{1}{4}Tr\,\tilde{F}_{\mu\nu}\,F_{\mu\nu} \\ &= \frac{1}{8}\tilde{F}_{\mu\nu}^{a}\,F_{\mu\nu}^{a} \end{aligned} \tag{2.4.14}$$

If a 4-dimensional volume V^4 in E^4, with boundary $\partial V^4 \sim S^3$ is considered, then

$$K_{\mu} = 0 \tag{2.4.15}$$

in a pure vacuum with

$$A_{\mu} = 0;\ F_{\mu\nu} = 0 \tag{2.4.16}$$

and the field equations (in the absence of matter)

$$D_{\mu}F_{\mu\nu} = 0 \tag{2.4.17}$$

are clearly satisfied over the whole region V^4, as is the Bianchi identity

$$D_{\mu}\tilde{F}_{\mu\nu} = 0 \tag{2.4.18}$$

Applying Gauss' theorem to equation (2.4.14) then gives

$$\int_{V^4} TrF_{\mu\nu} \tilde{F}_{\mu\nu} d^4x = 4\int_{V^4} \partial_\mu K_\mu d^4x$$

$$= 4\oint_{\partial V^4} K_\perp d^3x \qquad (2.4.19)$$

2.4.1. COSMOLOGICAL INSTANTONS

Equation (2.4.19) is the action solution for the non-linear gauge instanton. To find the corresponding action from the view point of cosmology, it is suggested that a universe that starts in the symmetric vacuum state is described by the Friedman-Robertson-Walker metric [Kolb, 1986]

$$d\tau^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \qquad (2.4.1.1)$$

together with the Friedman evolution equation

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi G}{3}\rho \qquad (2.4.1.2)$$

In this case, $\dot{S} = \frac{dS}{dt}$ and k is a parameter that can take the values -1, 0 or $+1$. The equation, for $k = +1$, is solved to yield the usual de Sitter solution

$$S(t) = H^{-1}\cosh(Ht) \qquad (2.4.1.3)$$

This solution (2.4.1.3) describes a universe that is contracting at $t < 0$, reaches its minimum size $S_{\min}(0) = H^{-1}$ at $t = 0$ and expands at $t > 0$. This behavior is analogous to that of a particle bouncing off a potential barrier at $S(t) = H^{-1}$ with S playing the role of the particle coordinate. Since particles can tunnel through potential barriers quantum mechanically, then the creation of the universe can be visualized as a quantum tunneling process [Shuryak, 1985], where it emerges having a finite size $S(0) = H^{-1}$ and zero velocity. Its following evolution is described by equation (2.4.1.2) with $t > 0$.

A semiclassical description of quantum tunneling process is given by the solution of equation (2.4.1.2) with t replaced by it (it is the complex imaginary time). This is because if potential energy is greater than the total energy, i.e.

$$V > E \tag{2.4.1.4}$$

then the process is classically $(\hbar = 0)$ forbidden, but the actual tunneling amplitude is [Michael et al., 1992]

$$\exp\left\{-\int_a^b 2m\sqrt{V-E}\,dx\right\} = \exp\left(-A_E\right) \tag{2.4.1.5}$$

A_E (the Euclidean action) is defined by the integral in brackets and for quantum tunneling to occur it must be the action for imaginary times. To show this, a case is considered where

$$E > V \tag{2.4.1.6}$$

and the transition is classically allowed. In this case, the "wave function" oscillates and the number of oscillations is normally given by [Bransdein and Joachain, 1989]

$$\int_a^b p\,dx = \int_a^b \left[2m\sqrt{E-V}\,dx\right] \tag{2.4.1.7}$$

For convenience, equation (2.4.17) may be written as

$$\begin{aligned}\int_a^b p\,dx &= \int_a^b p\,\dot{x}\,dt \\ &= \int_a^b (H_{TE} + L)dt \\ &= \int_a^b (E + L)dt \end{aligned} \tag{2.4.1.8}$$

where H_{TE} is the total energy or the Hamiltonian. If H_{TE} is normalized to zero, then

$$\begin{aligned}\int_a^b p\,dx &= \int_a^b L\,dt \\ &= A\end{aligned} \tag{2.4.1.9}$$

which is the total action for transition from vacuum state a to b. The only difference between equation (2.4.1.5) and (2.4.1.7) is that the sign of $E - V$ is reversed. However, the sign of V in the familiar equation of motion

$$m\ddot{x} = -\frac{\partial V}{\partial x} \tag{2.4.1.10}$$

is reversed if t is replaced by it. Hence, A_E is the action for imaginary times and the Euclidean version of equation (2.4.1.2) is [Kolb, 1986; Coleman, 1977; Curtis and Coleman, 1977]

$$\frac{k-\dot{S}^{2}}{S^{2}}=\frac{8\pi G}{3}\rho \qquad (2.4.1.11)$$

This equation (2.4.1.11) can then be solved to yield

$$S(t)=H^{-1}\cos(Ht) \qquad (2.4.1.12)$$

Equations (2.4.1.3) and (2.4.1.12) describe a four-sphere S^4 which is the de Sitter instanton [Vilenkin, 1982]. However, solution (2.4.1.12) does bounce at the classical turning point $S=H^{-1}$, but it does not approach any initial state at $t\to\pm\infty$. It is defined only for $|t|<\frac{2\pi}{H}$. The equation (2.4.1.12) describes the tunneling to the de Sitter space from a non classical space-time state and this can be represented pictorially as in figure (2.4.1.1) below:

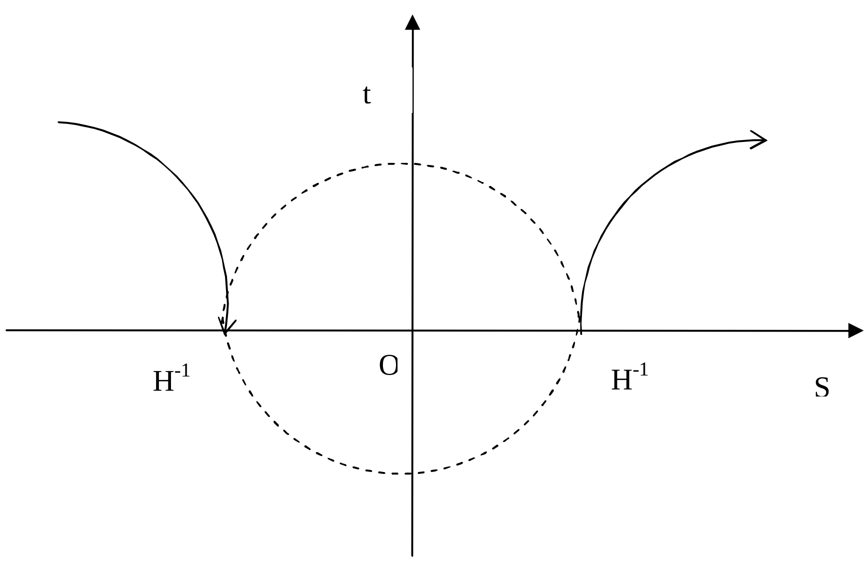


Figure 2.4.1.1: A pictorial representation of the creation of the inflationary Universe.

Physically, the connection between the instanton solution and the quantum tunneling of the universe from a non classical space-time state is not an obvious concept. To agree with the instanton solution, a hypothetical example from condensed matter physics is discussed in order to make the concept more obvious. Creation of an electron-positron pair in a constant field, say the electric field $\vec{E}$, is considered. It is then suggested that the electrons that are emitted from the emitter surface do not all of them have the same energy. This is because, they are emitted from varying depths of the emitter region, hence, lose different amounts of energy before emanating from the emitter surface. When the potential between the emitter and the collector is equal to the stopping potential, then even the most energetic electrons would just fail to reach the collector. If $v_{\max}$ is the maximum velocity of the emitted electrons at the emitter, then their kinetic energy at the emitter is $\frac{1}{2}m_e v_{\max}{}^2$, where m_e is the mass of the electron. As the electron travels from the emitter to the collector, its kinetic energy decreases and its potential energy increases such that the total energy is conserved. For those electrons that just manage to reach the collector, the velocity, and hence the kinetic energy becomes zero. The work-energy theorem then suggest that

$$\frac{m_o}{\sqrt{1-v^2}} = q\vec{E}\Delta\vec{x} \tag{2.4.1.13}$$

The system of units has been adopted in such a way that $c = \hbar = k_B = 1$. The equation (2.4.1.13) may be solved as follows: first v is replaced with $\frac{\Delta x}{\Delta t}$ and both sides of the equation squared to get

$$(m_o\Delta t)^2 = (qE\Delta x\Delta t)^2 - (qE\Delta x^2)^2 \tag{2.4.1.14}$$

which, when rearranged yields

$$(\Delta x^2)^2 - (\Delta x\Delta t)^2 - \left(\frac{m_o}{qE}\Delta t\right)^2 = 0 \tag{2.4.1.15}$$

Applying the quadratic formula, with $\Delta x^2 = x$, $\Delta t^2 = b$ and $\left(\frac{m_o}{qE}\Delta t\right)^2 = c$, solution of (2.4.1.15) to first-order is found to yield

$$(x - x_o)^2 + (t - t_o)^2 = R^2 \tag{2.4.1.16}$$

where

$$R^2 = -\frac{m_o{}^2}{q^2E^2} \tag{2.4.1.17}$$

Equation (2.4.1.16) describes a circular trajectory which, in this context, is an instanton. This is pictorially represented in figure (2.4.1.2) below

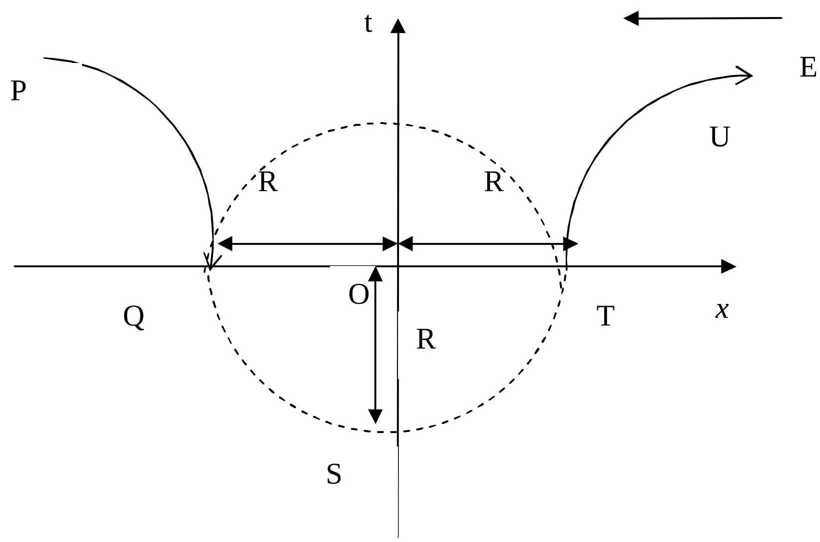


Figure 2.4.1.2: Particle-antiparticle pair creation in a constant electric field E [Kuo, 1976]

PQ and *TU* are the classically allowed trajectories: *PQ* describes a particle moving backwards in time (positron), the semicircle *QST* represents the instanton (2.4.1.16) and *TU* describes a particle moving forwards in time (electron) [Maumba et al., 2011].

2.4.2. TUNNELING PROBABILITY.

From the foregoing section (2.4.1), it has been established that the universe emerges quantum mechanically from a non-classical vacuum state with a finite size and begins to evolve along the inflationary lines. The singularity predicted by the big bang model is explicitly eliminated. However, the big challenge is to find out how a neutrino mass can influence this transition from a non-classical space-time state. To do so, it is suggested that the vacuum is an infinitely degenerate state consisting of an infinite number of non-equivalent vacua where the instanton will represent a transition from one vacuum state to another (i.e., the probability amplitude that the transition takes place). Classically, it is zero since the particle cannot penetrate the energy barrier. But, quantum mechanically, there is a barrier penetration factor i.e. the instanton solution can be used to estimate the barrier penetration amplitude which takes the standard form [Lewis, 1996]

$$P_b \propto e^{-A_E}\,, \qquad (2.4.2.1)$$

where, as before, A_E is the Euclidean action for imaginary time that is given as

$$A_E = \int \left[\frac{m_o}{\sqrt{1+(\Delta \dot{x})^2}} - q\vec{E}\Delta x \right] dt \qquad (2.4.2.2)$$

After the quantum tunneling, the universe may not be totally stable. This means that at early times the universe may have been extremely hot [Gamow, 1946] and has since cooled down to the presently observed temperature of $2.726K$ [Wick and Barry, 2000]. The analysis of the finite effective potential shows that initially $\bar{\phi} = 0$ is the only ground state of the theory. As time increases, the temperature of the universe decreases and at some critical temperature $T = T_c$, the symmetric vacuum ceases to be stable and a new energetically favored ground state appears. Hence, thermal, gravitationally induced or quantum fluctuations tends to force the field into the new asymmetric ground state which is the true vacuum. To study temperature effects, a real scalar field is considered that is described by the Lagrangian [Ellis et al., 1979]

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi), \qquad (2.4.2.3)$$

where

$$V(\phi) = -\frac{1}{2}M^2\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad (2.4.2.4)$$

For convenience, if the parameters M and λ are set to unity, then table 2.4.2.1 can be generated whose potential can also be sketched as in figure (2.4.2.1) below:

ϕ	-5	-4	-3	-2	-1	0	1	2	3	4	5
$-\frac{1}{2}\phi^2$	-12.5	-8	-4.5	-2	-0.5	0	-0.5	-2	-4.5	-8	-12.5
$\frac{1}{4}\phi^4$	156.3	64	20.3	4	0.25	0	0.25	4	20.3	64	156.3
$V(\phi)$	143.8	56	15.8	2	-0.25	0	-0.25	2	15.8	56	143.8

Table 2.4.2.1: Data table for the potential (2.4.2.4)[Maumba et al, 2011]

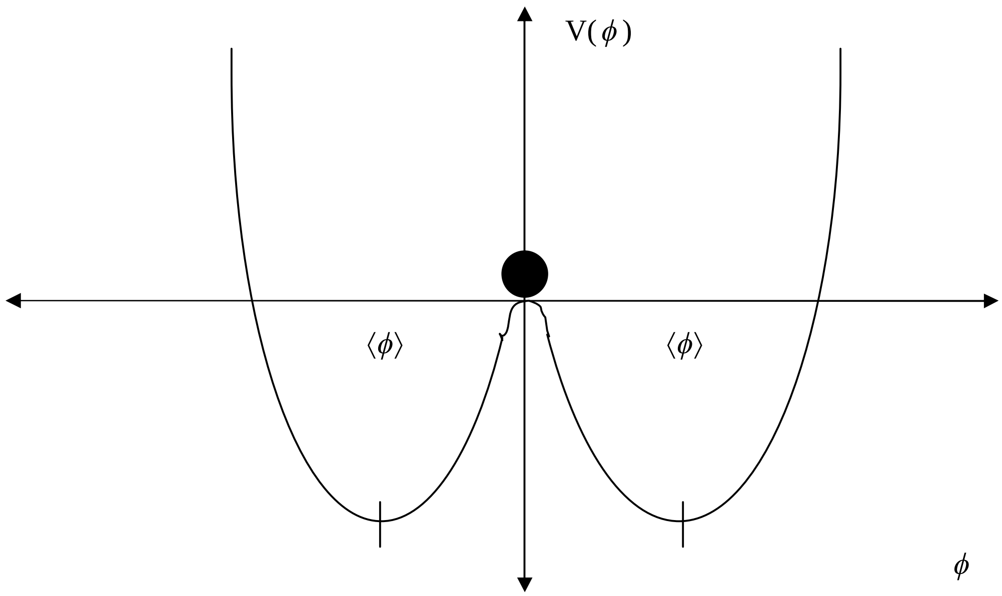


Figure 2.4.2.1: An example of the potential for a model with spontaneous symmetry breaking

By the condition that

$$\partial V/\partial\phi = 0 \tag{2.4.2.5}$$

the minimum of the potential and the value of the potential at the minimum are found to be

$$\langle\phi\rangle = \pm\sqrt{\frac{M^2}{\lambda}} \tag{2.4.2.6}$$

$$V(\langle\phi\rangle) = -\frac{M^2}{4\lambda} \tag{2.4.2.7}$$

The ground state of the system is either $+\langle\phi\rangle$ or $-\langle\phi\rangle$ and the reflection symmetry $\phi \leftrightarrow -\phi$ present in the Lagrangian is not respected by the vacuum state hence spontaneous symmetry breaking. From the definition of the stress energy-momentum tensor in terms of the Lagrangian [Kolb, 1986]

$$T_{\mu\nu} = -\partial_\mu\phi\,\partial_\nu\phi - Lg_{\mu\nu}\,, \tag{2.4.2.8}$$

the energy density of the vacuum should be

$$\begin{aligned}\langle T_{oo}\rangle &= \rho_V \\ &= -L \\ &= V(\phi) \\ &= -\frac{M^4}{4\lambda}\end{aligned} \tag{2.4.2.9}$$

The contribution of the vacuum energy to the total energy density today must be smaller than the critical density $\rho_c = 1.88 \times 10^{-29} h^2 g\, cm^{-3}$ [Coughlan, 1991]. Since this number is quite small, it is appropriate to require that $\rho_V = 0$. This can be accomplished by adding to the Lagrangian a constant factor of $\frac{+M^4}{4\lambda}$. The constant term will not affect the equations of motion but will only cancel the present vacuum energy.

High-temperature symmetry restoration requires that the effective finite temperature mass of ϕ be expressed as the zero-temperature mass $-M^2$ and an interaction mass, [Kolb, 1986]

$$M_{\text{int}}{}^2 \approx \alpha\lambda T^2, \qquad (2.4.2.10)$$

where α is a constant of order unity.

If

$$M_T{}^2 = -M^2 + M^2{}_{\text{int}} < 0, \qquad (2.4.2.11)$$

then the minimum of the potential will be at $\phi \neq 0$ (SSB), while if

$$M_T{}^2 = -M^2 + M^2{}_{\text{int}} > 0 \qquad (2.4.2.12)$$

the effective mass term will be positive and the minimum of the potential will be at $\phi = 0$, that is, symmetry is restored. However, there is a critical temperature

$$T_c = \frac{M}{\sqrt{\alpha\lambda}}, \qquad (2.4.2.13)$$

above which

$$\langle\phi\rangle = 0 \qquad (2.4.2.14)$$

A detailed approach to symmetry restoration is to account for the effect of the ambient background gas in the calculation of the higher-order quantum corrections to the classical potential; in particular, the finite temperature potential will include a temperature-dependent term that represents the free energy of ϕ particles at temperature *T*. To one loop, the full potential can be evaluated as follows:

First, the finite temperature one loop effective potential $V_{eff}^{(1)T}(\phi)$ is written as [Brandenberger, 1985]

$$V_{eff}^{(1)\beta}(\phi) = U(\phi) - \frac{i}{2}\left(\frac{1}{-i\beta}\right)\sum_{n=-\infty}^{\infty}\int\frac{d^3\vec{k}}{(2\pi)^3}\ln\left[1 - \frac{\lambda\phi^2}{2\left(\omega_n^2 - \vec{k}\right)}\right] + \text{const.}$$

(2.4.2.15)

$$= U(\phi) + \frac{1}{2\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3\vec{k}}{(2\pi)^3}\ln\left[1 + \frac{\lambda\phi^2}{2\left(\frac{4\pi^2}{\beta^2}n^2 - \vec{k}^2\right)}\right] + \text{const.}$$

$$V_{eff}^{(1)\beta}(\phi) = U(\phi) + \frac{1}{2\beta}\int\frac{d^3\vec{k}}{(2\pi)^3}\sum_{n=-\infty}^{\infty}\ln\left(\frac{4\pi^2}{\beta^2}n^2 + E_{\vec{k}}^2\right) + \text{const.} \qquad (2.4.2.16)$$

with

$$E_{\vec{k}}^2 = \vec{k}^2 + \frac{\lambda\phi^2}{2} \qquad (2.4.2.17)$$

The part containing E is calculated by first differentiating it with respect to E, then summing the resulting series using the identity

$$\sum_{n=1}^{\infty}\frac{\alpha}{\alpha^2 + n^2} = -\frac{1}{2\alpha} + \frac{\pi}{2}\coth\pi\alpha$$

(2.4.2.18)

$$= -\frac{1}{2\alpha} + \frac{\pi}{2} + \frac{\pi e^{-2\pi\alpha}}{1 - e^{-2\pi\alpha}}$$

and finally integrating the resulting function; this yields

$$V_{eff}^{(1)\beta}(\phi) = U(\phi) + \int\frac{d^3\vec{k}}{(2\pi)^3}\left[\frac{E_{\vec{k}}}{2} + \frac{1}{\beta}\ln\left(1 - e^{-\beta E_{\vec{k}}}\right)\right] + c\text{onst} \qquad (2.4.2.19)$$

Using the definition that

$$i\int_{-\infty}^{\infty}\frac{dx}{2\pi}\ln\left(-\varepsilon^2 + \alpha^2 - i\delta\right) = \alpha + \text{constant} \qquad (2.4.2.20)$$

then equation (2.4.2.19) becomes

$$V_{eff}^{(1)\beta}(\phi) = U(\phi) + \frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\ln(-k_o^2 + E_{\vec{k}}^2 - i\delta) + \frac{1}{\beta}\int\frac{d^3\vec{k}}{(2\pi)^3}\ln\left(1 - e^{-\beta E_{\vec{k}}}\right) \qquad (2.4.2.21)$$

When the second term on the right hand side of equation (2.4.2.21) is rotated to Euclidean space, the equation reduces to

$$V_{eff}^{(1)\beta}(\phi) = U(\phi) + \frac{1}{2}\int \frac{d^4k_E}{(2\pi)^4} \ln\left(1 + \frac{\lambda\phi^2}{2k_E^2}\right) + \frac{1}{2\pi^2\beta}\int dk\, k^2 \ln\left(1 - e^{-\beta E_k^-}\right) \qquad (2.4.2.22)$$

where the first two terms of (2.4.2.22) are the zero temperature terms [Edward, 1986]. The equation shows that introducing a finite temperature gives an extra term

$$\Delta V_{eff}^{(1)\beta}(\phi) = \frac{1}{2\pi^2\beta^4}\int \varepsilon^2 \ln\left(1 - e^{-\left\{\left(\varepsilon^2 + \frac{\lambda\phi^2}{2}\right)\beta^2\right\}^{\frac{1}{2}}}\right) d\varepsilon \qquad (2.4.2.23)$$

in the one loop effective potential; hence, the full potential takes the form

$$V_{eff}^{(1)\beta}(\phi) = V_{eff}^{(1)T=0}(\phi) + \frac{1}{2\pi^2\beta^4} I(\alpha) \qquad (2.4.2.24)$$

with

$$I(\alpha) = \int \varepsilon^2 \ln\left(1 - e^{-\sqrt{\varepsilon^2 + \alpha^2}}\right) d\varepsilon$$

and (2.4.2.25)

$$\alpha^2 = \frac{\lambda}{2}\phi^2\beta^2$$

For high temperatures, $I(\alpha)$ can be expanded into a power series in α^2 to get

$$I(\alpha) = \int_0^\infty \varepsilon^2 \ln\left(1 - e^{-\varepsilon}\right) d\varepsilon + \frac{\alpha^2}{2}\int_0^\infty \varepsilon \frac{e^{-\varepsilon}}{1 - e^{-\varepsilon}} d\varepsilon + O(\alpha^4)$$

$$= -\sum_{n=1}^\infty \frac{1}{n}\int_0^\infty \varepsilon^2 e^{-n\varepsilon} d\varepsilon + \frac{\alpha^2}{2}\int_0^\infty \frac{\varepsilon e^{-\varepsilon}}{1 - e^{-\varepsilon}} d\varepsilon + O(\alpha^4)$$

$$= -\frac{\pi^4}{45} + \alpha^2\frac{\pi^2}{12} + O(\alpha^4)$$

(2.4.2.26)

In this approximation, the scalar field theory result becomes

$$V_{eff}^{(1)\beta}(\phi) = V_{eff}^{(1)T=0}(\phi) + \frac{\lambda}{48\beta^2}\phi^2 - \frac{\pi^2}{90\beta^4} + O(\beta^4) \qquad (2.4.2.27)$$

The term proportional to T^4 is minus the leading contribution to the free energy and the second term is the interaction mass term for ϕ. Equation (2.4.2.27) has a critical temperature $T_c = \frac{2M}{\sqrt{\lambda}}$ above which the symmetry is restored and the phase transition from the symmetric to the broken phase can either be first order or higher order [Daniel, 1981]. If at T_c there is a barrier between $\phi = 0$ and the SSB minimum $\phi = \sigma$, then the change in ϕ will be discontinuous signaling a first order transition. However, if no barrier is present at T_c, then the change in ϕ will be continuous signaling a higher order transition. But at some temperature $T \le T_c$, the $\phi = 0$ phase is a metastable phase and this phase is terminated by the decay of the false vacuum by tunneling.

2.5. INFLATIONARY MODEL

2.5.1. INTRODUCTION

Grand unified models of particle physics predict that the state of thermal equilibrium of a quantum field will undergo a phase transition at a critical temperature T_c of order of the grand unification scale $\approx 10^{14}\,GeV$ [Kennedy et al., 1981]. Hence, if this quantum field began in an arbitrary hot state at the big bang singularity, then such a phase transition would have occurred in the early universe as the field cooled to below T_c as a result of the expansion of the universe. An appealing consequence that has been predicted so far is the possible existence of inflation. In 1981, Guth suggested this model as a panacea to the cosmological problems guided by the *SU(5)* grand unified theory [Guth, 1981]. He noted that if expansion of the early universe is speeded up in such a way that scale factor grows by a huge factor during a short period, then the cosmological problems could be avoided. The model has become so popular that work is still going on this idea. Here, the model is analyzed from the view point of the $k = 0$ curvature parameter. In particular, both the qualitative and quantitative aspects of the model are analyzed.

2.5.2. QUALITATIVE ASPECT OF THE INFLATIONARY MODEL

In the original scenario of inflation, Alan Guth suggested that the zero temperature potential energy of the field ϕ, known as the inflaton field, could have a local minimum with energy density ρ_v above that of the absolute minimum [Guth, 1981]. If the inflaton settled into the local minimum over a sufficiently large region of space as it cooled, then it could remain trapped there in a metastable state until it tunneled through to the true minimum. While in a false vacuum, the kinetic and spatial derivative energies of the field would be negligible compared with its potential energy ρ_v. Hence, the stress-energy tensor $T_{\alpha\beta}$ of the field would be dominated by the vacuum energy density ρ_v and the effect of the inflaton on the dynamics of the universe via Einstein's equations would be like having a large positive cosmological constant. In the Robertson-Walker model this produces the de-Sitter solution which yields an expansion of the universe on an exponential time scale.

However, Guth's model [Guth, 1981] suffers from the problem of not allowing the inflaton field to exit from the inflationary phase in such a way as to evolve to the presently observed universe. The new inflationary model [Linde, 1982; Albrechdt and Steinhardt, 1982], was proposed mainly to overcome the problem of obtaining a graceful exit from the inflationary phase. It is associated with the evolution of a weakly-coupled scalar field, the inflaton, which was initially displaced from the minimum of its potential. In particular, the inflaton field has the one-loop effective potential of the form shown in figure (2.5.2.1) below. It is assumed that, initially, the field is in thermal equilibrium at high temperature and in a symmetric minimum of V_T at $\phi = 0$ as in the curve (a). As the field cools due to expansion, it will transition into the metastable state shown in curve (c) by the local minimum of V_T at $\phi = 0$ for intermediate temperatures. At lower temperatures, the dip in V_T goes away and the dynamics of the field is governed by the classical evolution of ϕ in the zero-temperature potential of curve (d) with initial conditions $\phi \approx 0$ and $\dot{\phi} \approx 0$. If $V(\phi)$ is very flat near $\phi = 0$, then it will take a long time for ϕ to reach ϕ_c, i.e., as the temperature drops below T_c, the state of lowest energy shifts in the false vacuum at $\phi = 0$ until the field tunnels across the potential ($V(\phi) > 0$) barrier and rolls down the $V(0)$ slope to its true vacuum as shown in figure (2.5.2.1) below. During this time, the stress-energy tensor of the field ϕ will be dominated by ρ_v (ρ_v is vacuum energy above the true minimum of the flat part of the potential $V(\phi)$). Consequently, the universe has extra energy ρ_v at its disposal which will have dynamical effects via the Friedman equation

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi G}{3}\rho_v \qquad (2.5.2.1)$$

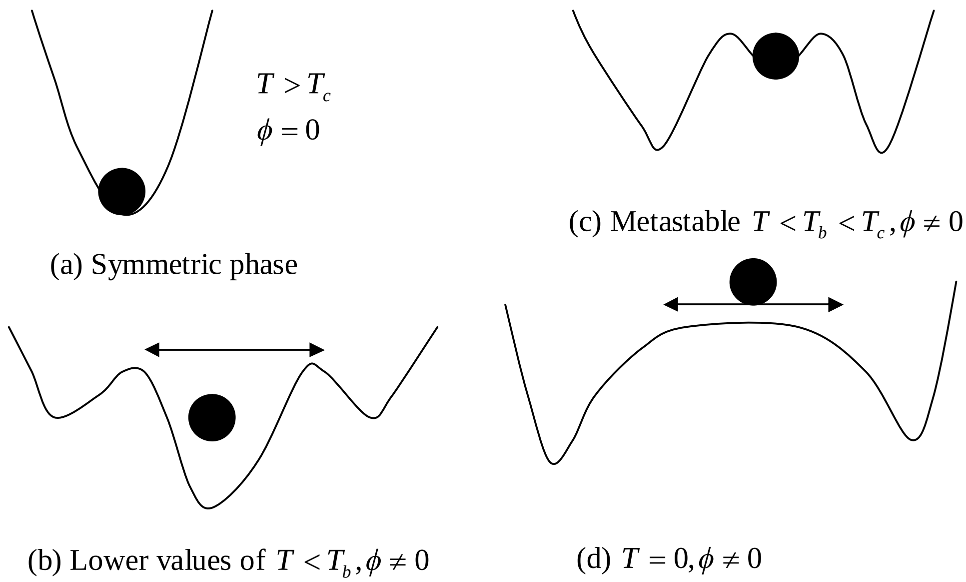


Fig 2.5.2.1: The behavior of the one-loop effective potential $V_T^{(1)}$.

Figure 2.5.2.1 shows the behavior of the one-loop effective potential $V_T^{(1)}$ in Coleman-Weinberg models [Sidney and Erick, 1973]. At high temperatures, $V_T^{(1)}$ has the form shown in curve (a), with a single minimum at $\phi = 0$. At lower values of T, $V_T^{(1)}$ develops side minima as shown in curve (b). At still lower temperatures $V_T^{(1)}$ has the form shown in curve (c). Finally curve (d) represents the effective potential at $T = 0$ which can also be represented by figure 2.5.2.2 below after symmetry breakdown.

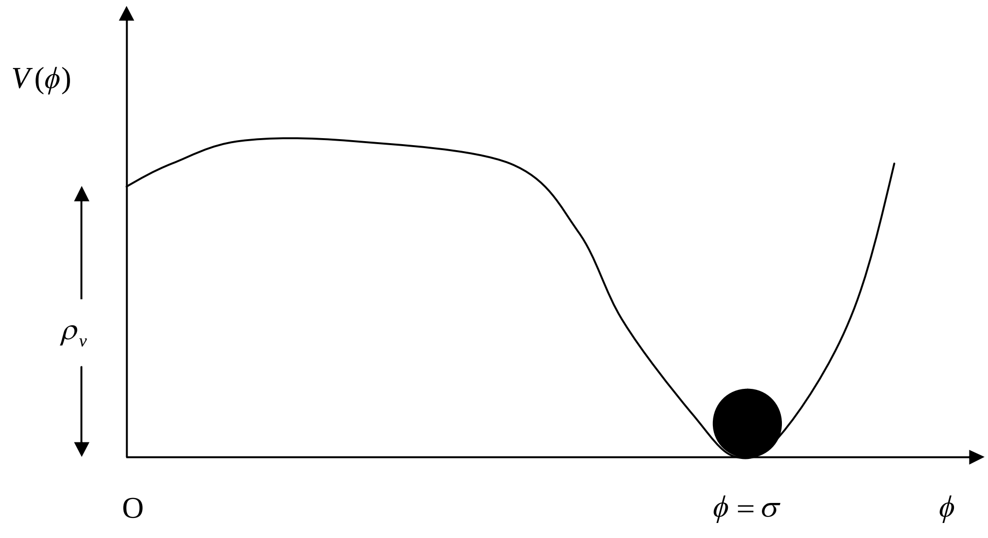


Fig 2.5.2.2: Effective potential for a field ϕ.

2.5.3. QUANTITATIVE ASPECT OF THE INFLATIONARY MODEL

Guth (1981) proposed the inflationary model of the early universe to provide a physical basis for the high-order accuracy that is observed in the present universe. Addressing to flatness and horizon puzzles of the big-bang theory, he noted that if expansion of the early universe speeded up in such a way that the scale factor grew by a factor 10^{28} within a short period, then the cosmological puzzles could be avoided. To achieve this, he argued that even though the cosmological term Λ which gives vacuum energy density

$$\rho_v = \frac{\Lambda}{8\pi G_N} \qquad (2.5.3.1)$$

is almost negligible in the present universe, it was very high in the early universe and could have dictated early cosmic dynamics. He considered that the Friedman equation should take the form [Guth, 1981]

$$\begin{aligned} \left(\frac{\dot{S}}{S}\right)^2 &= \frac{\Lambda}{3} \\ &= H_\Lambda^2 \qquad (2.5.3.2) \end{aligned}$$

as Λ term dominates the curvature term and the vacuum contains no particles. Equation (2.5.3.2) when integrated yields the de Sitter solution

$$S(t) = S(t_i)e^{H_\Lambda (t-t_i)}, \quad (2.5.3.3)$$

where t_i is the initial time. If inflationary period ends by time t_f, then $e^{H_\Lambda (t_f - t_i)}$ is required to be 10^{28}, i.e.,

$$H_\Lambda (t_f - t_i) \approx 65 \quad (2.5.3.4)$$

To explain how such a high cosmological term could arise in the early universe, Guth drew *SU(5)* grand unified theory into service and used the Higgs mechanism for SSB; he used the Higgs potential with thermal corrections as

$$V^T(\phi) \approx -\frac{1}{2}\mu^2\left(\phi^2 + \frac{1}{12}T^2\right) + \frac{1}{4}\lambda\phi^4 + \frac{1}{8}\lambda\phi^2 T^2 - \frac{\pi^2}{90}T^4 \quad (2.5.3.5)$$

for Higgs scalar ϕ satisfying the condition

$$\frac{1}{2}\dot{\phi}^2 << V(\phi) \quad (2.5.3.6)$$

The potential yields stationary points when

$$\begin{aligned}\frac{dV^T(\phi)}{d\phi} &= \phi\left(-\mu^2 + \lambda\phi^2 + \frac{\lambda}{4}T^2\right) \\ &= 0 \quad (2.5.3.7)\end{aligned}$$

from which the corresponding stationary points are found to be

$$\phi = 0 \quad (2.5.3.8)$$

and

$$\phi = \pm\frac{\sqrt{4\frac{\mu^2}{\lambda}\left(1 - \frac{T^2}{T_c^2}\right)}}{2} \quad (2.5.3.9)$$

where

$$T_c = \frac{2\mu}{\sqrt{\lambda}} \quad (2.5.3.10)$$

is the critical temperature. So ϕ tunnels through the barrier $T = T_c$ and, hence, $\phi = 0$ yields the false vacuum. However, when $T << T_c$, ϕ settles in the true vacuum states

$$\phi = \pm\frac{\mu}{\sqrt{\lambda}} \quad (2.5.3.11)$$

which is separated by a domain wall $\phi = 0$; this is a manifestation of spontaneous symmetry breaking. The vacuum energy density released when the inflaton field ϕ rolls down from the state $\phi = 0$ to states $\phi = \pm \frac{\mu}{\sqrt{\lambda}}$ is given by

$$\rho_v = V(\phi = 0) - V\left(\phi = \pm \frac{\mu}{\sqrt{\lambda}}\right) \tag{2.5.3.12}$$

$$= -\frac{1}{24}\mu^2 T^2 - \frac{\pi^2}{90}T^4 - \left[-\frac{1}{2}\mu^2\left(\frac{\mu^2}{\lambda} + \frac{1}{12}T^2\right) + \frac{\mu^4}{4\lambda} + \frac{1}{8}\mu^2 T^2 - \frac{\pi^2}{90}T^4\right]$$

$$= \frac{\mu^4}{4\lambda} - \frac{1}{8}\mu^2 T^2 \approx \frac{\mu^4}{4\lambda} \tag{2.5.3.13}$$

Then, at $T << T_c$, equation (2.5.3.1) reduces to

$$\Lambda = 8\pi G_N \rho_v$$

$$= 2\pi G_N \frac{\mu^4}{\lambda} \tag{2.5.3.13}$$

In this scenario, at the epoch of phase transition, vacuum energy is released as latent heat and super-cooled universe with temperature $T = 0$ is heated up to the temperature T_c.

Soon after the proposal of the original model [Guth, 1981], it was realized that the model suffers from the problem of exponential exit i.e. the model prompted the question “How did inflation end so that the observed particle-driven universe could emerge?” Earlier, Coleman [Coleman, 1977], Callan and Coleman [Callan and Coleman, 1977] as well as Coleman and De Luccia [Coleman and De Luccia, 1980] had obtained a mechanism for a transition from false vacuum to true vacuum. According to this mechanism, when potential barrier is large enough, transition from false to true vacuum takes place through nucleation of bubbles which expand with speed rapidly approaching that of light. The released energy during transition is transferred to walls of accelerating bubbles. As a result, the interior of the bubble approaches a vacuum and its nucleation rate takes the form [Michael and Robert, 1992]

$$\Im = Ce^{-A_E} \tag{2.5.3.14}$$

where A_E is the Euclidean action and $C \approx 10^{14} GeV$ as is the scale for GUT phase transition. Using the potential (2.5.3.5), the action becomes

$$A_E = \int d^4 x_E \sqrt{g}\left[\frac{1}{2}\dot{\phi} + V^T(\phi)\right] \tag{2.5.3.15}$$

Hence, nucleation rate of bubbles is very high in the false vacuum state $\phi = 0$ since $V^T(\phi) < 0$. This means that large number of bubbles are formed when $\phi \approx 0$ near $T \geq T_c$ with the Higgs potential of (2.5.3.5). The situation is analogous to the formation of large number of bubbles when water falls from a high altitude; however, when it flows on a flat surface, bubbles are not formed. This means that bubbles are created during fast motion, but in slow motion possibility of bubble creation is almost negligible. When the water bubbles expand after formation, energy inside it is transferred to their walls. These bubbles break when outside pressure is unable to balance inside pressure. Thus, this is what should happen when ϕ rolls down the hill tunneling through the temperature barrier $T = T_c$. Expansion speed of these bubbles is limited by the speed of light and the universe expands with superluminal speed, so that the bubbles move randomly without colliding with each other. The bubbles do not coalesce for mixing up of their interiors to yield true vacuum. Unless the transition from false to true vacuum takes place, universe cannot come out of the exponential phase. This is the inflationary exit problem.

In 1982 Linde [Linde, 1982] and, Albrecht and Steinhardt [Albrechdt and Steinhardt, 1982] realized that the graceful exit problem of Guth's model is caused by the Higgs potential as the field ϕ falls rapidly from the potential height. So they proposed another inflationary model, popularly known as the new inflationary model. In this model, temperature dependent Higgs potential is replaced by temperature-dependent Coleman-Weinberg potential

$$V^T(\phi) = \frac{18T^4}{\pi^2}\int_0^\infty dx x^2 \ln\left[1 - e^{-\psi}\right] + \frac{25g^4}{128\pi^2}\left[\phi^4 \ln\left(\frac{\phi}{\sigma}\right) - \frac{\phi^4}{4} + \frac{\sigma^4}{4}\right] \qquad (2.5.3.16)$$

where, $g^2 = 0.3$, $\sigma \approx 10^{14}\,GeV$ and $\psi = \sqrt{x^2 + \frac{5g^2}{12T^2}\phi^2}$ for GUTs. Hence, at $\phi = 0$, equation (2.5.3.16) becomes

$$V^T(\phi) = \frac{18\Gamma(3)\zeta(3)}{\pi^2}T^4 + \frac{25g^4}{512\pi^2}\sigma^4 \qquad (2.5.3.17)$$

They argued that near $\phi = 0$, Γ should vanish so that only one bubble is formed due to this potential in the false vacuum. Moreover, when ϕ tunnels through the temperature barrier, then

$$\frac{d^2V^T(\phi)}{d\phi^2} \approx 0 \qquad (2.5.3.18)$$

in the super-cooling stage. It shows that this potential remains almost flat for $\phi < \sigma$ and very near to $\phi = \sigma$ it falls to zero. So ϕ rolls down very slowly and, hence, only one bubble formed in the false vacuum state makes transition from the false to true vacuum state.

2.6. THE BOLTZMANN TRANSPORT EQUATION

The Boltzmann Equation, established by Ludwig Boltzmann in 1872 (a detailed account can be found in Cercignani, 1975), is an integro-differential equation that is well known to describe the behaviour of dilute gases. The equation still forms the basis for the kinetic theory of gases and has proved fruitful not only for a study of classical gases that Boltzmann had in mind, but also properly generalized, for studying electron transport in solids and plasmas, neutron transport in nuclear reactors, phonon transport in superfluids and radiative transfer in planetary and stellar atmospheres [Cercignani, 1975]. In spite of its relative importance to many areas of science and engineering, its relevance to neutrino interactions is not well known. Consequently, the present section is primarily intended to demonstrate how the equation can be extended to the description and, hence, its connection to neutrino interactions in the early expanding universe.

2.6.1. The Neutrino Boltzmann Equation

It is suggested that when considering a system of N particles of which the state is prescribed by its position and velocity coordinates $(\vec{x}_1,......,\vec{x}_N;\vec{u}_1,.........\vec{u}_N)$ respectively, the time evolution of the system can be studied using Newton's laws of motion or by Hamilton's equations. However, if N is large then it is not realistic to solve the equations of motion for all the position and velocity coordinates [Gupta, 1990]. Hence, a distribution function $f(\vec{x},\ \vec{u},\ t)$, which gives the particle number density in phase-space (six-dimensional space $(\vec{x},\ \vec{u})$) at time t, should be introduced. A dynamical theory at this level requires an equation which will govern the time-evolution of $f(\vec{x},\ \vec{u},\ t)$. The time derivative of $f(\vec{x},\ \vec{u},\ t)$ for a neutral system or fluid is given by the Boltzmann transport equation while, the corresponding equation for an ionized system (plasma) is the Vlasov equation [Choudhuri, 1998]. However, in this study the main interest will be the establishment of a neutrino Boltzmann transport equation.

For a statistical treatment of a neutrino system, it is useful to introduce the concept of an ensemble [Gupta, 1990], which is a set of many replicas of the same system that are identical in all other respects apart from being in different states at an instant of time. Hence, each member of the ensemble can be represented by a point in the phase space at an instant of time and their evolutions will correspond to different trajectories in the phase space. If the density $\rho_{ens}\left(q_s(t),\ p_s(t),\ t\right)$ is measured as a function of time, then Liouville's theorem [Cercignani, 1975] will require the time derivative of this density along the trajectory to be zero, i.e.,

$$\frac{D\rho_{ens}}{Dt} = 0\,, \tag{2.6.1.1}$$

where D/Dt is the time derivative operator along the trajectory. If $(q_s,\ p_s)$ and $(q_s + \delta q_s,\ p_s + \delta p_s)$ denote the states of the neutrino density at times t and $t + \delta t$ on this trajectory respectively, then

$$\frac{D\rho_{ens}}{Dt} = \lim_{\delta t \to 0} \frac{\rho_{ens}(q_s + \delta q_s, p_s + \delta p_s, t + \delta t) - \rho_{ens}(q_s, p_s, t)}{\delta t} \tag{2.6.1.2}$$

Expansion in a Taylor series to linear terms in small quantities gives [Thomas/Finney, 1984]

$$\rho_{ens}(q_s + \delta q_s, p_s + \delta p_s, t + \delta t) = \rho_{ens}(q_s, p_s, t) + \sum_s \delta q_s \frac{\partial \rho_{ens}}{\partial q_s} + \sum_s \delta p_s \frac{\partial \rho_{ens}}{\partial p_s} + \delta t \frac{\partial \rho_{ens}}{\partial t} \tag{2.6.1.3}$$

Substituting equation (2.6.1.3) in equation (2.6.1.1) gives

$$\frac{D\rho_{ens}}{Dt} = \frac{\partial \rho_{ens}}{\partial t} + \sum_s \dot{q}_s \frac{\partial \rho_{ens}}{\partial q_s} + \sum_s \dot{p}_s \frac{\partial \rho_{ens}}{\partial p_s} \tag{2.6.1.4}$$

To be consistent with Liouville's theorem, it has to be shown that the right hand side of equation (2.6.1.4) is equal to zero. This can be done by considering the equation of continuity which applies to ordinary fluids in ordinary space or to ensemble point distributions in phase space where the total number of ensemble points is conserved in time. If ρ be the density of the neutrino fluid in some space, then the neutrino mass flux $\int \rho dV$ within a volume can change only due to its mass flux across the surface bounding that volume, i.e.,

$$\frac{\partial}{\partial t} \int \rho dV = -\int \rho \vec{u}.d\vec{A} \tag{2.6.1.5}$$

where $\int \rho \vec{u}.d\vec{A}$ is the outward neutrino mass flux through the bounding surface with the negative sign implying that the outward mass flux reduces the mass within the bounded volume. The surface integral can be transformed into a volume integral by using Gauss's theorem [Gupta, 2004] so that

$$\int \left[\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{u}) \right] dV = 0 \qquad (2.6.1.6)$$

Since this is true for any arbitrary volume, then

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{u}) = 0 \qquad (2.6.1.7)$$

which is the continuity equation [Jackson, 1999]. This equation is then applied to neutrino fluid in phase space by taking $\rho = \rho_{ens}$ and $\vec{u} = \left(\dot{q}_s ,\ \dot{p}_s \right)$ so that

$$\frac{\partial \rho_{ens}}{\partial t} + \sum_s \frac{\partial}{\partial q_s}(\rho_{ens}\ \dot{q}_s) + \sum_s \frac{\partial}{\partial p_s}(\rho_{ens}\ \dot{p}_s) = 0 \qquad (2.6.1.8)$$

i.e.

$$\frac{\partial \rho_{ens}}{\partial t} + \sum_s \dot{q}_s \frac{\partial \rho_{ens}}{\partial q_s} + \sum_s \dot{p}_s \frac{\partial \rho_{ens}}{\partial p_s} + \rho_{ens} \sum_s \left(\frac{\partial \dot{q}_s}{\partial q_s} + \frac{\partial \dot{p}_s}{\partial p_s} \right) = 0 \qquad (2.6.1.9)$$

Using Hamilton's equations [Gupta, 1990], it is found that

$$\frac{\partial \dot{q}_s}{\partial q_s} + \frac{\partial \dot{p}_s}{\partial p_s} = \frac{\partial}{\partial q_s}\left(\frac{\partial H}{\partial p_s} \right) - \frac{\partial}{\partial p_s}\left(\frac{\partial H}{\partial q_s} \right) = 0 \qquad (2.6.1.10)$$

Hence, the right hand side of equation (2.6.1.4) is zero and Liouville's theorem is established.

Considering that the ensemble points initially inside the phase-space volume element

$$d^n q_s d^n p_s \qquad (2.6.1.11)$$

fills the volume element

$$d^n q_s^{'} d^n p_s^{'} \qquad (2.6.1.12)$$

after some time *t*, then from the conservation of ensemble points

$$\rho_{ens} d^n q_s d^n p_s = \rho_{ens}^{'} d^n q_s^{'} d^n p_s^{'}, \qquad (2.6.1.13)$$

where ρ_{ens} and $\rho_{ens}^{'}$ are the corresponding densities in the phase-space volume elements. Since Liouville's theorem demands that

$$\rho_{ens} = \rho_{ens}^{'}, \qquad (2.6.1.14)$$

then

$$d^n q_s d^n p_s = d^n q_s^{'} d^n p_s^{'} \qquad (2.6.1.15)$$

A state of N neutrinos is prescribed by $6N$ position and velocity coordinates. The corresponding $6N$-dimensional phase space is referred to as the $\Gamma-\text{space}$ and a state of the system is represented by a point in this $\Gamma-\text{space}$. Also a six-dimensional space with six dimensions corresponding to the position and velocity $(x,\ t)$ of an individual neutrino can be introduced. This is referred to as the $\mu-\text{space}$ and each of the N neutrinos would be represented by a point in this $\mu-\text{space}$ at an instant of time. Hence, N points in the $\mu-\text{space}$ will be required to represent a state of a system of N neutrinos. Thus, there is a correspondence between the representations in the $\Gamma-\text{space}$ and $\mu-\text{space}$, i.e., the state of the system represented by one point in the $\Gamma-\text{space}$ gets mapped into a configuration of N points in the $\mu-\text{space}$ and the time evolution of the system will give rise to a trajectory in the $\Gamma-\text{space}$ that gets mapped to N trajectories of the N points in the $\mu-\text{space}$. To quantitatively define the distribution function $f(x,u,t)$ in $\mu-\text{space}$, a small number δn of points in a small volume element δV of the $\mu-\text{space}$ is considered so that

$$f(x,u,t)=\lim_{\delta V\to 0}\frac{\delta n}{\delta V} \tag{2.6.1.16}$$

If the N neutrinos do not interact with each other but move under the influence of some external potential $\phi(x)$ alone, then one can introduce a Hamiltonian

$$H(x,u,t)=\frac{1}{2}u^2+\phi(x) \tag{2.6.1.17}$$

appropriate for $\mu-\text{space}$. The Hamiltonian can then be used in Hamilton's equations to give the neutrino equations of motion. However, to incorporate interactions amongst the particles can cause a problem, i.e., if a particle with coordinates (x,u) interacting with a nearby particle $(x^{'},u^{'})$ is considered, then the interaction can be described by a potential of the form $\phi(x,x^{'})$ and, hence, incorporating it in the Hamiltonian for the $\Gamma-\text{space}$ would not be difficult. But it cannot be written in the form $\phi(x)$ and, therefore, cannot be incorporated in the Hamiltonian for the $\mu-\text{space}$. Hence, a Hamiltonian formulation of the dynamics of N particles is possible in the $\Gamma-\text{space}$; however, the same can be possible in the $\mu-\text{space}$ only if the mutual interactions amongst the particles can be neglected. If the mutual interactions are negligible, then the system is collisionless and, thus,

$$\frac{Df}{Dt}=0 \tag{2.6.1.18}$$

Therefore, the total derivative can be put in the form

$$\frac{Df}{Dt} \equiv \frac{\partial f}{\partial t} + u.\nabla f + \dot{u}.\nabla_u f \tag{2.6.1.19}$$

Thus, from equations (2.6.1.18) and (2.6.1.19), it follows that

$$\frac{\partial f}{\partial t} + u.\nabla f + \dot{u}.\nabla_u f = 0 \tag{2.6.1.20}$$

This is the familiar collisionless Boltzmann equation [Choudhuri, 1998] and the value of the distribution function $f(\vec{x}, \vec{u}, t)$ does not change along the trajectory of a particle provided collisions are neglected.

However, collisions can produce changes in $f(\vec{x}, \vec{u}, t)$ due to two reasons:

1. Some neutrinos originally having velocity $\vec{u}$ may have other velocities after collisions; this causes a decrease in $f(\vec{x}, \vec{u}, t)$.
2. Some neutrinos originally having other velocities may have the velocity $\vec{u}$ after collisions, thereby causing an increase in $f(\vec{x}, \vec{u}, t)$.

Consequently, equation (2.6.1.18) must be modified to the suggested form

$$\frac{Df}{Dt} d^3x\, d^3u = -C_{out} + C_{in} \tag{2.6.1.21}$$

where C_{out} and C_{in} are the rates at which neutrinos leave and enter the elementary volume d^3xd^3u of the $\mu-$space due to collisions. To describe neutrino binary collisions, it is considered that two neutrinos with initial velocities u and u_1 acquire velocities $u^{'}$ and $u_1^{'}$ after a collision. Since all neutrinos are assumed similar, i.e., have the same mass *m*, the conservation laws of momentum and energy imply [Blatt, 1986]

$$u + u_1 = u^{'} + u_1^{'} \tag{2.6.1.22}$$

$$u^2 + {u_1}^2 = (u^{'})^2 + (u_1^{'})^2 \tag{2.6.1.23}$$

To calculate the final velocities $u^{'}$ and $u_1^{'}$ from the initial velocities, six scalar equations are required since $u^{'}$ and $u_1^{'}$ have six scalar components. Four of these are provided by the equations (2.6.1.22) through (2.6.1.23) and the fifth condition comes from the fact that collisions are coplanar if the force of interaction between the two particles is always radial ,i.e., $u^{'}$ will have to lie in the plane of u and u_1, forcing $u_1^{'}$ also to lie in the same plane. A sixth condition is still needed. This comes from the nature of the interaction between neutrinos as the outcome of neutrino collisions is not expected to be independent of the nature of interaction. Since the interest is in a statistical treatment, then knowledge of the probability of neutrino deflection in different directions becomes necessary. This can be determined by introducing the concept of a differential scattering cross-section. To quantitatively define this quantity, a beam of neutrinos of number density n_1 and velocity u_1 colliding with a second beam of number density n and velocity u is considered. A neutrino in the second beam will experience a flux

$$I = |u - u_1| n_1 \tag{2.6.1.24}$$

of neutrinos from the first beam and the number of collisions per unit volume per unit time which will deflect neutrinos from the second beam into a solid angle $d\Omega$ is δn_c. This number is proportional to [Bransden and Joachain, 1989]:

- the number density *n* of neutrinos in the second beam;

- the flux *I* these neutrinos are exposed to and;

- the solid angle $d\Omega$

This information can be put together to yield

$$\delta n_c = \sigma(u, u_1 | u', u_1').n.|u - u_1| n_1.d\Omega, \tag{2.6.1.25}$$

where the constant of proportionality $\sigma(u, u_1 | u', u_1')$ is the differential scattering cross-section. The conservation laws (2.6.1.22) and (2.6.1.23) along with the condition that neutrinos from the second beam are required to go into the solid angle $d\Omega$ completely determines the final velocities u' and u_1'.

To evaluate the term C_{out}, a stream of neutrinos having their velocity vectors within d^3u and neutrinos of velocity vectors within d^3u_1 is suggested. The first stream makes up a beam with number density $n = f(x, u, t)d^3u$ and velocity u, whereas the second constitutes a beam with number density $n = f(x, u_1, t)d^3u_1$ and velocity u_1. Substituting for n and n_1 in equation (2.6.1.25) leads to

$$\delta n_c = \sigma(u, u_1 | u', u_1') |u - u_1| f(x, u, t) f(x, u_1, t).d\Omega\, d^3u\, d^3u_1 \tag{2.6.1.26}$$

Since C_{out} is the total number of collisions per unit time within the volume $d^3x\, d^3u$, it is obtained by multiplying δn_c with d^3x and then integrating the result over all Ω and u_1. That is,

$$C_{out} = d^3x\, d^3u \int d^3u_1 \int d\Omega\, \sigma(u, u_1 | u', u_1') |u - u_1|\, f(x, u, t)\, f(x, u_1, t) \qquad (2.6.1.27)$$

To evaluate C_{in}, reverse collisions between neutrinos with velocities in d^3u' and neutrinos with velocities in d^3u_1' such that their velocities after collisions lie within d^3u and d^3u_1, respectively, is considered. In analogy with equation (2.6.1.26) the number of such collisions per unit volume per unit time is

$$\delta n_C' = \sigma(u', u_1' | u, u_1) |u' - u_1'|\, f(x, u', t)\, f(x, u_1', t). d\Omega\, d^3u'\, d^3u_1' \qquad (2.6.1.28)$$

Since

$$|u - u_1| = |u' - u_1'| \qquad (2.6.1.29)$$

and

$$d^3u\, d^3u_1 = d^3u'\, d^3u_1', \qquad (2.6.1.30)$$

then

$$\delta n_C' = \sigma(u, u_1 | u', u_1') |u - u_1|\, f(x, u', t)\, f(x, u_1', t). d\Omega\, d^3u\, d^3u_1 \qquad (2.6.1.31)$$

Thus the term C_{in} is obtained by multiplying $\delta n_C'$ by d^3x and the result integrated over Ω and u_1. That is,

$$C_{in} = d^3x\, d^3u \int d^3u_1 \int d\Omega\, \sigma(u, u_1 | u', u_1') |u - u_1|\, f(x, u', t)\, f(x, u_1', t) \qquad (2.6.1.32)$$

Hence, substitution of equations (2.6.1.27) and (2.6.1.32) into (2.6.1.21) yields

$$\frac{\partial f}{\partial t} + \vec{u}.\vec{\nabla} f + \frac{\vec{F}}{m}.\vec{\nabla}_u f = \int d^3u_1 \int d\Omega |u - u_1| \sigma(\Omega) (f' f_1' - f\, f_1), \qquad (2.6.1.33)$$

where

$$f = f(x, u, t),\ \ f_1 = f(x, u_1, t),\ \ f' = f(x, u', t),\ \ f_1' = f(x, u_1', t) \text{ and } \vec{F} = m\dot{u}$$
$$(2.6.1.34)$$

The term $\vec{F} = m\dot{u}$ incorporates any force field the neutrinos may be subjected to. The equation (2.6.1.33) with the collision integral for binary collisions is, a non-linear integro-differential equation for neutrino distribution function $f(x, u, t)$.

For a homogenous and isotropic universe, the second and third terms on the left hand side of equation (2.6.1.33) vanish.

Hence,

$$\frac{\partial f}{\partial t} = \int d^3u_1 \int d\Omega |u - u_1| \sigma(\Omega)(f' f_1' - f\ f_1) \tag{2.6.1.35}$$

Using the definition that

$$f = \frac{n}{Volume\ V}, \tag{2.6.1.36}$$

then

$$\begin{aligned}\frac{\partial f}{\partial t} &= \frac{1}{S^3}\frac{\partial n}{\partial t} - \frac{3n}{S^4}\frac{\partial S}{\partial t} \\ &= \int d^3u_1 \int d\Omega |u - u_1| \sigma(\Omega)(f' f_1' - f\ f_1)\end{aligned} \tag{2.6.1.37}$$

Taking $V \approx S^3$, equation (2.6.1.37) reduces to

$$\frac{\partial n}{\partial t} - 3n\frac{\dot{S}}{S} = \sigma v \left(n'^2 - n^2\right) \tag{2.6.1.38}$$

where $v = |u - u_1|$. In equilibrium, $\frac{\partial n}{\partial t} \to 0$ and, if no new neutrinos are being created then $n'^2 = 0$; hence, equation (2.6.1.38) reduces to

$$3\frac{\dot{S}}{S} = \sigma\, v\, n \tag{2.6.1.39}$$

2.7. COSMOLOGICAL NEUTRINO MASS BOUNDS

In the context of the electroweak model, it is established that the neutrino can acquire mass through the standard Higgs mechanism through a modified standard Lagrangian. However, the result is inadequate for the reason that it fails to predict the actual numerical mass value. Hence, it is suggested that the investigations be extended to the very early and expanding universe where the neutrino weak interactions were very dominant. Essentially, physical conditions in an expanding universe change with time. Consequently, to understand the observational features of the universe today, it is important that the past history of the universe is well understood. By extension, the physical processes that occur in the early universe when the temperature T is very high require knowledge of physics of particle interactions at high energies. Based on present knowledge of the latter, the universe can be divided into three different phases:

- Present understanding of particle interactions is reasonably complete for energies below $100\,GeV$ [Weinberg, 1967]. Correspondingly, one should be able to follow the evolution of the universe from the temperature of $T \approx 100\,GeV \approx 1.2 \times 10^{13} K$ downwards with reasonable accuracy.

- There are theoretical models which attempt to describe the particle interactions in the energy range $100\,GeV$ to $10^{16} GeV$ [Langacker, 1980]. These models are comparatively more speculative with the uncertainties increasing with energy, i.e., given a specific particle physics model, the evolution of the universe in the range $10^{16} GeV \approx 1.2 \times 10^{29} K$ to $1 GeV \approx 1.2 \times 10^{13} K$ can be worked out. Since the models are not unique, obtaining unique predictions is most unlikely.

- The physics at energies above $10^{16} GeV$ is very uncertain. Quantum gravitational effects will be very significant at energies, especially $E \geq M_{Planck} \approx 1.22 \times 10^{19} GeV$ [Misner, 1957; Conradi and Zeh, 1990]. The very basis for most of the present discussions, classical general relativity, breaks down (at these energies) and the uncertainties in the knowledge of particle interactions at high energies will prevent one from predicting a unique material content for the universe. To make any progress, reasonable assumptions should be made about this material content of the universe at some moment in time and the consequences then worked out. Chief among these assumptions is the distribution functions for the various particles.

2.7.1. DISTRIBUTION FUNCTIONS IN THE EARLY UNIVERSE

The contents of the universe at early epochs will be in a form very different from that in the present universe. This is because, atomic and nuclear structures have respective binding energies of the order of a few eV and MeV [Padmanabhan, 1993], so that when the temperature of the universe is higher than these values, such systems cannot exist as bound objects. Further, when the temperature T of the universe becomes higher than the rest mass m of a charged particle, say an electron or muon, the photon energy will be large enough to produce these particles and their antiparticles in large numbers [Wataghin, 1965]. For example, when $T >> m_{electron} \approx 0.5 MeV \approx 5.8 \times 10^{9} K$ there will be a large number of positrons in the universe. The typical energy of these particles will be T making them ultra relativistic. Thus, depending on the temperature T, the early universe would be populated by different kinds of elementary particles at different times. To work out the physical processes at time t, the distribution function $f_i(\vec{x},\vec{p},t) = f_i(\vec{p},t)$ of these particles should be known; the dependence of f_i on the space coordinates is ruled out because of the homogeneity of the universe [Padmanabhan, 1993]. To determine the form of $f_i(\vec{p},t)$, it is considered that different species of particles will be interacting constantly through various forces, scattering off each other and exchanging energy and momentum. If the rate of these reactions $\Gamma(t)$ is much higher than the rate of expansion of the universe

$$H(t) = \left(\frac{\dot{S}}{S}\right), \tag{2.7.1.1}$$

then these interactions can produce and maintain thermodynamic equilibrium among the interacting particles with some temperature $T(t)$. Therefore, the role of interactions will be limited to providing a mechanism for thermal equilibrium and, hence, the participating neutrinos can be treated as ideal Fermi gas with the distribution function [Srivastava, 2008]

$$f_i(\vec{p},t)d^3\vec{p} = \frac{g_i}{(2\pi)^3}\left\{\exp\left[\frac{(E_p - \mu_i)}{T_i(t)}\right] + 1\right\}^{-1} d^3\vec{p} \tag{2.7.1.2}$$

where g_i is the spin degeneracy factor of the neutrino species, $\mu_i(T)$ is the chemical potential, *E(p)* is the energy given as

$$E(\vec{p}) = (\vec{p}^{\,2} + m^2)^{1/2} \tag{2.7.1.3}$$

for $\hbar = c = 1$ and $T_i(t)$ is the temperature of the neutrino species i at time *t*.

At any time, the universe will also contain a blackbody distribution of photons with some characteristic temperature $T_\gamma(t)$. Its distribution will be given by

$$f_\gamma(\vec{p},t)d^3\vec{p} = \frac{g_\gamma}{(2\pi)^3}\left\{\exp\left[\frac{(E_p - \mu_\gamma)}{T_\gamma(t)}\right] - 1\right\}^{-1} d^3\vec{p} \tag{2.7.1.4}$$

If the neutrino species i couples to the photon and the rate of these $i-\gamma$ interactions is high enough i.e. $(\Gamma_{i-\gamma} >> H)$, then the interacting neutrino will have the same temperature as that of the photons and the photon temperature $T_i = T_\gamma$ will be the universal temperature of the very early and hot universe. As the universe evolves, the temperature $T(t)$ will change due to expansion in a timescale of the order of [Hughes, 1991]

$$t = H^{-1}(t)$$
$$= \left(\frac{\dot{S}}{S} \right)^{-1} \tag{2.7.1.5}$$

It could happen that, at some given instant, the total interaction rate $\Gamma_i(t)$ of the neutrino species falls below the expansion rate $H(t)$ $(\Gamma_i(t) \leq H(t))$ but the interaction rate among all the other species Γ_{other} could still be much higher than the expansion rate $\Gamma_{other} > H$. In such a case, the distribution functions of all species other than that of the neutrino will still be given by equation (2.7.1.2) with a common temperature T after the neutrino has completely decoupled.

Once the neutrino is completely decoupled, it will be traveling a long a geodesic in the space-time. This enables one to obtain the decoupling temperature distribution function, f_{dec}, after the neutrino has decoupled from the known form of the equilibrium distribution function, f_{equil}, before decoupling (the subscript *dec* implies decoupling). For simplicity, it is assumed that the decoupling occurs instantaneously at some time $t = t_{dec}$ when the temperature is T_{dec} and the scale factor is S_D. For $t < t_{dec}$, the distribution function is given by equation (2.7.1.2) and at some later time $t > t_{dec}$, the distribution function can be $f_{dec}(\vec{p}, t)$. Because of the redshift in momentum, all neutrinos with momentum $\vec{p}$ at time t must have had momentum $p\left[\frac{S(t)}{S(t_{dec})}\right]$ at $t = t_{dec}$. Therefore,

$$f_{dec}(\vec{p}, t) = f_{equil}\left(\vec{p} \frac{S(t)}{S(t_{dec})}, t_{dec} \right) \tag{2.7.1.6}$$

for $t > t_{dec}$, where f_{equil} is the equilibrium distribution function of the neutrino species i. Thus, as long as the neutrino was in equilibrium at some time t, its distribution at later times can easily be determined from equation (2.7.1.6). Consequently, from the distribution function (2.7.1.2), the number density n, energy density ρ and the pressure P for the neutrino species can be defined by the expressions [Murugeshan, 2003]:

$$n = \int f(\vec{p}) d^3\vec{p}$$

$$= \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{\left(E^2 - m^2\right)} E dE}{\exp\left[\frac{1}{T}\left(E - \mu\right)\right] + 1} \qquad (2.7.1.7)$$

$$\rho = \int E f(\vec{p}) d^3\vec{p}$$

$$= \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{\left(E^2 - m^2\right)} E^2 dE}{\exp\left[\frac{1}{T}\left(E - \mu\right)\right] + 1} \qquad (2.7.1.8)$$

$$P = \int \frac{|P|^2}{3E} f(\vec{P}) d^3\vec{P}$$

$$= \frac{g}{6\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{\frac{3}{2}} dE}{\exp\left[\frac{1}{T}\left(E - \mu\right)\right] + 1} \qquad (2.7.1.9)$$

When the neutrinos are highly relativistic $(T >> m)$ and nondegenerate $(T >> \mu)$, then their energy density and that of photons becomes

$$\rho \cong \frac{g}{2\pi^2} \int_0^\infty \frac{E^3 dE}{e^{E/T} \pm 1}$$

$$= \begin{cases} g_B \dfrac{\pi^2}{30} T^4 \\ \dfrac{7}{8} g_F \dfrac{\pi^2}{30} T^4 \end{cases} \qquad (2.7.1.10)$$

Hence, the total energy density contributed by all the relativistic species should be

$$\rho_{total} = \sum_{i=boson} g_i \left(\frac{\pi^2}{30}\right) T_i^4 + \sum_{i=fermion} \frac{7}{8} g_i \left(\frac{\pi^2}{30}\right) T_i^4$$

$$= g_{total} \frac{\pi^2}{30} T^4 \qquad (2.7.1.11)$$

where

$$g_{total} = \sum_{boson} g_B \left(\frac{T_B}{T}\right)^4 + \sum_{fermion} \frac{7}{8} g_F \left(\frac{T_F}{T}\right)^4 \qquad (2.7.1.12)$$

In writing g_{total}, the possibility that all the species may have a thermal distribution but may not have the same temperature has explicitly been taken into account. But, if they have the same temperature, then

$$q = g_{boson} + \frac{7}{8} g_{fermion} \tag{2.7.1.13}$$

and $q = g_{total}$. Therefore, the pressure due to relativistic neutrino species is

$$\begin{aligned} P &\approx \frac{\rho}{3} \\ &= g\frac{\pi^2}{90}T^4 \end{aligned} \tag{2.7.1.14}$$

and its entropy density j is [Kolb, 1986]

$$\begin{aligned} j &= \frac{1}{T}(\rho + P) \\ &= \frac{2\pi^2}{45}qT^3 \end{aligned} \tag{2.7.1.15}$$

with

$$\begin{aligned} q &= g_{total} \\ &= \sum_{boson} g_B \left(\frac{T_B}{T}\right)^3 + \frac{7}{8}\sum_{fermion} g_F \left(\frac{T_F}{T}\right)^3 \end{aligned} \tag{2.7.1.16}$$

whereas, the number density of all the relativistic species is

$$\begin{aligned} n &= \frac{g}{2\pi^2}\int_0^\infty \frac{E^2 dE}{e^{E/T} \pm 1} \\ &= \begin{cases} \dfrac{\xi(3)}{\pi^2} g_B T^3 \\ \dfrac{3\xi(3)}{4\pi^2} g_F T^3 \end{cases}, \end{aligned} \tag{2.7.1.17}$$

with $\xi(3) \approx 1.202$ being the Riemann-Zeta function of order 3 [Gupta, 2004]. Combining equation (2.7.1.17) with (2.7.1.10), it is found that the mean energy density of the relativistic particles

$$\langle E \rangle \equiv \left(\frac{\rho}{n}\right) \tag{2.7.1.18}$$

is about $2.7T$ for bosons and $3.15T$ for fermions. In the limit $T << m$, (thermal energy less than rest energy) the exponential in equation (2.7.1.2) is large compared to unity. Therefore, for both bosons and fermions, it is found that

$$n \approx \frac{g}{2\pi^2}\int_0^\infty p^2 dp \exp\left[-\frac{(m-\mu)}{T}\right]\exp\left(-\frac{p^2}{2mT}\right)$$

$$= g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[-\frac{1}{T}(m-\mu)\right] \qquad (2.7.1.19)$$

A comparison of (2.7.1.17) and (2.7.1.19) shows that the number and energy density of nonrelativistic neutrinos are exponentially damped by the factor $\exp-\left(\frac{m}{T}\right)$ with respect to that of the relativistic neutrinos so that, for $t < t_{equil}$ in the radiation dominated phase, the contribution of nonrelativistic neutrinos to ρ can be ignored. During the radiation dominated phase, the scale factor is proportional to the square root of time

$$S(t) \propto \sqrt{t} \qquad (2.7.1.20)$$

Equation (2.7.1.20) is differentiated with respect to time to yield

$$\left(\frac{\dot{S}}{S}\right)^2 = H^2(t)$$

$$= \frac{1}{4t^2}$$

$$= \frac{8\pi G}{3}\rho$$

$$= \frac{8\pi G}{3} g\left(\frac{\pi^2}{30}\right)T^4 \qquad (2.7.1.21)$$

The last two steps of equation (2.7.1.21) follow, after applying equations (2.7.1.14) and (2.7.1.10) respectively. The equation (2.7.1.21) can also be written in terms of the Planck mass [Coughlan, 1991]

$$m_{Pl} = \frac{1}{\sqrt{G}}, \qquad (2.7.1.22)$$

$$= 1.22 \times 10^{19}\, GeV$$

so that

$$H(T) \cong 1.66\sqrt{g}\left(\frac{T^2}{m_{Pl}}\right) \qquad (2.7.1.23)$$

$$t \cong \frac{0.3}{\sqrt{g}}\left(\frac{m_{Pl}}{T^2}\right) \approx \frac{1s}{\sqrt{g}}\left(\frac{T}{1MeV}\right)^{-2} \qquad (2.7.1.24)$$

The factor g in these expressions counts the degrees of freedom of those neutrinos which are still relativistic at temperature T. As the temperature decreases, more neutrinos will become nonrelativistic and, g and q will decrease as functions of temperature;

$$g = g(T) \text{ and } q = q(T) \tag{2.7.1.25}$$

,i.e., are slowly varying functions of T. In particular, $g \approx 10^2$ at $T \geq 300GeV$, $g \approx 10$ for $T \approx (100-1)MeV$ and $g \approx 3$ for $T < 1MeV$. The slow variation of $q(T)$ has the consequence that the expression for the conserved entropy J in the radiation dominated phase is

$$J \propto q(T)T^3S^3 \tag{2.7.1.26}$$

The equation shows that the temperature T will decrease as S^{-1} only if q is constant and if the number of degrees of freedom changes, then T will decrease slightly more slowly than S^{-1} so that, from(2.7.1.26), the correct relation is

$$q^{1/3}(T)T \propto S^{-1} \tag{2.7.1.27}$$

For a neutrino that decouples while still relativistic $(T_{dec} >> m)$, the distribution function takes the form

$$\begin{aligned} f_{dec} &= f_{equi}\left(p\frac{S(t)}{S(t_{dec})}, T_{dec}\right) \\ &= \frac{g}{(2\pi)^3}\left[\exp\frac{1}{T}\left(p\frac{S(t)}{S(t_{dec})}\right) \pm 1\right]^{-1} \end{aligned} \tag{2.7.1.28}$$

This has the same form as the f_{equil} for a relativistic neutrino species with the temperature

$$T(t) = T_{dec}\left[\frac{S(t_{dec})}{S(t)}\right] \tag{2.7.1.29}$$

even though the neutrino species is not in thermodynamic equilibrium any longer. The temperature in this distribution falls strictly as S^{-1} while the entropy J of the particles is conserved separately. The number density of the decoupled neutrinos is then given by

$$n = g_{eff}\left(\frac{\xi(3)}{\pi^2}\right)T_{dec}{}^3\left(\frac{S_{dec}}{S}\right)^3, \tag{2.7.1.30}$$

where

$$g_{eff} = \frac{3g}{4} \tag{2.7.1.31}$$

for fermion and

$$g_{eff} = g \tag{2.7.1.32}$$

for boson. This number density will be comparable to the number density of photons at any given time. In particular, any such neutrino species will continue to exist in our universe today as a relic background, with number densities comparable to that of photons. On the other hand, for a neutrino species which decouples when it is already nonrelativistic, its distribution changes to

$$\begin{aligned} f_{dec}(p) &= f_{equil}\left[p\left(\frac{S}{S_{dec}}\right), T_{dec}\right] \\ &= \frac{g}{(2\pi)^3} e^{-\frac{m}{T_{dec}}} e^{-\frac{p^2}{2mT_{dec}}\left(\frac{S}{S_{dec}}\right)^2} \end{aligned} \tag{2.7.1.33}$$

which has the same form as the nonrelativistic Maxwell-Boltzmann distribution with a temperature

$$T(t) = T_{dec}\left(\frac{S_{dec}}{S}\right)^2 \tag{2.7.1.34}$$

which decreases as the square of the scale factor. Then the corresponding number density becomes

$$n \cong g\left(\frac{mT_{dec}}{2\pi}\right)^{3/2}\left(\frac{S_{dec}}{S}\right)^3 \exp\left(-\frac{m}{T_{dec}}\right) \tag{2.7.1.35}$$

from which, the number density is found to be

$$n \propto S^{-3} \tag{2.7.1.36}$$

and the energy density as

$$\rho \approx nm \tag{2.7.1.37}$$

To any neutrino species which is not being created or destroyed, a conserved number N is assigned to it so that

$$\begin{aligned} N &\propto nS^3 \\ &= \frac{n}{j} \end{aligned} \tag{2.7.1.38}$$

From equations (2.7.1.15), (2.7.1.16) and (2.7.1.19), for $\mu << T$, it follows that for photons and neutrinos, N should be

$$N = \begin{cases} \dfrac{45\xi(3)}{2\pi^4}\left[\dfrac{g_{eff}}{q}\right] \cong 0.278\left(\dfrac{g_{eff}}{q}\right) \\ \\ \dfrac{45}{2\pi^4}\left(\dfrac{\pi}{8}\right)^{1/2}\left(\dfrac{g}{q}\right)\left(\dfrac{m}{T}\right)^{3/2} e^{-m/T} \cong 0.145\left(\dfrac{g}{q}\right)\left(\dfrac{m}{T}\right)^{3/2} e^{-m/T} \end{cases} \tag{2.7.1.39}$$

respectively. At the decoupling temperature ($T \le 10^{12} K$) a significant number density (i.e., number density comparable to that of photons) of relativistic electrons (e) and positrons $\bar{e}$, whose rest mass energy is[Mani and Mehta, 1988]

$$m_e \approx 0.5 MeV$$

$$= 5.8 \times 10^9 K \tag{2.7.1.40}$$

were also in equilibrium with the photons. In addition, neutrons and protons contained in the present universe must have existed at $T = 10^{12} K$ as well, since these particles could not have been produced at $T < 10^{12} K$. Therefore, the ratio between the number density of baryons (n_B) and the number density of photon (n_γ) remains approximately constant from the temperature $T \approx 10^{12} K$ to the present. At $T \approx 10^{12} K$, the energy density of the universe is dominantly contributed by e, $\bar{e}$, ν, $\bar{\nu}$ and photons. Since the interactions among them maintain the required equilibrium, they will all have the same temperature and by taking

$$\begin{aligned} g_B &= g_\gamma \\ &= 2 \\ g_e &= \bar{g}_e \\ &= 2 \\ g_\nu &= \bar{g}_\nu \\ &= 1 \end{aligned} \tag{2.7.1.41}$$

and including three flavours of neutrinos [DiLella, 1987], the total number of degrees of freedom becomes

$$g_{total} = g_B + \frac{7}{8} g_F \tag{2.7.1.42}$$

$$= 2 + \frac{7}{8}(2 + 2 + 2 \times 3)$$

$$= 10.75 \tag{2.7.1.43}$$

The $g-$values for electrons and positrons represent the two possible spin states for massive spin one-half fermions. Photons have two accessible states (corresponding to two states of polarization) giving $g_\gamma = 2$ and massless spin one-half neutrinos exist only in left-handed or right-handed states, making $g_\nu = 1$. From equations (2.7.1.23) and (2.7.1.24), the precise time-temperature relationship for this phase of evolution then becomes

$$H(T) = 5.44\left(\frac{T^2}{m_{Pl}}\right) \qquad (2.7.1.44)$$

and

$$t = 0.09\left(\frac{m_{Pl}}{T^2}\right) \qquad (2.7.1.45)$$

Since neutrinos have no electric charge, they have no direct coupling with photons. Also their interaction with baryons can be ignored because of the low density of baryons. So they are essentially kept in equilibrium through reactions of the form

$$\nu\,\overline{\nu} \leftrightarrow e\,\overline{e} \qquad (2.7.1.46)$$

and

$$\nu\, e \leftrightarrow \nu\, e \qquad (2.7.1.47)$$

The cross section $\sigma(E)$, for these weak interaction processes is of the form [Padmanabhan, 1993]

$$\sigma(E) = \left(\frac{\alpha^2 E^2}{m_x{}^2}\right) \qquad (2.7.1.48)$$

where $\alpha \approx 2.8\times10^{-2}$ is related to the gauge coupling constant g by

$$\alpha = \frac{g^2}{4\pi} \qquad (2.7.1.49)$$

and

$$m_x \approx 90 GeV \qquad (2.7.1.50)$$

is the mass of the gauge vector boson mediating the weak interaction [Hollik, 1992]. Defining the Fermi coupling constant as

$$\begin{aligned} G_F &= \frac{\alpha}{m_x{}^2} \\ &= 1.17\times10^{-5}(GeV)^{-2} \\ &= (293 GeV)^{-2} \end{aligned} \qquad (2.7.1.51)$$

and using the fact that $E \approx T$, then

$$\sigma = {G_F}^2 E^2$$

$$= {G_F}^2 T^2 \qquad (2.7.1.52)$$

Since the number density of the interacting neutrinos is

$$n = \frac{3}{4}\xi(3)\frac{g}{\pi^2}T^3$$

$$= 1.096T^3 \qquad (2.7.1.53)$$

and

$$v \approx c = 1 \qquad (2.7.1.54)$$

then the rate for neutrino interactions becomes

$$\Gamma = n\,\sigma\,v$$

$$= 1.3\,{G_F}^2\,T^5 \qquad (2.7.1.55)$$

Hence, from equations (2.7.2.45) through (2.7.1.55), equilibrium neutrino Boltzmann transport equation is obtained to be

$$\frac{\Gamma}{H} \approx 0.24\,T^3\left(\frac{m_{Pl}}{{G_F}^{-2}}\right)$$

$$= \left(\frac{T}{1.4MeV}\right)^3$$

$$= \left(\frac{T}{1.6\times10^{10}K}\right)^3 \qquad (2.7.1.56)$$

Equation (2.7.1.56) shows that the interaction rates of neutrinos become lower than the expansion rate when the temperature drops below $T_D \approx 1MeV$. At lower temperatures, the neutrinos are completely decoupled from the rest of matter. Since they are assumed to be massless in the standard electroweak model they are, therefore, relativistic at the time of decoupling. Their distribution function at later times is given by equation (2.7.1.33) with $T_v \propto S^{-1}$. Consequently, the present day universe should contain a relic background of these neutrinos.

At the time of decoupling, the photons, neutrinos and the rest of the matter had the same temperature. As long as the photon temperature decreases as S^{-1}, neutrinos and photons will continue to have the same temperature even though the neutrinos have decoupled. However, the photon temperature will decrease at slightly lower rate if the g-factor is changing. In this case, T_γ will become higher than T_ν as the universe cools and the change in the value of g will occur when the temperature of the universe falls below [Mani and Mehta, 1988]

$$\begin{aligned} T &\approx m_e \\ &= 0.5 MeV \\ &= 5.8 \times 10^9 K \end{aligned} \tag{2.7.1.57}$$

Thus, when the temperature becomes lower than $5 \times 10^9 K$, the mean energy of the photons falls below the energy required to create $e\bar{e}$ pairs and, hence, the backward reactions in the process

$$e\,\bar{e} \leftrightarrow \gamma\,\gamma \tag{2.7.1.58}$$

becomes suppressed while the forward reactions continues to occur, resulting in the disappearance of the $e\bar{e}$ pairs. This process clearly changes the value of g, i.e., at $T_D > T \geq m_e$, neutrinos decouple and their entropy is separately conserved but the photons $(g = 2)$ are in equilibrium with electrons $(g = 2)$ and positrons $(g = 2)$ so that equation (2.7.1.43) becomes

$$\begin{aligned} g(\gamma, e, \bar{e}) &= 2 + \frac{7}{8} \times 4 \\ &= \frac{11}{2} \end{aligned} \tag{2.7.1.59}$$

However, for $T << m_e$, the $e\bar{e}$ annihilation is complete and, therefore, the only relativistic species left in this set is the photon $(g = 2)$. It is known that the conservation of entropy

$$J = q(ST)^3 \tag{2.7.1.60}$$

implies that the quantity

$$q(ST)^3 = g(ST_\gamma)^3 \tag{2.7.1.61}$$

remains constant during expansion as photons, electrons and positrons have same temperature in order for $g = q$. Further, since g decreases during the $e\bar{e}$ annihilation, then the value of $(ST_\gamma)^3$ after the $e\bar{e}$ annihilation will be higher than its value before, that is,

$$\begin{aligned} \frac{(ST_\gamma)^3{}_{after}}{(ST_\gamma)^3{}_{before}} &= \frac{g_{before}}{g_{after}} \\ &= \frac{11}{4} \end{aligned} \tag{2.7.1.62}$$

The neutrinos do not participate in this process as they are already decoupled. They are characterized by a temperature $T_\nu(t)$ which falls as S^{-1} and their entropy $(j_\nu S^3)$ is conserved separately. If the temperature of the neutrino is parameterized to

$$T_\nu = KS^{-1}, \tag{2.7.1.63}$$

then originally before $e\bar{e}$ annihilation, the photons and the neutrinos had the same temperature so that, from (2.7.1.63), it is found that

$$\begin{aligned}(ST_\nu)_{before} &= (ST_\gamma)_{before} \\ &= K\end{aligned} \tag{2.7.1.64}$$

Consequently,

$$\begin{aligned}(ST_\nu)_{after} &= \left(\frac{11}{4}\right)^{1/3}(ST_\gamma)_{before} \\ &= \left(\frac{11}{4}\right)^{1/3}(ST_\nu)_{before} \\ &= \left(\frac{11}{4}\right)^{1/3}(ST_\nu)_{after} \\ &= 1.4(ST_\nu)_{after}\end{aligned} \tag{2.7.1.65}$$

The first equality in equation (2.7.1.65) follows from equation (2.7.1.62), whereas the second is from the fact that $T_\gamma = T_\nu$ at $T \geq m_e$ and the third from strict constancy of (ST_ν). Hence, the $e\bar{e}$ annihilations increase the temperature of photons compared to that of neutrinos by a factor

$$\left(\frac{11}{4}\right)^{1/3} \approx 1.401 \tag{2.7.1.66}$$

The photons released by the process

$$(e\bar{e} \to \gamma\gamma) \tag{2.7.1.67}$$

get thermalized rapidly due to the scattering with charged particles. After the $e\bar{e}$ annihilations, the g factor does not change. Both T_γ and T_ν fall as S^{-1} and the ratio

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \cong 0.714 T_\gamma \tag{2.7.1.68}$$

should be maintained till today. The relic neutrino background today should have the distribution given by equation (2.7.1.28) with

$$(T_\nu)_{now} \cong 0.714 \times 2.726K = 1.946K \qquad (2.7.1.69)$$

Thus, the species of particles which remain relativistic today will be photons $(g_\gamma = 2)$ with a temperature $T_\gamma \approx 2.726K$ and three flavors of massless neutrinos and antineutrinos $(g_F = 3+3=6)$ with a temperature

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \qquad (2.7.1.70)$$

From equations (2.7.1.11) and (2.7.1.16), it is found that

$$g(now) = 2 + \frac{7}{8} \times 6 \times \left(\frac{4}{11}\right)^{4/3} \approx 3.36$$

$$(2.7.1.71)$$

$$q(now) = 2 + \frac{7}{8} \times 6 \times \frac{4}{11} \approx 3.91$$

Therefore, the energy and entropy densities of these relativistic neutrinos in the present day universe should be

$$\rho_{rel} = \frac{\pi^2}{30} g T^4 = 8.09 \times 10^{-34} gcm^{-3}$$

$$(2.7.1.72)$$

$$j_{rel} = \frac{2\pi^2}{45} q T^3 = 2.97 \times 10^3 cm^{-3}$$

2.7.2. BIG BANG NUCLEOSYNTHESIS

The standard hot big bang model of the universe [Srivastava, 2008] provides a reliable framework for understanding the origin and evolution of the universe. One of the features of the present universe which is naturally explained in this model is relative abundance and formation of the light element especially helium-4 (^4He). The formation of these elements is known as nucleosynthesis and the success of primordial nucleosynthesis in predicting the large primordial abundance of ^4He and deuterium D is the strongest evidence that the universe can be described by a Friedmann-Limatre-Robertson-Walker cosmology at very early times [Duane et al., 1982]. Because of this concordance, it is attractive to assume that the Friedmann-Limatre-Robertson-Walker cosmology was applicable at the time of nucleosynthesis and then demand that the resulting primordial abundances of the light elements be within bounds extrapolated from present observations. This approach results in a limit on the contribution to the energy density from additional particles present in the universe at temperature $T \leq 1\,MeV$ such as additional neutrino species [Lidsey, 2000].

At very high temperatures where $T >> 1\,MeV$, the weak interaction rates for the following processes

$$n + \nu_e \leftrightarrow p + e^-$$

$$n + e^+ \leftrightarrow p + \bar{\nu}_e \qquad (2.7.2.1)$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

were all in equilibrium, i.e., $\Gamma_W > H$. Thus, it is expected that, initially the number of neutrons to protons is unity ($(n/p) = 1$) and this ratio is essentially controlled by the Boltzmann factor so that [Carr, 1985]

$$\left(\frac{n}{p}\right) = e^{-\frac{\Delta m}{T}} \qquad (2.7.2.2)$$

where $\Delta m = m_n - m_p$ is the neutron-proton mass difference and $k_B = c = 1$. At temperatures $T >> 1\,MeV$, nucleosynthesis cannot occur even though the rate for forming the first isotope, deuterium, through the reaction [Keith, 1984]

$$n + p \leftrightarrow D + \gamma \qquad (2.7.2.3)$$

is possible. At $T >> 1\,MeV$, deuterium is photodissociated because $E_\gamma > 2.2\,MeV$ (the binding energy of deuterium). But, since the density of photons is very high

$$\frac{n_\gamma}{n_B} \sim 10^{10}, \qquad (2.7.2.4)$$

then the onset of nucleosynthesis will depend on the quantity $\eta^{-1} e^{-\frac{2.2\,MeV}{T}}$, where

$$\eta \equiv \frac{n_B}{n_\gamma} \qquad (2.7.2.5)$$

is the baryon to photon ratio. When this quantity becomes of order $\leq O(1)$, the rate for $p + n \to D + \gamma$ finally becomes greater than the rate for dissociation $D + \gamma \to p + n$. This is found to occur when $T \sim 0.1\,MeV$ [Kolb, 1986]. Since nucleosynthesis begins when $T < 1\,MeV$, then the rates for processes which control the neutron-to-proton ratio equation (2.7.2.1) as well as those which keep neutrinos in equilibrium are frozen out. Neutrinos are effectively at a lower temperature at $T \le 0.5\,MeV$ and taking this into account, the expansion rate becomes

$$H = \sqrt{\frac{8\pi G_N \rho}{3}}$$
$$= \sqrt{\frac{8\pi^3}{90} g(T)}\,\frac{T^2}{M_{Pl}}$$
$$= 1.66\sqrt{g(T)}\,\frac{T^2}{M_{Pl}} \qquad (2.7.2.6)$$

where

$$g(T) = g_\gamma + \left(\frac{4}{11}\right)^{\frac{4}{3}} g_\nu$$

Once deuterium is produced by a reaction process (2.7.2.3), then tritium can be produced by a process

$$D + D \leftrightarrow T + p \qquad (2.7.2.7)$$

which, on further reaction, produces ^4He by a process

$$D + T \leftrightarrow {}^4He + n \qquad (2.7.2.8)$$

^4He has, in addition, several other processes which go towards its production [Keith A O, 1984]

$$D + D \leftrightarrow {}^4He + \gamma$$
$${}^3He + n \leftrightarrow {}^4He + \gamma$$
$${}^3He + {}^3He \leftrightarrow {}^4He + 2p \qquad (2.7.2.9)$$
$$T + p \leftrightarrow {}^4He + \gamma$$

Additional processes for producing T and ^3He include [Hughes, 1991]

$$n + D \leftrightarrow T + \gamma$$
$$p + D \leftrightarrow {}^3He + \gamma \qquad (2.7.2.10)$$
$$D + D \leftrightarrow {}^3He + n$$

The nuclear chain is temporarily halted at this point because there are gaps at masses $A = 5$ and $A = 8$, i.e., there are no stable nuclei with those masses [Keith, 1984]. However, there is some further production which accounts for the abundance of ^6Li and ^7Li through

$$^3He + ^4He \leftrightarrow ^7Be \left| \begin{array}{l} +\gamma \\ \rightarrow ^7Li + e^- + \bar{\nu}_e \end{array} \right.$$

$$T + ^4He \leftrightarrow ^7Li + \gamma \qquad (2.7.2.11)$$

$$p + ^7Li \leftrightarrow ^4He + ^4He$$

Due to the gap at $A = 8$, there is very little subsequent nucleosynthesis in the big bang. A second chief factor in the ending of nucleosynthesis is that during this whole process the universe continues to expand and cool. At lower temperatures it becomes exponentially difficult to overcome the coulomb barriers in nuclear collisions. Hence, the contribution by baryons and photons from the big bang nucleosynthesis to the number density in the present universe is [Keith, 1984]

$$\begin{aligned} n_B &= \frac{\rho_B}{m_B} \\ &= \frac{\Omega_B \rho_c}{m_B} \\ &= 1.13 \times 10^{-5} \Omega_B h_o^2 cm^{-3} \end{aligned} \qquad (2.7.2.12)$$

where ρ_B is the energy density of baryons, m_B is the nucleon mass and Ω_B is that part of $\Omega \equiv \frac{\rho}{\rho_c}$ which is in the form of baryons with

$$\begin{aligned} \rho_c &= \frac{3H^2}{8\pi G_N} \\ &= 1.88 \times 10^{-29} h_o^2 \ g \ cm^{-3} \end{aligned} \qquad (2.7.2.13)$$

is the critical energy density. The number density of photons is then given by

$$\begin{aligned} n_\gamma &= \int dn_\gamma \\ &= \frac{2\xi(3)}{\pi^2} T^3 \\ &= 400 \left(\frac{T_p}{2.726} \right)^3 cm^{-3} \end{aligned} \qquad (2.7.2.14)$$

where T_p is the present temperature of the microwave background radiation. The baryon-to-photon ratio is usually expressed as [Michael, 1987]

$$\eta = 2.81\times10^{-8}\Omega_B h_p^2\left(\frac{2.726}{T_p}\right)^3 \quad (2.7.2.15)$$

Hence, it is possible to determine η if Ω_B, h_p and T_p are known. But, turning around equation (2.7.2.15) gives

$$\Omega_B = 3.56\times10^{7}\eta h_p^{-2}\left(\frac{T_p}{2.726}\right)^3 \quad (2.7.2.16)$$

The limits of η consistent with the abundances of deuterium D, helium-3 ^3He and lithium-7 ^7Li is [Mohapatra and Lai, 1981]

$$(3-4)\times10^{-10} \le \eta \le (7-10)\times10^{-10} \quad (2.7.2.17)$$

Combining equations (2.7.2.15) through (2.7.2.17) and using the limits

$$0.5 \le h_p \le 1 \quad (2.7.2.18)$$

and

$$2.7 \le T_p \le 3K\,, \quad (2.7.2.19)$$

yields

$$0.01 \le \Omega_B \le 0.03 \quad (2.7.2.20)$$

Since $\Omega = 1$ for a closed universe, then from (2.7.2.20), it is evident that baryons can negligibly contribute to the energy density of the universe.

2.7.3. RELIC BACKGROUND OF MASSIVE NEUTRINOS

From the outgoing section (s), it has been found that a massless neutrino has negligible contribution to the total energy density of the universe. In particular, a massless neutrino has been found to contribute energy density of $8.09\times10^{-34}gcm^{-3}$ as compared to $1.88\times10^{-29}h_o^2\ g\,cm^{-3}$ for a closed universe. Hence, a massive neutrino that decouples while still relativistic ($T_{dec} >> m$ with m being the mass of the particle and T_{dec} the decoupling temperature) is suggested in this section. Essentially, the massive neutrino should be characterized by the conserved physical quantity

$$N = 0.28\left(\frac{g_{eff}}{q}\right)_{T=T_{dec}}$$

$$= 0.21\left(\frac{g}{q(T_{dec})}\right) \tag{2.7.3.1}$$

where

$$g_{eff} = \frac{3g}{4} \tag{2.7.3.2}$$

for a neutrino. They will have a number density

$$n_o = Nj_o$$

$$= 2.97\times10^3\, Ncm^{-3}$$

$$= 619g(q(T_{dec}))^{-1}cm^{-3} \tag{2.7.3.3}$$

in the present universe and would have been non-relativistic at some temperature $T_{nonrel} \approx m$ in the past for $m > T_{dec}$. If not, then they will be relativistic even today and will behave just like the massless neutrino case analyzed earlier. The energy that must be contributed by each of this stable relativistic neutrino in the present universe is

$$E \approx m \tag{2.7.3.4}$$

Using equation (2.7.3.3), the total energy density of these massive neutrinos is

$$\rho = n_o m \tag{2.7.3.5}$$

For a weakly interacting massive neutrino that decouple when it is non-relativistic, the conserved number of neutrinos N is modified to the form

$$N = 0.145\frac{g}{q(T_{dec})}\left(\frac{m_v}{T_{dec}}\right)^{3/2} e^{-m/T_D} \tag{2.7.3.6}$$

Equation (2.7.3.6) shows that N depends strongly on mass m_v. In this case, to determine the mass *m*, the decoupling temperature T_{dec} has to be determined by the equilibrium condition that

$$\Gamma \sim 3H \tag{2.7.3.7}$$

The reactions which are capable of changing the number N of the weakly interacting massive neutrinos are of the form

$$A\overline{A} \leftrightarrow X\overline{X}\ , \tag{2.7.3.8}$$

where X is some generic species particle.

The average value of the reaction rates σv for the annihilation processes involving this generic particle is found to take the form [Kolb, 1986]

$$\langle \sigma v \rangle = \sigma_o \left(\frac{T}{m} \right)^k \tag{2.7.3.9}$$

where k is set to unity and the cross section σ_o is defined as [Gary S, 1984]

$$\sigma_o \cong \frac{c}{2\pi} {G_F}^2 m^2 \tag{2.7.3.10}$$

with the constant c depending on the type of the fermions; that is Fermions with spin one-half can be either Dirac type or Majorana type and for Dirac-type fermions, $c \approx 5$ so that the equation (2.7.3.10) becomes

$$\sigma_o \cong \frac{5}{2\pi} {G_F}^2 m^2 \tag{2.7.3.11}$$

whereas for Majorana type, $c \approx 1$ and equation (2.7.3.10) becomes

$$\sigma_o \cong \frac{1}{2\pi} {G_F}^2 m^2 \tag{2.7.3.12}$$

For very heavy neutrinos, their interaction behavior is modified drastically so that the reaction rate becomes

$$\begin{aligned} \Gamma &= n \langle \sigma v \rangle \\ &= g_A \left(\frac{mT}{2\pi} \right)^{3/2} e^{-\frac{m}{T}} \sigma_o \left(\frac{T}{m} \right)^k \\ &= \frac{\sigma_o g_A}{(2\pi)^{3/2}} T^3 \left(\frac{m}{T} \right)^{\frac{3}{2} - k} e^{-\frac{m}{T}} \end{aligned} \tag{2.7.3.13}$$

Since the expansion rate is given by

$$H = 1.66 \sqrt{g} \frac{T^2}{m_{Pl}}, \tag{2.7.3.14}$$

then from the calculated Boltzmann equation, the equilibrium condition (2.7.3.7) gives

$$\frac{\Gamma}{3H} \sim 1 \tag{2.7.3.15}$$

This leads to

$$\frac{\Gamma}{3H} = 3.827 \times 10^{-2} \frac{g_A}{\sqrt{q}} \left(\frac{m}{T} \right)^{\frac{1}{2} - k} e^{-\frac{m}{T}} (\sigma_o m m_{Pl}) \approx 1 \tag{2.7.3.16}$$

when (2.7.3.13) and (2.7.3.14) are applied. Hence, when equation (2.7.3.16) is solved for $\exp\left(-\frac{m}{T}\right)$ and the result substituted into equation (2.7.3.6), it is found to yield

$$N = \frac{3.79}{\sqrt{q}}\left(\frac{m}{T_D}\right)^{k+1}(\sigma_o m m_{Pl})^{-1}$$

$$= \frac{2.87\times10^{-9}}{\sqrt{q}}\left(\frac{m}{T_D}\right)^{k+1}\left(\frac{m}{1GeV}\right)^{-3} \quad (2.7.3.17)$$

Equation (2.7.3.17) corresponds to a number density

$$n_o = Nj_o$$

$$= \frac{8.523\times10^{-6}}{\sqrt{q}}\left(\frac{m}{T_D}\right)^{k+1}\left(\frac{m}{1GeV}\right)^{-3} \quad (2.7.3.17)$$

and the density parameter $(\Omega h^2)_{wimp}$ now becomes

$$(\Omega h^2)_{wimp} = \frac{0.81}{\sqrt{q}}\left(\frac{m}{T_D}\right)^{k+1}\left(\frac{m}{1GeV}\right)^{-2} \quad (2.7.3.18)$$

2.8. A NEUTRINO OSCILLATION

In this section, the possibility of a neutrino oscillating from one quantum state to the other as it propagates through space and in matter is examined. It is suggested that knowledge or information on neutrino oscillations may form an indirect but reliable and sensitive way of experimentally searching for a non-zero neutrino mass. This is because the oscillation probability can be expressed as a function of mass difference which is the focus of the study.

2.8.1. NEUTRINO OSCILLATIONS IN VACUUM

The idea of a neutrino oscillation was put forward by Bruno Pontecorvo [Pontecorvo, 1957] who pointed out that oscillations can occur if neutrino states of definite mass do not coincide with the weak interaction eigenstates. Using the Kaon oscillation analogy, Pontecorvo took nearly ten years to develop the quantitative formalism of neutrino oscillation in a vacuum. He considered that for Dirac neutrino mass, the part of the Lagrangian that describes the lepton (the electron plus the neutrino) masses and charged current interactions is

$$L_{W+m} = \frac{g}{\sqrt{2}} \bar{e}'_{aL} \gamma^{\mu} \nu'_{aL} W_{\mu}^{-} + (m_l)_{ab} \bar{e}'_{aL} e'_{bR} + (m_D)_{ab} \bar{\nu}'_{aL} \nu'_{bR} + h.c \qquad (2.8.1.1)$$

The mass matrix of charged leptons m_l and the neutrino mass matrix m_D in equation (2.8.1.1) are complex matrices that can be diagonalized by unitary transformations

$$\begin{aligned} e'_L &= V_L e_L & \nu'_L &= U_L \nu_L \\ e'_R &= V_R e_R & \nu'_R &= U_R \nu_R \end{aligned} \qquad (2.8.1.2)$$

where the matrices V_L, V_R, U_L and U_R are unitary; hence, the diagonalized form of the mass matrices of charged leptons and neutrinos under the transformation (2.8.1.2) are

$$V_L^* m_l V_R = (m_l)_{diag},$$

$$U_L^* m_D U_R = (m_D)_{diag} \qquad (2.8.1.3)$$

respectively, whereas the unprimed fields e_{iL}, e_{iR}, ν_{iL} and ν_{iR} are the components of the Dirac mass eigenstate fields

$$e_i = e_{iL} + e_{iR} \qquad (2.8.1.4)$$

and

$$\nu_i = \nu_{iL} + \nu_{iR} \qquad (2.8.1.5)$$

Therefore, the Lagrangian (2.8.1.1) under the transformation (2.8.1.2) becomes

$$L_{W+m} = \frac{g}{\sqrt{2}} \bar{e}_i \gamma^{\mu} (V_L^* U_L)_{ij} \nu_{Lj} W_{\mu}^{-} + m_{li} \bar{e}_{Li} e_{Ri} + m_{Di} \bar{\nu}_{Li} \nu_{Ri} + h.c \qquad (2.8.1.6)$$

where m_{li} are the charged lepton masses and m_{Di} are the neutrino masses. However, the matrix

$$U = V_L^* U_L \qquad (2.8.1.7)$$

is the lepton mixing matrix or Maki-Nakagawa-Sakata (MNS) matrix [Maki et al., 1962], the leptonic analog of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [Cabibbo, 1963; Kobayashi and Maskawa, 1973]. It relates a neutrino flavour eigenstate $|\nu'_a\rangle$ produced or absorbed alongside with the corresponding charged lepton, to the mass eigenstate $|\nu_i\rangle$

$$|\nu'_a\rangle = U_{ai}^* |\nu_i\rangle \qquad (2.8.1.8)$$

The challenge is to know the probability of finding a neutrino in a state $|\nu_b\rangle$ at a later time t given that, at a time $t = 0$, the neutrino flavor eigenstate $|\nu_a\rangle$ was produced. If its initial state ($t = 0$) is

$$|\nu(0)\rangle = |\nu_a\rangle$$

$$= U^*_{aj}|\nu_j\rangle \qquad (2.8.1.9)$$

then, at a later time t, its state will be given by

$$|\nu(t)\rangle = U^*_{aj}\, e^{-iE_j t}|\nu_j\rangle \qquad (2.8.1.10)$$

By definition, the probability amplitude for finding a particle at time t in a flavor state $|\nu_b\rangle$ is [Hughes, 1991]

$$A(\nu_a \to \nu_b; t) = \langle \nu_b|\nu(t)\rangle$$

$$= U_{bi}U^*_{aj}e^{-iE_i t}\langle \nu_i|\nu_j\rangle$$

$$= U_{bj}e^{-iE_i t}U^*_{aj} \qquad (2.8.1.11)$$

where the sum over intermediate states j is implied. Hence, the neutrino oscillation probability, i.e., the probability of transforming ν_a into ν_b is given by

$$P(\nu_a \to \nu_b; t) = |A(\nu_a \to \nu_b; t)|^2$$

$$= \left|U_{bj}e^{-iE_j t}U^*_{aj}\right|^2 \qquad (2.8.1.12)$$

If neutrinos have a Majorana mass term, then equation (2.8.1.1) has to be modified, i.e., the term $(m_D)_{ab}\overline{\nu}'_{aL}\nu'_{bR} + h.c$ is replaced by [Peccei, 1988]

$$(m_M)_{ab}\overline{\nu^C_{aL}}'\nu'_{bR} + h.c = (m_M)_{ab}\nu'^T_{aL}C\nu'_{bR} + h.c \qquad (2.8.1.13)$$

The mass term (2.8.1.13) breaks not only the individual lepton flavours, but also the total lepton number; further, the symmetric Majorana mass matrix $(m_M)_{ab}$ is diagonalized by the transformation

$$U^T_L m_M U_L = (m_M)_{diag} \qquad (2.8.1.14)$$

so that the field transformation operators in equation (2.8.1.2) are used. Thus, the structure of the charged current interactions is the same as in the case of the Dirac neutrinos or the oscillation probability in the case of Majorana mass term is the same as in the case of the Dirac mass term; this means that one cannot distinguish between Dirac and Majorana neutrinos by studying neutrino oscillations. This is because the total lepton number is not violated by the neutrino flavor oscillations.

2.8.2. NEUTRINO OSCILLATIONS IN MATTER

In the preceding section (2.8.1), neutrino oscillation in a vacuum has been analyzed. But since the medium through which it propagates is not empty, it is likely that the oscillations in matter may differ from those in the vacuum. Therefore, to model neutrino oscillations as they propagate through medium, it is important that matter effects be taken into account. When propagating through matter, neutrinos can be absorbed by the matter constituents, or scattered off them changing their momentum and energy. To come up with an evolution equation, neutrinos flavors ν_e, ν_μ and ν_τ are considered to interact with the electrons, protons and neutrons of matter through neutral current (NC) interactions mediated by Z^0 bosons [Peccei, 1989]. Electron neutrinos, in addition, have charged current (CC) interactions with electrons of the medium which are mediated by the $W^\pm$ exchange as in figure 2.8.2.1 below.

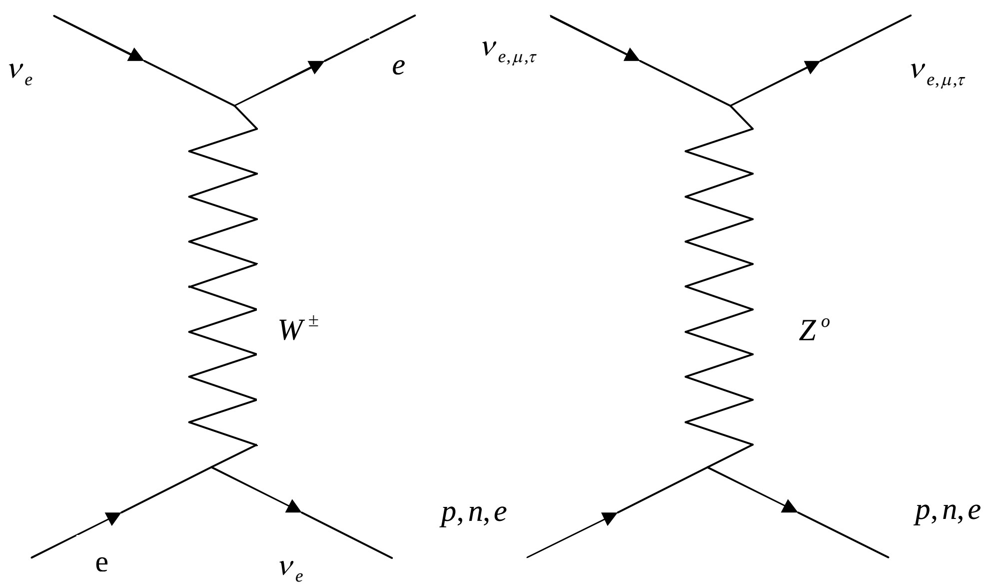


Figure 2.8.2.1 Neutrino Feynmann diagrams

At low neutrino energies, the charged current interactions are described by the effective Hamiltonian [Munoz and Paredes, 2007].

$$H_{CC} = \frac{G_F}{\sqrt{2}}\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)e\right]$$

$$= \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma_\mu (1-\gamma_5) e\right]\left[\bar{\nu}_e \gamma^\mu (1-\gamma_5)\nu_e\right], \qquad (2.8.2.1)$$

where the Fierz transformation has been used [Lewis, 1996]. In order to obtain the coherent forward scattering contribution to the energy of the electron-neutrino ν_e in matter, i.e. the matter-induced potential for ν_e, the variables corresponding to ν_e are fixed so that [Akhmedov, 1999]

$$H_{eff}(\nu_e) = \langle H_{CC}\rangle_{electron}$$

$$= \bar{\nu}_e V_e \nu_e \qquad (2.8.2.2)$$

For convenience, the following definitions

$$\langle \bar{e}\gamma_0 e\rangle = \langle e^* e\rangle = N_e \ , \ \langle \bar{e}\gamma e\rangle = \langle \nu_{\mathbf{e}}\rangle \ , \ \langle \bar{e}\gamma_0\gamma_5 e\rangle = \langle \frac{\sigma_e p_e}{E_e}\rangle \ , \ \langle \bar{e}\gamma\gamma_5 e\rangle = \langle \sigma_e \rangle$$

(2.8.2.3)

are made, and for unpolarized medium of zero total momentum, only the first term survives so that

$$(V_e)_{CC} = V_{CC}$$

$$= \sqrt{2} G_F N_e \qquad (2.8.2.4)$$

Neutral current (NC) contributions V_{NC} to the matter-induced neutrino potentials can also be included. But, in an electrically neutral medium, the number densities of protons and electrons coincide so that the corresponding contributions to V_{NC} cancel. The contribution due to the NC scattering of neutrinos off the neutrons is then

$$(V_a)_{NC} = -G_F N_n / \sqrt{2} \qquad (2.8.2.5)$$

where N_n is the neutron number density. Equations (2.8.2.5) and (2.8.2.4) combine to yield

$$V_e = \sqrt{2} G_F \left(N_e - \frac{N_n}{2} \right)$$

$$\mathrm{V}_\mu = V_\tau = \sqrt{2} G_F \left(-\frac{N_n}{2} \right) \qquad (2.8.2.6)$$

However, in the absence of matter, the evolution equation in the mass eigenstate basis is

$$i \frac{d}{dt} |\nu\rangle = H |\nu\rangle \qquad (2.8.2.7)$$

where

$$H = diag(E_1,\ E_2) \qquad (2.8.2.8)$$

This gives the evolution equation in the flavour basis as

$$i\left(\frac{d}{dt}\right)|\nu_{fl}\rangle = H_{fl}|\nu_{fl}\rangle$$

$$= UHU^{*}|\nu_{fl}\rangle \tag{2.8.2.9}$$

Since the matrix

$$U = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \tag{2.8.2.10}$$

and

$$U^{*} = \begin{pmatrix} c^{*} & -s^{*} \\ s^{*} & c^{*} \end{pmatrix} \tag{2.8.2.11}$$

then

$$UHU^{*} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}\begin{pmatrix} c^{*} & -s^{*} \\ s^{*} & c^{*} \end{pmatrix} \tag{2.8.2.12}$$

$$= \begin{pmatrix} cc^{*}E_1 + ss^{*}E_2 & -cs^{*}E_1 + sc^{*}E_2 \\ -sc^{*}E_1 + cs^{*}E_2 & ss^{*}E_1 + cc^{*}E_2 \end{pmatrix} \tag{2.8.2.13}$$

For relativistic neutrinos,

$$E_i \approx p + \frac{m_i^2}{2E} \tag{2.8.2.14}$$

hence,

$$UHU^{*} = \begin{pmatrix} c^2\left(p + \frac{{m_1}^2}{2E}\right) + s^2\left(p + \frac{{m_2}^2}{2E}\right) & -cs\left(p + \frac{m_1^2}{2E}\right) + cs\left(p + \frac{m_2^2}{2E}\right) \\ -sc\left(p + \frac{m_1^2}{2E}\right) + cs\left(p + \frac{m_2^2}{2E}\right) & s^2\left(p + \frac{m_1^2}{2E}\right) + c^2\left(p + \frac{m_2^2}{2E}\right) \end{pmatrix}$$

$$
= \begin{pmatrix} p + \frac{m_1^2}{2E}c^2 + \frac{m_2^2}{2E}s^2 & -\frac{m_1^2}{4E}\sin 2\theta + \frac{m_2^2}{4E}\sin 2\theta \\ -\frac{m_1^2}{4E}\sin 2\theta + \frac{m_2^2}{4E}\sin 2\theta & p + \frac{m_1^2}{2E}s^2 + \frac{m_2^2}{2E}c^2 \end{pmatrix} \tag{2.8.2.15}
$$

By considering that

$$
\begin{aligned} c^2 &= \cos^2\theta \\ s^2 &= \sin^2\theta \\ \Delta m^2 &= m_2^2 - m_1^2 \end{aligned} \tag{2.8.2.16}
$$

and using the trigonometric relations

$$
\begin{aligned} \cos^2\theta &= \frac{1}{2}(1 + \cos 2\theta) \\ \sin^2\theta &= \frac{1}{2}(1 - \cos 2\theta) \end{aligned} \tag{2.8.2.17}
$$

equation (2.8.2.9) then yields

$$
i\frac{d}{dt}\nu = \begin{pmatrix} \left(p + \frac{m_1^2 + m_2^2}{4E}\right) - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \left(p + \frac{m_1^2 + m_2^2}{4E}\right) + \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\nu \tag{2.8.2.18}
$$

The expressions in brackets in the diagonal elements of equation (2.8.2.18) coincide. This means that they can only modify the common phase of the neutrino states and, therefore, have no effect on neutrino oscillations which depend on the phase differences. Hence, they can be omitted and the evolution equation describing neutrino oscillations in vacuum will then take the form

$$
i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta \mathrm{m}^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \tag{2.8.2.19}
$$

To arrive at the full neutrino evolution equation in matter, the matter-induced potential V_e is added to the diagonal elements of the effective Hamiltonian H_{fl} in equation (2.8.2.19) to obtain

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}=\begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta+\sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \qquad (2.8.2.20)$$

2.8.2.1. MATTER OF CONSTANT DENSITY

For the case of constant matter density and a certain chemical composition ($N_e = constant$), diagonalization of the effective Hamiltonian in (2.8.2.20) requires that the neutrino eigenstates in matter be given by

$$\nu_A = \nu_e \cos\theta_M + \nu_\mu \sin\theta_M$$

$$\nu_B = -\nu_e \sin\theta_M + \nu_\mu \cos\theta_M\,, \qquad (2.8.2.1.1)$$

where the mixing angle θ_M is defined by [Wick and Barry, 2000]

$$\tan 2\theta_M = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta-\sqrt{2}G_F N_e} \qquad (2.8.2.1.2)$$

It is different from the vacuum mixing angle θ and, therefore, the matter eigenstates ν_A and ν_B do not coincide with mass eigenstates ν_1 and ν_2. The difference of the neutrino eigenenergies in matter will then be given by [Akhmedov, 1999]

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2}{2E}\cos 2\theta-\sqrt{2}G_F N_e\right)^2+\left(\frac{\Delta m^2}{2E}\right)^2\sin^2 2\theta} \qquad (2.8.2.1.3)$$

and the probability of $\nu_e \leftrightarrow \nu_\mu$ oscillations will take the form

$$P(\nu_e \to \nu_\mu;\ \mathrm{L}) = \sin^2 2\theta_M \sin^2\left(\pi\frac{L}{l_M}\right), \qquad (2.8.2.1.4)$$

where l_M is defined by

$$l_M = \frac{2\pi}{E_A - E_B} = \frac{2\pi}{\sqrt{\left(\frac{\Delta m^2}{2E}\cos 2\theta-\sqrt{2}G_F N_e\right)^2+\left(\frac{\Delta m^2}{2E}\right)^2\sin^2 2\theta}} \qquad (2.8.2.1.5)$$

The following oscillation amplitude

$$\sin^2 2\theta_M = \frac{\left(\frac{\Delta m^2}{2E}\right)^2 \sin^2 2\theta}{\left(\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\right)^2 \sin^2 2\theta} \qquad (2.8.2.1.6)$$

has a resonance-like form, with the maximum value $\sin^2 2\theta_M = 1$ being achieved when the condition

$$\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta \qquad (2.8.2.1.7)$$

is satisfied; this is similar to the Mikheyev-Smirnov-Wolfenstein (MSW) [Wolfenstein, 1978; Mikheyev and Smirnov, 1985] resonance condition for neutrino oscillations in matter. From equation (2.8.2.1.5), it follows that when the condition (2.8.2.1.7) is fulfilled, mixing in matter is maximal for $\theta_M = 45^o$ and is independent of the vacuum mixing angle θ. Thus, the probability of neutrino flavour oscillation in matter can be large even if the mixing angle is very small. However, for resonance enhancement of neutrino oscillations in matter to be possible, the right hand side of equation (2.8.2.1.7) must be positive so that

$$\Delta m^2 \cos 2\theta = (m_2^2 - m_1^2)(\cos^2\theta - \sin^2\theta) > 0, \qquad (2.8.2.1.8)$$

i.e., if ν_2 is heavier than ν_1, then $\cos^2\theta > \sin^2\theta$ and vice versa. From equation (2.8.2.1.1) it can then be interpreted that the condition (2.8.2.1.8) is equivalent to the requirement that, of the two mass eigenstates ν_1 and ν_2, the lower-mass one has a larger ν_e component.

2.8.2.2. MATTER OF VARYING DENSITY

In this section, neutrino oscillation in matter of varying density is studied. Typically, a beam of non-monochromatic neutrinos (neutrinos with some energy distribution) propagating in a medium with a density profile is considered. In particular, a case where an electron neutrino produced in matter of very high density (e.g. in the core of the sun and propagates in matter whose density decreases along the neutrino trajectory) is considered. As it propagates toward regions of smaller matter density, the mixing increases and becomes maximal at the resonance point, where $\theta = 45^o$. As it propagates further toward smaller densities, the mixing angle continues to decrease, reaching the value $\theta_f = \theta$ at densities $N_e << (N_e)_{MSW}$, where

$(N_e)_{MSW}$ is the resonance value of the electron number density given by equation (2.8.2.1.7). If the matter density changes slowly enough (that is, adiabatically along the neutrino path), the neutrino system will have enough time to adjust itself to the changing external conditions. In the adiabatic regime, the effective Hamiltonian $H_{fl}(t)$ will then be diagonalized by a unitary transformation

$$\nu_{fl} = \tilde{U}(t)\nu \qquad (2.8.2.2.1)$$

$$\tilde{\mathrm{U}}(\mathrm{t})^* H_{fl}(t)\tilde{U}(t) = \tilde{H}_d(t)$$

$$= diag(E_A(t),\ \mathrm{E_B}(t)), \qquad (2.8.2.2.2)$$

where $E_A(t)$ and $E_B(t)$ are instantaneous eigenstate values of $H_{fl}(t)$ and the matrix $\tilde{U}(t)$ has the same form as (2.8.2.10) except that the vacuum mixing angle θ has to be replaced by the mixing angle $\theta(t)$, with $N_e = N_e(t)$. The evolution equation in the basis of the instantaneous eigenstates can then be written as

$$i\left(\frac{d}{dt}\right)\nu = \left[\tilde{H}_d - i\tilde{U}^*\left(\frac{d\tilde{U}}{dt}\right)\right]\nu \qquad (2.8.2.2.3)$$

$$= \left[\begin{pmatrix} E_A(t) & 0 \\ 0 & E_B(t) \end{pmatrix} - i\begin{pmatrix} c & -s \\ s & c \end{pmatrix}\frac{d}{dt}\begin{pmatrix} c & s \\ -s & c \end{pmatrix}\right]\nu$$

$$= \left[\begin{pmatrix} E_A(t) & 0 \\ 0 & E_B(t) \end{pmatrix} - i\begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} \dot{c} & \dot{s} \\ -\dot{s} & \dot{c} \end{pmatrix}\right]\nu$$

$$= \left[\begin{pmatrix} E_A(t) & 0 \\ 0 & E_B(t) \end{pmatrix} - i\begin{pmatrix} c\dot{c}+s\dot{s} & c\dot{s}-s\dot{c} \\ s\dot{c}-c\dot{s} & s\dot{s}+c\dot{c} \end{pmatrix}\right]\nu \qquad (2.8.2.2.4)$$

$$= \left[\begin{pmatrix} E_A(t) & 0 \\ 0 & E_B(t) \end{pmatrix} - i\begin{pmatrix} -cs\dot{\theta}(t)+sc\dot{\theta}(\mathrm{t}) & c^2\dot{\theta}(t)+s^2\dot{\theta}(t) \\ -s^2\dot{\theta}(t)-c^2\dot{\theta}(\mathrm{t}) & sc\dot{\theta}(t)-cs\dot{\theta}(t) \end{pmatrix}\right]\nu$$

$$= \left[\begin{pmatrix} E_A(t) & 0 \\ 0 & E_B(t) \end{pmatrix} - i \begin{pmatrix} 0 & \dot{\theta}(t) \\ -\dot{\theta}(\mathrm{t}) & 0 \end{pmatrix} \right] v \tag{2.8.2.2.5}$$

which may also be expressed as

$$i\frac{d}{dt}\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} E_A(t) & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & E_B(t) \end{pmatrix}\begin{pmatrix} v_A \\ v_B \end{pmatrix} \tag{2.8.2.2.6}$$

with

$$\dot{\theta}(t) = \frac{d\theta}{dt} \tag{2.8.2.2.7}$$

and

$$v = \begin{pmatrix} v_A \\ v_B \end{pmatrix} \tag{2.8.2.2.8}$$

The effective Hamiltonian in this basis change is not diagonal since the mixing angle θ is not constant; the matter eigenstate basis changes with time. If the off-diagonal terms are small so that

$$\left|\dot{\theta}\right| << \left|E_A - E_B\right|, \tag{2.8.2.2.9}$$

then the transitions between the instantaneous eigenstates v_A and v_B are suppressed. This corresponds to the adiabatic condition defined by

$$\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta \tag{2.8.2.2.10}$$

From this relation, it is explicit that

$$\cos 2\theta = \frac{2E}{\Delta m^2}\sqrt{2}G_F N_e \tag{2.8.2.2.11}$$

and when the equation (2.8.2.2.11) is differentiated with respect to time, it yields

$$\dot{\theta} = -\frac{E_v}{\Delta m^2}\frac{\sqrt{2}G_F}{\sin 2\theta}\dot{N}_e \tag{2.8.2.2.12}$$

This then means that the adiabaticity parameter γ may be written as

$$\gamma^{-1} = \frac{2\left|\dot{\theta}\right|}{\left|E_A - E_B\right|}$$

$$= \frac{\sin 2\theta \dfrac{\Delta m^2}{2E}}{\left|E_A - E_B\right|^3}\left|\dot{V}_{CC}\right| << 1 \qquad (2.8.2.2.13)$$

where $E_A - E_B$ and V_{CC} are given by equations (2.8.2.1.3) and (2.8.2.4) respectively. If at a time $t = t_i$, the electron neutrino is produced with the field

$$\nu(t_i) = \nu_e(t)$$
$$= \cos\theta_i(t)\nu_A + \sin\theta_i(t)\nu_B, \qquad (2.8.2.2.14)$$

then (in the adiabatic approximation) the neutrino state at time t_f is

$$\nu(t_f) = \cos\theta_i e^{-i\int_{t_i}^{t_f} E_A(t)dt}\nu_A + \sin\theta_i e^{-i\int_{t_i}^{t_f} E_B(t)dt}\nu_B \qquad (2.8.2.2.15)$$

Considering that $t = t_f$ and mixing angle $\theta(t_f) \equiv \theta_f$ is different from θ_i, then

$$\langle\nu_e|\nu_\mu\rangle = \left(c_i\langle\nu_A| + s_i\langle\nu_B|\right)\left(-s_f e^{-i\int_{t_i}^{t_f} E_A(t)dt}|\nu_A\rangle + c_f e^{-i\int_{t_i}^{t_f} E_B(t)dt}|\nu_B\rangle\right)$$
$$= e^{-i\varepsilon}\left(-c_i s_f + s_i c_f e^{i\delta}\right), \qquad (2.8.2.2.16)$$

where

$$\varepsilon = \int_{t_i}^{t_f} E_A(t)dt \qquad (2.8.2.2.17)$$

and

$$\delta = \int_{t_i}^{t_f}\left(E_A(t) - E_B(t)\right)dt \qquad (2.8.2.2.18)$$

Therefore, the probability for an electron-neutrino transforming into a muon-neutrino is

$$P(\nu_e \to \nu_\mu) = \left|<\nu_e|\nu_\mu>\right|^2$$

$$= \left\{-c_i s_f + s_i c_f e^{i\delta}\right\}\left\{-c_i^* s_f^* + s_i^* c_f^* e^{-i\delta}\right\}$$
$$= c_i c_i^* s_f s_f^* - c_i s_f s_i^* c_f^* e^{-i\delta} - s_i c_f c_i^* s_f^* e^{i\delta} + s_i c_f s_i^* c_f^* \qquad (2.8.2.2.19)$$

Taking

$$c = c^*$$

$$s = s^*$$

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad (2.8.2.2.20)$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)\,,$$

then equation (2.8.2.2.19) becomes

$$P(\nu_e \to \nu_\mu) = \frac{1}{2} - \frac{1}{2}\cos 2\theta_i \cos 2\theta_f - \frac{1}{2}\sin 2\theta_i \sin 2\theta_f \cos\delta \qquad (2.8.2.2.21)$$

To include all possible transitions between matter eigenstates ν_A and ν_B due to violation of adiabaticity, P' (known as the hopping probability) is considered as the probability that $\nu_A \leftrightarrow \nu_B$ transitions have occurred in the course of the evolution of the neutrino system. Then

$$P(\nu_e \to \nu_\mu) \approx 1 - P' \qquad (2.8.2.2.22)$$

But, from the Landau-Zener approximation [Zener, 1932], the hopping probability P' is defined by

$$P' \approx e^{-\frac{\pi}{2}\gamma_r} \qquad (2.8.2.2.23)$$

Therefore, the adiabaticity parameter at the MSW resonance becomes

$$\gamma_r = \left(\frac{\Delta m^2}{2E}\sin 2\theta\right)^2 \frac{1}{\left|\dot{V}_{CC}\right|_{res}}\,,$$

$$= \frac{\sin^2 2\theta}{\cos 2\theta}\frac{\Delta m^2}{2E} L_\rho \qquad (2.8.2.2.24)$$

where

$$L_\rho = \left|\dot{V}_{CC}/V_{CC}\right|^{-1}_{res}$$

$$= \left|\dot{N}_e/N_e\right|^{-1}_{res} \qquad (2.8.2.2.25)$$

is the length scale at resonance (the characteristic distance over which the electron number density varies significantly in the resonance region). Hence, by defining

$$\Delta r = 2\tan 2\theta L_\rho \qquad (2.8.2.2.26)$$

and applying the oscillation length relation (2.8.2.1.5) defined as

$$(l_m) = \frac{2\pi}{\left|E_A - E_B\right|_{res}} = \frac{4\pi E}{\Delta m^2}\frac{1}{\sin 2\theta}, \qquad (2.8.2.2.27)$$

at the resonance, then the adiabaticity parameter (2.8.2.2.24) becomes

$$\gamma_r = \pi \frac{\Delta r}{(l_m)_{res}} \qquad (2.8.2.2.28)$$

i.e., $\gamma_r > 3$ is the condition that at least one oscillation length fits into the resonance region. From (2.8.2.2.10) and (2.8.2.2.24), both the MSW resonance condition and the adiabaticity parameter at the resonance γ_r depend on neutrino energy. This shows that the efficiency of the matter-enhanced neutrino flavor conversion is energy dependent. In addition, heavy neutrinos may have a chance to decay into lighter neutrinos via, for example, the process shown in figure 2.8.2.2 below which is proportional to the amount of mixing.

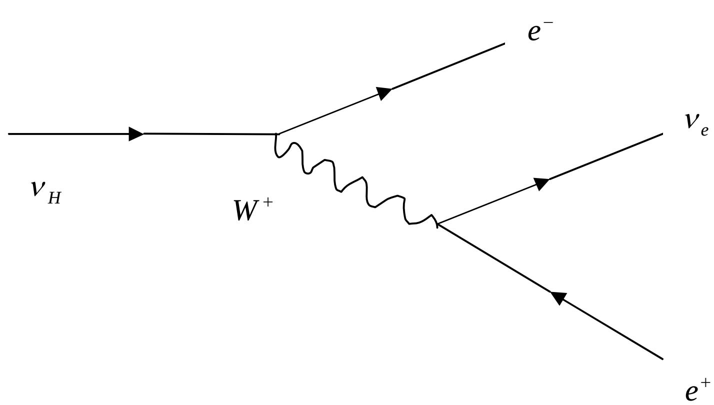


Figure 2.8.2.2: Heavy neutrino decay via mixing

To model neutrino decay, a beam of independent massive neutrinos each having a probability λ of decaying per unit time is considered. The number decaying per unit time dt is taken to be [Murugeshan, 2003]

$$dN = -\lambda N(t)dt, \qquad (2.8.2.2.29)$$

where $N(t)$ is the number of neutrinos at time t. Integration of equation (2.8.2.2.29) yields

$$N(t) = N(0)e^{-\lambda t} \qquad (2.8.2.2.30)$$

In one half-life, half of all neutrinos present will decay. Since the mean life is the average time a neutrino exists before it decays, it is connected to λ and $t_{1/2}$ by

$$\tau = \frac{1}{\lambda}$$

$$= \frac{t_{1/2}}{\ln 2}$$

$$= 1.44\, t_{1/2} \qquad (2.8.2.2.31)$$

To relate the exponential properties of a decaying neutrino, the time dependence of neutrino spinor at rest, that is, $\vec{P} = 0$, is expressed explicitly as

$$\psi(t) = \psi(0)e^{-\frac{iEt}{\hbar}} \qquad (2.8.2.2.32)$$

If the energy E of this state is real, then the probability of finding the particle is not a function of time because

$$|\psi(t)|^2 = |\psi(0)|^2 \qquad (2.8.2.2.33)$$

A particle described by a wave function of the type in equation (2.8.2.2.33) with real energy E does not decay. To introduce a decay of a state described by $\psi(t)$, a small imaginary term is added to the energy, so that

$$E = E_0 - \frac{1}{2}i\Gamma \qquad (2.8.2.2.34)$$

where E_0 and Γ are real, and the factor $\frac{1}{2}$ is chosen for convenience. With equation (2.8.2.2.34), the probability of decaying becomes

$$|\psi(t)|^2 = |\psi(0)|^2 e^{\frac{-\Gamma t}{\hbar}} \qquad (2.8.2.2.35)$$

Equation (2.8.2.2.35) agrees with equation (2.8.2.2.30) if

$$\Gamma = \lambda\hbar \qquad (2.8.2.2.36)$$

With definitions (2.8.2.2.33) and (2.8.2.2.34), the wave function of a decaying neutrino is found to be of the form

$$\psi(t) = \psi(0)e^{-i\frac{E_0 t}{\hbar}} e^{-\frac{\Gamma t}{2\hbar}}, \qquad (2.8.2.2.37)$$

where the addition of a small imaginary part to the energy allows a description of an exponentially decaying state. But from equation (2.8.2.2.37), shows $\psi(t)$ is a function of time rather than energy. Hence, a change from $\psi(t)$ to $\psi(E)$ is effected by Fourier transform [Gupta, 2004]; this is done by considering a function $f(t)$ expressed as an integral

$$f(t) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{+\infty} d\omega \, g(\omega) \, e^{-i\omega t} \tag{2.8.2.2.38}$$

which inverts to

$$g(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} dt f(t) e^{+i\omega t} \tag{2.8.2.2.39}$$

If $f(t)$ is set equal to $\psi(t)$ in equation (2.8.2.2.37) and the decay starts at time $t = 0$, then the lower limit on the integral can be set equal to zero. Therefore, $g(\omega)$ becomes

$$g(\omega) = (2\pi)^{-1/2} \psi(0) \int_0^{\infty} dt e^{i\left(\omega - \frac{E_0}{\hbar}\right)t} e^{-\Gamma t/2\hbar} \tag{2.8.2.2.40}$$

which on integration, yields

$$g(\omega) = \frac{\psi(0)}{\sqrt{2\pi}} \frac{i\hbar}{(\hbar\omega - E_0) + i\Gamma/2} \tag{2.8.2.2.41}$$

Since $E = \hbar\omega$, then the probability density $P(E)$ of finding a neutrino with energy E is proportional to $|g(\omega)|^2 = g^{\bullet}(\omega) g(\omega)$, that is,

$$\begin{aligned} P(E) &= const \times g^{\bullet}(\omega) g(\omega) \\ &= const \times \frac{\hbar^2}{2\pi} \frac{|\psi(0)|^2}{(E - E_0)^2 + \Gamma^2/4} \end{aligned} \tag{2.8.2.2.42}$$

Therefore, from the condition

$$\int_{-\infty}^{+\infty} P(E) dE = 1 \tag{2.8.2.2.43}$$

it is found that

$$const. = \frac{\Gamma}{\hbar^2 |\psi(0)|^2} \tag{2.8.2.2.44}$$

and hence, combination of equations (2.8.2.2.42) and (2.8.2.2.44) leads to

$$P(E)=\frac{\Gamma}{2\pi}\frac{1}{(E-E_0)^2+(\Gamma/2)^2} \tag{2.8.2.2.45}$$

The relation (2.8.2.2.45) shows that the energy of a decaying neutrino state is not sharp. The small imaginary part in equation (2.8.2.2.34) leads to decay, which introduces a broadening of the state. With definitions (2.8.2.2.32) and (2.8.2.2.36), the product of lifetime and width becomes

$$\tau\,\Gamma=\hbar \tag{2.8.2.2.46}$$

This relation can be interpreted as a Heisenberg uncertainty relation

$$\Delta t\Delta E\geq\hbar \tag{2.8.2.2.47}$$

Thus, to determine the mass or energy of the neutrino to within an accuracy of $\Delta E=\Gamma$, a time $\Delta t=\tau$ is required. This shows that even if a longer time is used, an uncertainty in the energy or mass of a decaying neutrino is inevitable.

CHAPTER THREE

NEUTRINO MASS RESULTS

3.1. INTRODUCTION

In this chapter, results for neutrino mass investigation are presented. In particular, the result in regard to the existence of the neutrino is presented and the process by which the neutrino can acquire mass through a modified standard electroweak Lagrangian presented. The result for massive neutrino in the big bang model is also presented. Essentially, the equilibrium condition between the weak interactions and the expansion rate generate a Neutrino Boltzmann transport equation which is solved by successive approximation technique to yield a neutrino mass value. This value, when used in the calculated seesaw relation, it is found to give a unification-energy-scale result that is interesting in the context of grand unified models. Further, a result on neutrino oscillation as a consequence of its propagation in matter is presented. By using the binomial expansion theorem, it is explicitly found that the phenomenon of neutrino oscillations can only occur if a neutrino has mass.

3.2. EXISTENCE OF THE NEUTRINO

In a decay process, if *N(T)* is taken as the relative number of particles emitted with kinetic energy *T*, then the number of particles *N(T)* emitted against their kinetic energies *T* can theoretically be represented by a graph like the one below:

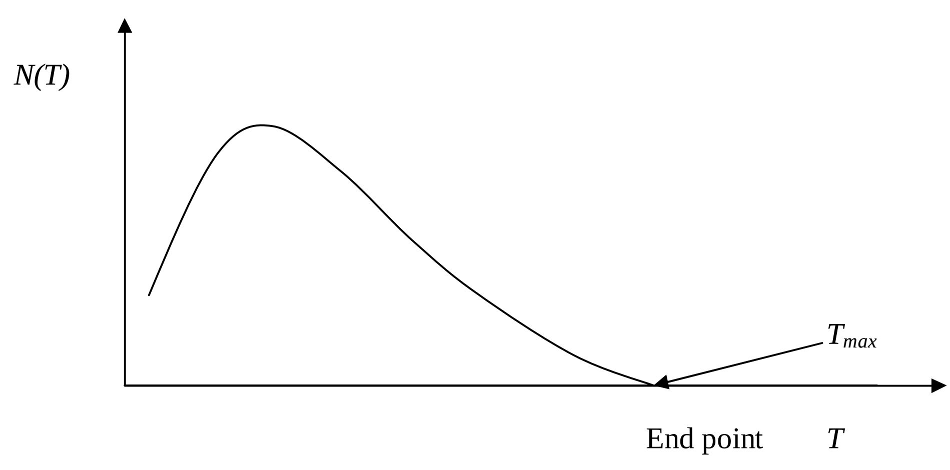


Figure 3.2.1: Number of particles emitted with energy T

N(T) is zero above an end point energy T_{max}, which reflects the fact that the kinetic energy of the particles is limited by the differences in rest masses of the parent nucleus *A* and the daughter nucleus *B*. In a typical beta (β) decay process such as the one suggested below

$$A \to B + e + \cdots\cdots \qquad (3.2.1)$$

the visible particles are *A*, *B* and *e*. If these were the only particles, then one would be dealing with a two-body decay process

$$A \to B + e \qquad (3.2.2)$$

which, in the rest frame of *A*, would have momenta that add up to zero and are uniquely determined by the rest masses m_A, m_B and m_e. In this case, *N(T)* versus *T* would generate the graph given below and all the particles would have the same kinetic energy.

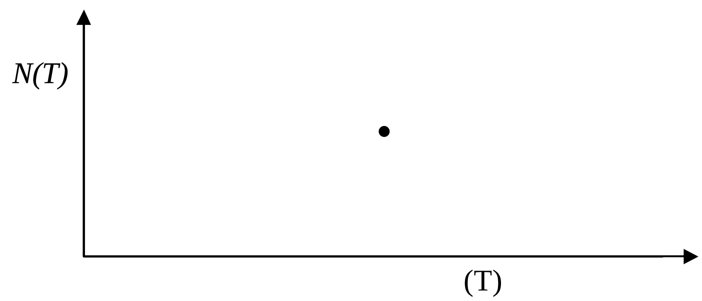


Figure 3.2.2: Electron number density for a two-body decay

Away from the general case, the decay process that is sampled for investigation in this study involves the a related beta (β) decay process of the neutron given in equation (3.2.3) below

$$n \to p + e^- \qquad (3.2.3)$$

This equation is physically inadequate especially from the viewpoint of conservation laws. Further, the justification for this is found by generating the following data that gives the quantum numbers for the various particles participating in this decay process (3.2.3):

Quantum No. \ Equation	Left-hand side (n)	Right-hand side (p + e)	
Spin (s)	$\frac{1}{2}$	$\frac{1}{2}+\frac{1}{2}=1$	**?**
Charge (q)	0	$\frac{1}{2}+\left(-\frac{1}{2}\right)=0$	ok
Baryon Number (B)	1	1 + 0 = 1	ok
Lepton Number (L)	0	0 + 1 =1	**?**

Table 3.2.1: Particle Quantum Numbers

From this data, it is evident that the above process is lacking some information or something is wrong with it. In order for the process to describe physical reality, the equation is modified to the form

$$n \rightarrow p + e^{-} + X \qquad (3.2.4)$$

and the proposed particle X, henceforth known as the neutrino, is found to have the following quantum numbers:

Equation / Quantum No.	Left-hand side (n)	Right-hand side (p + e)	? = X	$p + e^{-} + \overline{X}$
Spin (s)	$\frac{1}{2}$	$\frac{1}{2}+\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$
Charge (q)	0	$\frac{1}{2}+\left(-\frac{1}{2}\right)$	0	0
Baryon Number (B)	1	1 + 0	0	1
Lepton Number (L)	0	0 + 1	-1	0

Table 3.2.2: Neutrino Quantum Numbers

To investigate the role this neutrino may play in an evolving universe, it is suggested that the universe is a uniformly distributed collection of mass points defined by a mass density ρ_m. By considering a sphere of radius *S(t)* anywhere in the universe, the gravitational field acting on the mass m located at A is found to depend on the masses within the sphere as in the figure below:

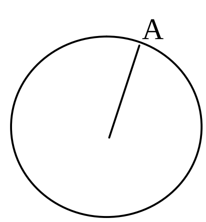


Figure 3.2.3: Sphere of radius S(t)

The equation of motion for the mass m_A is then written as

$$m\ddot{S} = -\frac{GmM}{S^2} \qquad (3.2.5)$$

S is a function of time since the universe is expanding and M is constant in time, which is the mass within the sphere. Multiplying both sides of equation (3.2.5) by $\dot{S}$ gives

$$\dot{S}\ddot{S} = -\frac{GM}{S^2}\dot{S} \qquad (3.2.6)$$

which is the same as

$$\frac{d}{dt}\left(\frac{\dot{S}^2}{2}\right) = GM\frac{d}{dt}\left(\frac{1}{S}\right) \qquad (3.2.7)$$

Equation (3.2.7) is integrated to yield

$$\frac{\dot{S}^2}{2} - \frac{GM}{S} = E \qquad (3.2.8)$$

where E is a constant of integration. Since *M(S)* is the mass contained within the spherical volume, it can also be calculated by the relation

$$M(S) = \frac{4}{3}\pi S^3 \rho_m \qquad (3.2.9)$$

so that the equation (3.2.8) now becomes

$$\frac{\dot{S}^2}{2} - \frac{4\pi}{3}GS^2\rho_m = E \qquad (3.2.10)$$

This equation has the same form as the energy conservation law in a gravitational potential, namely:

Kinetic energy + Potential energy = Total energy (3.2.11)

From equation (3.2.10), the following interesting observations can be made:

If

- $E > 0$, there should be no bound on *S*.
- $E < 0$, *S* is bound in an orbit.
- $E = 0$, *S* can just 'escape'.

But here, *S* stands for the universal scale factor, which means that

- $E > 0$, the universe will expand for ever;
- $E < 0$, the universe will stop expanding at some finite time in the future;
- $E = 0$, the universe is just free to continue expanding.

In the context of general relativity, the conditions can be related to the curvature of space. In particular, the third condition means that space is 'flat'. This is what is being referred to as the "flatness problem"; the "problem" being to explain why $E = 0$ or nearly so.

3.3. THE FLATNESS PROBLEM

A neutrino-dominated and evolving universe can be considered to be described by the dynamical Friedman equation

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi G}{3}\rho, \qquad (3.3.1)$$

where $S(t)$ is the scale factor, k is the curvature parameter and ρ is the energy density. In the $k = 0$ model, a resulting relationship between S and t can be established; from the elementary kinetic theory of gases, an equation of state for a free neutrino gas can be written as

$$P = \frac{\rho}{3} \qquad (3.3.2)$$

The equation (3.3.2) is used in the energy conservation law

$$\dot{\rho} = -3\frac{\dot{S}}{S}(\rho + P) \qquad (3.3.3)$$

to give

$$\dot{\rho} = -4\frac{\dot{S}}{S}\rho \qquad (3.3.4)$$

This is integrated to yield

$$\rho \,\alpha\, S^{-4} \qquad (3.3.5)$$

Applying equation (3.3.5), equation (3.3.2) can be found to take the form

$$\frac{k}{S^2} = (\Omega - 1)\frac{\dot{S}^2}{S^2} \qquad (3.3.6)$$

$$= \frac{\Omega - 1}{4t^2}$$

where

$$\Omega = \frac{\rho}{\rho_c} \qquad (3.3.7)$$

Ω is the relative cosmological energy density parameter. For the present epoch, equation (3.3.6) can be written as

$$\frac{k}{S_p^{\,2}} = (\Omega_p - 1)\frac{\dot{S}_p^{\,2}}{S_p^{\,2}} \qquad (3.3.8)$$

Using the simple fact that

$$S(t) \propto T^{-1} \qquad (3.3.9)$$

it can be established that, for $k = \pm 1$,

$$(\Omega - 1) = (\Omega_p - 1)4H_p^{\,2}t^2\frac{T^2}{T_p^{\,2}} \quad , \qquad (3.3.10)$$

where T is the corresponding value of temperature. Equation (3.3.10) is then used to generate the following data for the important phases in the early universe:

PHASE	TIME (s) (Hughes, 1991)	TEMPERATURE (K) (Hughes, 1991)	COSMOLOGICAL ENERGY DENSITY $(\Omega - 1) = (\Omega_p - 1)4H_p^{\,2}t^2\frac{T^2}{T_p^{\,2}}$ (Maumba, et al. 2008)
Planck	10^{-43}	10^{32}	$(\Omega - 1) = 3.108\times10^{-61}h^2(\Omega_p - 1)$
GUT	10^{-35}	10^{28}	$(\Omega - 1) = 4.626\times10^{-53}h^2(\Omega_p - 1)$
Electroweak symmetry breaking	10^{-10}	10^{15}	$(\Omega - 1) = 4.626\times10^{-27}h^2(\Omega_p - 1)$
Quark confinement	10^{-5}	10^{12}	$(\Omega - 1) = 5.140\times10^{-22}h^2(\Omega_p - 1)$
Neutrino Decoupling	10^{0}	10^{10}	$(\Omega - 1) = 4.626\times10^{-17}h^2(\Omega_p - 1)$

Table 3.3.1: Data for Flatness Problem

From the data, it is interesting to observe that the coefficient on the right-hand side of equation (3.3.10) keeps on changing, i.e. at the Planck time, the approximate value of 10^{-61} leads the force of gravity to break away from the strong and electroweak interactions, whereas the value 10^{-53} leads to the strong force breaking from its counterpart electroweak interaction and 10^{-27} leads to the electroweak symmetry breaking. However, when equation (3.3.1) is rewritten as

$$-k = \dot{S}^2 - \frac{8\pi G}{3}\rho S^2 \tag{3.3.11}$$

then, $-k$ can be interpreted as the total energy of the universe with the kinetic energy term being represented by $\dot{S}^2$ and the gravitational potential energy by the term containing ρ. If the total energy is positive ($k = -1$), then the kinetic energy is great enough (the initial velocity is greater than the escape velocity) and the universe will continue to expand forever i.e. the universe is open. If the total energy is negative ($k = +1$), the universe will recollapse i.e. the universe is closed. If the total energy is zero ($k = 0$), the universe is at the escape velocity and it will expand indefinitely. Since the universe has been expanding at almost the critical rate, it is expected that some mechanism or processes in the early universe could have driven it into its present state. Rewriting equation (3.3.1) in the form

$$\frac{\dot{S}^2 + k}{S^2} = \frac{8\pi G}{3}(\rho_v + \rho_r) \tag{3.3.12}$$

analogous results for the flatness problem can be obtained. Here, $\rho_r \propto S(t)^{-4}$ is the energy density of radiation and since ρ_r falls as the universe expands while ρ_v stays constant, the vacuum energy density dominates. Therefore, ρ_r is neglected and the equation (3.3.12) is solved for the cosmic models $k = 0$, $k = +1$ and $k = -1$ as follows:

The $k = 0$ model

In this model, equation (3.3.12) reduces to

$$\frac{\dot{S}^2}{S^2} = \frac{8\pi G}{3}\rho_v \tag{3.3.13}$$

or

$$\begin{aligned}\frac{\dot{S}}{S} &= \sqrt{\frac{8\pi G}{3}\rho_v} \\ &= H \end{aligned} \tag{3.3.14}$$

where H is Hubble's constant. This integrates to give

$$S \propto \exp Ht \tag{3.3.15}$$

The $k = +1$ model

In this case, equation (3.3.12) reduces to

$$\frac{\dot{S}^2 + 1}{S^2} = \frac{8\pi G}{3}\rho_v \tag{3.3.16}$$

or

$$\dot{S} = \sqrt{\frac{8\pi G}{3}\rho_v S^2 - 1} \tag{3.3.17}$$

which can be expressed as

$$\begin{aligned} \frac{dS}{dt} &= \frac{1}{\alpha}\sqrt{S^2 - \beta^2} \\ &= \frac{\beta}{\alpha}\sqrt{\frac{S^2}{\beta^2} - 1} \\ &= \sqrt{u^2 - 1}, \end{aligned} \tag{3.3.18}$$

with

$$\alpha = \sqrt{\frac{3}{8\pi G\rho_v}} \tag{3.3.19}$$

$$\beta = \alpha^{-1}$$

and

$$u = \frac{S}{\beta} \tag{3.3.20}$$

Using equation (3.3.20), equation (3.3.18) then becomes

$$\frac{du}{\sqrt{u^2 - 1}} = \frac{dt}{\beta} \tag{3.3.21}$$

which integrates to yield

$$\begin{aligned} S &= \beta \cosh \alpha t \\ &= \alpha^{-1} \cosh \alpha t \end{aligned} \tag{3.3.22}$$

or

$$S = H^{-1} \cosh Ht \tag{3.3.23}$$

when equations (3.3.14), (3.3.19) and (3.3.20) are applied. Thus, equation (3.3.23) imply

$$S \propto \cosh Ht \tag{3.3.24}$$

The $k = -1$ model

For $k = -1$, equation (3.3.12) reduces to

$$\frac{\dot{S}^2 - 1}{S^2} = \frac{8\pi G}{3}\rho_v \tag{3.3.25}$$

or

$$\dot{S} = \sqrt{\frac{8\pi G}{3}\rho_v S^2 + 1} \tag{3.3.26}$$

which can be rewritten as

$$\begin{aligned} \frac{dS}{dt} &= \frac{1}{\alpha}\sqrt{(S^2 + \beta^2)} \\ &= \sqrt{\frac{S^2}{\beta^2} + 1} \end{aligned} \tag{3.3.27}$$

Making use of the substitutions in equations (3.3.19) and (3.3.20), then equation (3.3.27) becomes

$$\frac{du}{\sqrt{u^2 + 1}} = \frac{1}{\alpha}dt \tag{3.3.28}$$

which, when integrated yields

$$S = H^{-1}\sinh Ht \tag{3.3.29}$$

This implies that

$$S \propto \sinh Ht \tag{3.3.30}$$

In the approximation that

$$Ht \to \infty, \tag{3.3.31}$$

both equations (3.3.24) and (3.3.30) reduce to equation (3.3.15), which is the flat model. If the total energy density ρ_t is represented by $(\rho_v + \rho_r)$, then, for $\rho_r \propto T^4$ and $T \propto S^{-1}$,it is found that

$$\rho \propto S^{-4} \tag{3.3.32}$$

Equation (3.3.32) can be expanded into a Taylor series to yield

$$\rho_t(x) = a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + ... + a_n x^n \tag{3.3.33}$$

where $x = S^{-4}$. When the relevant coefficients are inserted, equation (3.3.33) reduces to

$$\rho_t(x) = \rho_o + S^{-4} - 4S^{-9} + 20S^{-14} - 120S^{-19} + ... \tag{3.3.34}$$

3.4. PARTICULATE NATURE OF THE UNIVERSE

To model the early universe as a particle, the various possible quantum numbers should be calculated. One of the important quantum numbers calculated is the size or volume of the very early universe. The main idea is that; current observations indicate that the universe is expanding. When extrapolated backwards, it is natural to expect the volume and/or size of the universe to vanish at zero time. Consequently, to make meaningful studies/investigations on the physical processes (especially the weak interaction processes) in the very early universe, it is mandatory that the eminent singularity be addressed first. In this particular study, the idea of instantons is suggested as a possible solution to this puzzle. It is considered that the creation of the universe is a spontaneous tunneling process that occurs by the nucleation of bubbles whose tunneling probability is calculated by solving the simple equation of motion

$$\Delta_E \phi - V^{'}(\phi) \equiv \frac{d^2\phi}{dt^2} + \nabla^2\phi - V^{'}(\phi) = 0 \tag{3.4.1}$$

subject to boundary conditions

$$\phi = 0 \text{ at } r^2 = \vec{x}^2 + t^2 = \infty \tag{3.4.2}$$

and

$$\frac{d\phi}{dr} = 0 \text{ at } r = 0 \tag{3.4.3}$$

The probability of the bubble nucleation per unit volume per unit time is defined by the relation

$$\Im = C\exp(-A_E), \tag{3.4.4}$$

where the action A_E is defined by the relation

$$A_E(\phi) = \int d^4x \left[\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right] \tag{3.4.5}$$

and ***C*** is a constant. The Euler-Lagrange equations of motion are then applied to equation (3.4.5) to yield

$$\Delta_E\phi - V^{'}(\phi) = \frac{\partial^2\phi}{\partial t^2} + \nabla^2\phi - V^{'}(\phi) = 0 \tag{3.4.6}$$

When the following coordinate transformations

$$R^2 \equiv r^2$$

$$x^2 - x_0^2 \equiv \vec{x}^{\,2} \tag{3.4.7}$$

$$t^2 - t_0^2 \equiv t^2$$

are made, equation (2.4.1.16) reduces to

$$\begin{aligned} r^2 &= \vec{x}^{\,2} + t^2 \\ &= x^2 + y^2 + z^2 + t^2 \end{aligned} \tag{3.4.8}$$

This equation (3.4.8) is then differentiated to give

$$\frac{\partial r}{\partial x} = \frac{x}{\sqrt{\vec{x}^{\,2} + t^2}} \tag{3.4.9}$$

$$\frac{\partial r}{\partial t} = \frac{t}{\sqrt{\vec{x}^{\,2} + t^2}} \tag{3.4.10}$$

$$\begin{aligned} \frac{\partial\phi}{\partial x} &= \frac{\partial r}{\partial x}\frac{\partial\phi}{\partial r} \\ &= \frac{x}{\sqrt{\vec{x}^{\,2} + t^2}}\frac{\partial\phi}{\partial r} \end{aligned} \tag{3.4.11}$$

This means that

$$\frac{\partial^2\phi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\left\{ \frac{x}{\sqrt{\vec{x}^2 + t^2}} \frac{\partial \phi}{\partial r} \right\}$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{\vec{x}^2 + t^2}} \right) \frac{\partial \phi}{\partial r} + \frac{x}{\sqrt{\vec{x}^2 + t^2}} \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial r} \right)$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{\vec{x}^2 + t^2}} \right) \frac{\partial \phi}{\partial r} + \frac{x}{\sqrt{\vec{x}^2 + t^2}} \frac{\partial}{\partial r} \frac{\partial r}{\partial x}\left(\frac{\partial \phi}{\partial r} \right)$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{\vec{x}^2 + t^2}} \right) \frac{\partial \phi}{\partial r} + \frac{x^2}{\vec{x}^2 + t^2} \frac{\partial^2 \phi}{\partial r^2} \tag{3.4.12}$$

For the first term of the last step on the right hand side of (3.4.12), the following quotient rule

$$\frac{\partial}{\partial x}\left(\frac{f(x)}{g(x)} \right) = \frac{g(x) f^{'}(x) - f(x) g^{'}(x)}{g^2}, \tag{3.4.13}$$

is used to yield

$$\frac{\partial}{\partial x}\left(\frac{x}{\sqrt{\vec{x}^2 + t^2}} \right) \frac{\partial \phi}{\partial r} = \left\{ \frac{1.\left(\vec{x}^2 + t^2\right)^{\frac{1}{2}} - x^2 \left(\vec{x}^2 + t^2\right)^{-\frac{1}{2}}}{\left(\vec{x}^2 + t^2\right)} \right\} \frac{\partial \phi}{\partial r}$$

$$= \frac{1}{\sqrt{\vec{x}^2 + t^2}} \frac{\partial \phi}{\partial r} - \frac{x^2}{\left(\vec{x}^2 + t^2\right)^{\frac{3}{2}}} \frac{\partial \phi}{\partial r} \tag{3.4.14}$$

Equation (3.4.14) is now substituted into equation (3.4.12) to give

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\sqrt{\vec{x}^2 + t^2}} \frac{\partial \phi}{\partial r} - \frac{x^2}{\left(\vec{x}^2 + t^2\right)^{\frac{3}{2}}} \frac{\partial \phi}{\partial r} + \frac{x^2}{\left(\vec{x}^2 + t^2\right)} \frac{\partial^2 \phi}{\partial r^2} \tag{3.4.15}$$

By repeating the procedure for $\frac{\partial^2 \phi}{\partial y^2}$, $\frac{\partial^2 \phi}{\partial z^2}$ and $\frac{\partial^2 \phi}{\partial t^2}$, it is found that

$$\frac{\partial^2 \phi}{\partial y^2} = \frac{1}{\sqrt{\vec{x}^2 + t^2}} \frac{\partial \phi}{\partial r} - \frac{y^2}{\left(\vec{x}^2 + t^2\right)^{\frac{3}{2}}} \frac{\partial \phi}{\partial r} + \frac{y^2}{\left(\vec{x}^2 + t^2\right)} \frac{\partial^2 \phi}{\partial r^2} \tag{3.4.16}$$

$$\frac{\partial^2\phi}{\partial z^2} = \frac{1}{\sqrt{\vec{x}^2 + t^2}}\frac{\partial\phi}{\partial r} - \frac{z^2}{\left(\vec{x}^2 + t^2\right)^{\frac{3}{2}}}\frac{\partial\phi}{\partial r} + \frac{z^2}{\left(\vec{x}^2 + t^2\right)}\frac{\partial^2\phi}{\partial r^2} \tag{3.4.17}$$

and

$$\frac{\partial^2\phi}{\partial t^2} = \frac{1}{\sqrt{\vec{x}^2 + t^2}}\frac{\partial\phi}{\partial r} - \frac{t^2}{\left(\vec{x}^2 + t^2\right)^{\frac{3}{2}}}\frac{\partial\phi}{\partial r} + \frac{t^2}{\left(\vec{x}^2 + t^2\right)}\frac{\partial^2\phi}{\partial r^2}, \tag{3.4.18}$$

respectively. The combination of equations (3.4.15) through (3.4.18) yields

$$\frac{d^2\phi}{dr^2} + 3r^{-1}\frac{d\phi}{dr} - V^{'}(\phi) = 0 \tag{3.4.19}$$

Equation (3.4.19) has no general solution but can be solved approximately. The approximation is made that if the difference in energy between the metastable and true vacua is small compared to the height of the barrier, then the damping term can be neglected

$$\frac{d\phi}{dr} \to 0 \tag{3.4.20}$$

and equation (3.4.19) reduces to

$$\frac{d^2\phi}{dr^2} - V'(\phi) = 0 \tag{3.4.21}$$

Since

$$\begin{aligned} V'(\phi) &= \frac{dV(\phi)}{d\phi} \\ &= \frac{dV}{dr}\frac{dr}{d\phi} \end{aligned} \tag{3.4.22}$$

then

$$\begin{aligned} \frac{d}{dr}\left(\frac{d\phi}{dr}\right) &= \frac{d^2\phi}{dr^2} \\ &= \frac{dV}{dr}\frac{dr}{d\phi} \end{aligned} \tag{3.4.23}$$

When equation (3.4.23) is cross-multiplied by $\frac{d\phi}{dr}dr$, it yields

$$\frac{d\phi}{dr}d\left(\frac{d\phi}{dr}\right) = dV \tag{3.4.24}$$

Using the substitution that

$$\frac{d\phi}{dr} = \mathrm{y} \tag{3.4.25}$$

makes equation (3.4.24) to take the form

$$ydy = dV \tag{3.4.26}$$

which integrates to give

$$\frac{y^2}{2} = V(\phi) + V_0 \tag{3.4.27}$$

Since the potential energy level is always arbitrary, the integration constant V_0 can be set to zero to yield

$$\frac{y^2}{2} = V(\phi) \tag{3.4.28}$$

Hence, substituting equation (3.4.25) into (3.4.28) leads to

$$\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 = V(\phi) \tag{3.4.29}$$

Since, from equations (3.4.20) and (3.4.29) the action is

$$A_E(\phi) = \int \left\{ \frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi) \right\} dr\,, \tag{3.4.30}$$

then

$$A_E(\phi) = \int \left\{ \left(\frac{d\phi}{dr}\right)^2 \right\} dr \tag{3.4.31}$$

But, in terms of V, equation (3.4.29) can also be expressed as

$$dr = \frac{d\phi}{\sqrt{2V}} \tag{3.4.32}$$

From equations (3.4.31) and (3.4.27), the action can be obtained as

$$A_E = \int_0^{\sigma} d\phi \sqrt{2V(\phi)} \tag{3.4.33}$$

Equation (3.4.33) has a familiar form from the theory of instantons and to agree with the physical interpretation of the instanton solution (as probability of barrier penetration), the equation (3.4.33) is solved by making a coordinate transformation. It is suggested that

$$\begin{aligned} x - x_o &= \Delta x \\ &= k\cos\theta \qquad (3.4.34) \\ t - t_o &= \Delta t \end{aligned}$$

$$= k\sin\theta \tag{3.4.35}$$

and

$$k = \frac{m_o}{qE} \tag{3.4.36}$$

Then equation (2.4.2.2) becomes

$$\int_0^{\pi}\left[\frac{m_o}{\sqrt{1+\left(\Delta\dot{x}\right)^2}} - q\vec{E}.\Delta\vec{x}\right] = \int_0^{\pi}\left(\frac{m_o k\sin\theta}{\sqrt{k^2(\cos^2\theta+\sin^2\theta)}} - qEk\cos\theta\right)k\cos\theta\, d\theta$$

$$= \int_o^{\pi} m_o k\sin\theta\cos\theta\, d\theta - qEk^2\int_o^{\pi}\cos^2\theta\, d\theta$$

$$= \frac{m_o k}{2}\int_0^{\pi}\sin 2\theta\, d\theta - qEk^2\int_0^{\pi}\cos^2\theta\, d\theta$$

$$= -\frac{m_o k}{4}\cos 2\theta\,\Big|_0^{\pi} - qEk^2\theta\,\Big|_0^{\pi}$$

$$= -\frac{m_o^2}{qE}\pi \tag{3.4.37}$$

Thus, combination of equations (2.4.2.1) and (3.4.37) yields

$$A_E = \frac{\pi\, m_o{}^2}{\left|q\vec{E}\right|} \tag{3.4.38}$$

From equations (3.4.4) and (3.4.38), a result for barrier penetration probability is obtained as

$$P_b \propto \exp\left(-\frac{\pi\, m_o{}^2}{\left|q\vec{E}\right|}\right) \tag{3.4.39}$$

3.5. NUMERICAL NEUTRINO MASS VALUE

To generate a mass term for the neutrino, existence of the right-handed neutrino ν_R is suggested which leads to a total of 16 helicity states. With this additional neutrino helicity, it is possible to generate the following flavor results from figure 2.2.3.1:

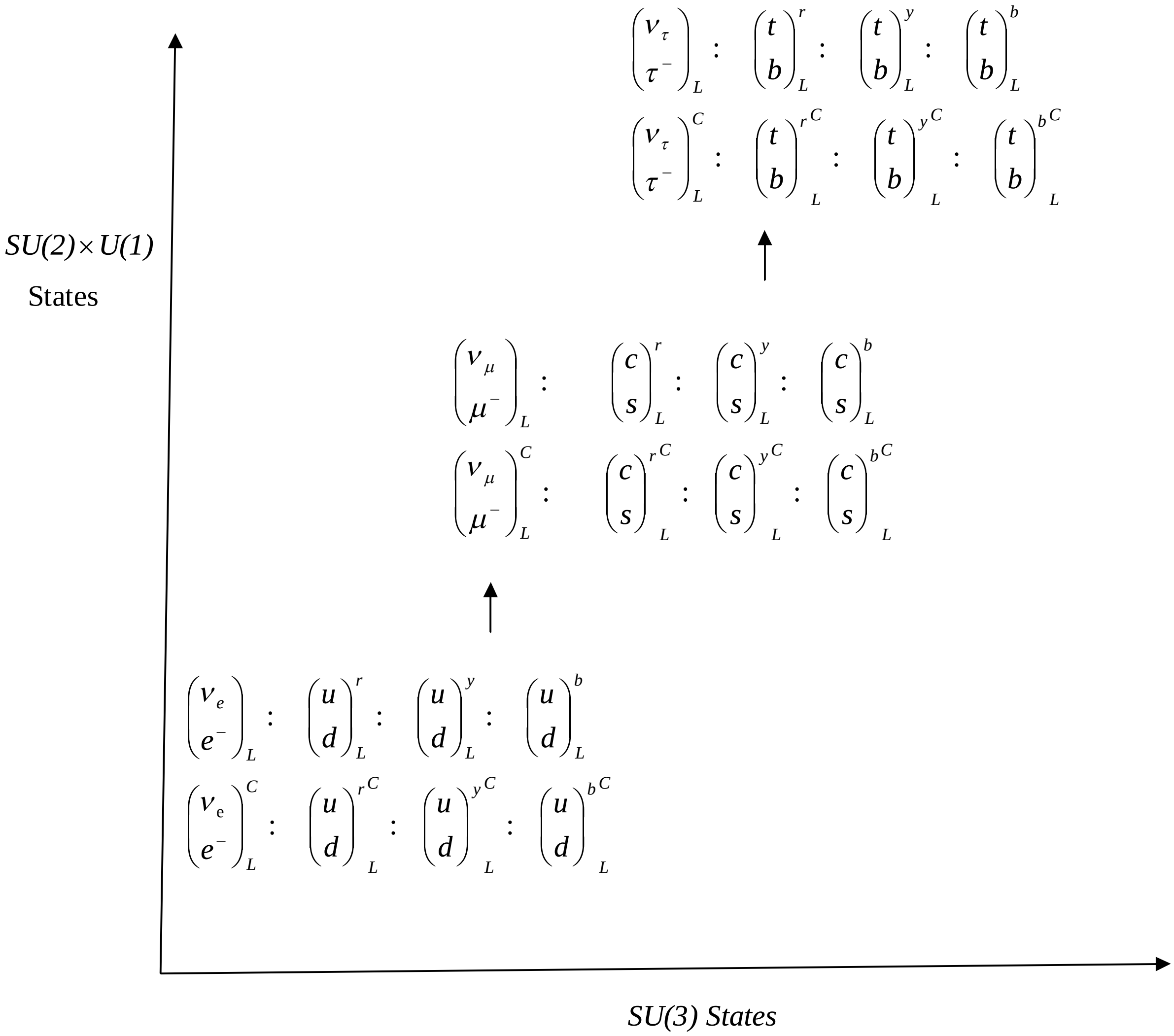


Figure 3.5.1: Sixteen helicity states

In particular, to generate a mass term for the neutrino, the standard electroweak Lagrangian without the mass term is first written as

$$L = \overline{\psi_L}\gamma^\mu\left[i\partial_\mu - g\vec{\tau}.\vec{W}_\mu - \frac{g^{'}}{2}YB_\mu\right]\psi_L + \overline{\psi}_R\gamma^\mu\left[i\partial_\mu - \frac{g^{'}}{2}YB_\mu\right]\psi_R - \frac{1}{4}\vec{W}_{\mu\nu}.\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{3.5.1}$$

Then, the following gauge invariant term is added to the Lagrangian

$$L = -g_\nu\left[\bar{\ell}\Phi R + \bar{R}\Phi\ell\right] \tag{3.5.2}$$

where

$$\ell = \begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, \quad \Phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} \text{ and } R = \begin{pmatrix}\nu_e \\ e\end{pmatrix}_R \tag{3.5.3}$$

The first term in brackets on the right hand side of (3.5.2) is expanded out to the form

$$\left(\bar{\nu}_e \quad \bar{e}_L\right)\begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} = \bar{\nu}_e\,\phi^+ + \bar{e}_L\,\phi^0 \tag{3.5.4}$$

Then the symmetry is then spontaneously broken by substituting

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ \sigma + H_p\end{pmatrix} \tag{3.5.5}$$

into the Lagrangian (3.5.2) to give

$$L = -\frac{g_\nu\,\sigma}{\sqrt{2}}\left(\bar{\nu}_R\,\nu_L + \bar{\nu}_L\,\nu_R\right) - \frac{g_\nu}{\sqrt{2}}\left(\bar{\nu}_R\,\nu_L + \bar{\nu}_L\,\nu_R\right)H_p \tag{3.5.6}$$

where H_p represents the Higgs fluctuations around the ground state $\phi^0 = \sigma$. Comparison of the result (3.5.6) with the standard mass term of the electroweak model

$$-m\bar{f}\;f = -m\bar{f}\left\{\frac{1}{2}\left(1-\gamma^5\right)+\frac{1}{2}\left(1+\gamma^5\right)\right\}f = -m\left(\bar{f}_R\,f_L + \bar{f}_L\,f_R\right) \tag{3.5.7}$$

shows that the first term in equation (3.5.6) has precisely the form of a fermion mass term with

$$m_{\nu_e} = \frac{g_\nu \sigma}{\sqrt{2}} \tag{3.5.8}$$

In terms of m_{ν_e}, the Lagrangian (3.5.6) can then be written as

$$L = -m_{\nu_e}\bar{\nu}\,\nu - \frac{m_{\nu_e}}{\sigma}\bar{\nu}\,\nu\,H_p \tag{3.5.9}$$

instead of

$$L = -g_e\left[\bar{\ell}\Phi R + \bar{R}\bar{\Phi}\ell\right] \tag{3.5.10},$$

where $R = (e)_R$. This leads to both a mass term and an interaction term for the neutrino. The results (3.5.9) above shows that the neutrino can acquire mass through the same mechanism as the other leptons, only if the standard electroweak Lagrangian is modified. The modification generates a mass term whose actual numerical value is yet to be determined once the mass of the Higgs boson is observed experimentally. But before the issue of the Higgs boson is resolved by the yet-to-be-built powerful particle accelerators (such as the super LHC), it is important that the very high temperatures that existed in the very early universe be utilized. In particular, it is suggested that the energy that must be contributed by each of the stable relativistic neutrino in the present universe should follow the simple Einsteinium law

$$E \approx m \tag{3.5.11}$$

for a relativistic particle. But for non-relativistic and massive neutrino, the equation (3.5.11) should be modified so as to yield the total energy density of massive neutrinos as

$$\rho = n_o m$$

$$= 6.19 \times 10^3 \frac{g}{q(T_{dec})}\left(\frac{m}{10eV}\right) eVcm^{-3} \tag{3.5.12}$$

On substituting

$$\rho_c = \frac{3H^2}{8\pi G}$$

$$= 10^4 h^2 eVcm^{-3} \tag{3.5.13}$$

and

$$\Omega = \frac{\rho}{\rho_c} \tag{3.5.14}$$

into equation (3.5.12), the following relation is found

$$(\Omega h^2) = 0.619\left(\frac{m}{10eV}\right)\frac{g}{q(T_{dec})} \tag{3.5.15}$$

For a closed universe, the constraint condition that $\Omega h^2 \leq 0.1296$ in equation (3.5.15) gives abound on m as

$$m \leq 2.26 \frac{q(T_{dec})}{g} eV \tag{3.5.16}$$

The relation (3.5.16) shows that the value of m depends on the value of q at the time of decoupling; neutrinos with mass less than about $1\,MeV$ decouple at $T_D \approx (1-3)\,MeV$ when the total number of degrees of freedom is

$$q = 10.75 \tag{3.5.17}$$

This implies that for a massive Majorana neutrino ($g = 2$), equation (3.5.15) yields

$$\Omega_\nu h^2 \geq \frac{m_\nu}{91.10\,eV} \tag{3.5.18}$$

from which it is clear that

$$m_\nu \leq 11.84\,eV\,, \tag{3.5.19}$$

while for a Dirac massive neutrino ($g = 4$), equation (3.5.15) leads to

$$\Omega_\nu h^2 \geq \frac{m_\nu}{45.5eV} \tag{3.5.20}$$

from which it is found that

$$m_\nu \leq 5.92\, eV \tag{3.5.21}$$

Actually, the limit (3.5.18) is additive in the sense that if more than one flavor of light neutrinos exist, then the limit becomes

$$\sum_\nu m_\nu \leq 91.10 \Omega_\nu h^2 eV \tag{3.5.22}$$

Thus, for Majorana-type neutrinos, the limit (3.5.19) reduces to

$$m_\nu \leq 3.94\, eV\,, \tag{3.5.23}$$

whereas for Dirac-type, it becomes

$$m_\nu \leq 1.97\, eV \tag{3.5.24}$$

The value of q will be higher at higher decoupling temperature T_D; for example, at temperatures above 300GeV, the $SU(5)$ grand unified model of particle interactions predicts that 8 gluons, one photon, three weak gauge bosons (W^+, W^- and Z), one Higgs doublet and three generations of quarks and leptons will all be relativistic so that

$$q \approx 106.75 \tag{3.5.25}$$

For a neutrino decoupling at $T_D \geq 300\, GeV$, the corresponding mass bound is

$$(\Omega h^2)_{wimp} \geq \left(\frac{m_\nu}{904.7 eV} \right) \tag{3.5.26}$$

Then, equation (3.5.26) leads to

$$m_\nu \leq 112.09\, eV \tag{3.5.27}$$

for a massive Majorana neutrino or

$$m_\nu \leq 56.05\, eV \tag{3.5.28}$$

for a massive Dirac neutrino. However, for three species of neutrinos, the value of mass m_ν is

$$m_\nu \leq 37.36\, eV \tag{3.5.29}$$

$$m_\nu \leq 18.68\, eV \tag{3.5.30}$$

for Majorana and Dirac neutrinos, respectively; but, decoupling at such high energies is possible only if the neutrinos have non-standard interactions.

For very massive and non-relativistic neutrinos, a numerical estimate is made i.e., the Boltzmann equation (2.7.3.16) is solved for the decoupling temperature T_D by taking the natural logarithms of the equation (2.7.3.16) to give

$$\frac{m}{T_D} = 17.966 + \ln\left(\frac{g_A}{\sqrt{q}}\right) + \left(\frac{1}{2} - k\right)\ln\left(\frac{m}{T_D}\right) + 3\ln\left(\frac{m}{1GeV}\right) \tag{3.5.31}$$

Since g is a slowly varying function of temperature T, then equation (3.5.31) is to be solved approximately. By successive approximation technique this gives the result

$$\frac{m}{T_D} \approx 17.966 \tag{3.5.32}$$

Starting with $k = 0$ in equation (3.5.32), the $\ln\left(\frac{m}{T_D}\right)$ term corrects the result to

$$\frac{m}{T_D} \approx 19.41 \tag{3.5.33}$$

from which the decoupling temperature becomes

$$T_D \approx 52MeV \tag{3.5.34}$$

At this temperature (3.5.34), the total number of degrees of freedom $q = 106.75$; hence, equation (3.5.31) yields

$$\ln\left(\frac{g}{\sqrt{q}}\right) \approx \ln 5$$

$$= 1.61 \tag{3.5.35}$$

which corrects $\frac{m}{T_D}$ in (3.5.31) to

$$\frac{m}{T_D} \approx 17.8 \tag{3.5.36}$$

Thus, to this order, it is found that

$$\frac{m}{T_D} = 17.8 \tag{3.5.37}$$

On substituting equation (3.5.37) into equation (2.7.1.39), then

$$N \approx 5.11 \times 10^{-9}\left(\frac{m}{1GeV}\right)^{-3} \tag{3.5.38}$$

and

$$\Omega_A h^2 \approx 1.44\left(\frac{m}{1GeV}\right)^{-2} \tag{3.5.39}$$

The neutrino A and its antineutrino $\overline{A}$ will contribute twice this value to Ω, so that

$$\Omega_{A\overline{A}} h^2 = 2.88\left(\frac{m}{1GeV}\right)^{-2} \tag{3.5.40}$$

From equation (3.5.40), the constraint condition that $\Omega h^2 \leq 0.1296$ gives the mass bound as

$$m_\nu \geq 4.71\, GeV \tag{3.5.41}$$

and for three neutrino flavors, the result reduces to

$$m_\nu \geq 1.57\, GeV \tag{3.5.42}$$

3.6. NEUTRINO MASS AND GRAND UNIFICATION

The neutrino mass can aid in the unification of the fundamental interactions if the corresponding Seesaw relation can be found. Most of the standard models use or follow complicated methods, especially the renormalization method, to arrive at the seesaw relation. A simple approach that is formulated in this study, involves the mathematical theory of matrices. In particular, the eigenvalue equation (2.2.2.18) is solved by simply putting it into the form

$$|K - \lambda I| = 0 \tag{3.6.1}$$

where ***I*** is 2×2 unitary matrix and $\lambda's$ are the distinct eigenvalues of the matrix ***K***. The corresponding values of the matrices ***K*** and ***I*** are substituted in (3.6.1) to give

$$\begin{aligned} |K - \lambda I| &= \begin{vmatrix} -\lambda & m_D \\ m_D & M - \lambda \end{vmatrix} \\ &= \lambda^2 - \lambda M - {m_D}^2 \end{aligned} \tag{3.6.2}$$

From equation (3.6.1), equation (3.6.2) takes the form

$$\lambda^2 - \lambda M - {m_D}^2 = 0 \tag{3.6.3}$$

To first order, application of the quadratic formula on equation (3.6.3) gives

$$\begin{aligned} \lambda_1 &= m_1 \approx -\frac{m_D^2}{M_M} \\ \lambda_2 &= m_2 \approx M_M \end{aligned} \tag{3.6.4}$$

In this case, one neutrino is superheavy

$$m_2 \approx M >> m_D \tag{3.6.5}$$

while the other is superlight

$$m_1 \approx \frac{m_D^2}{M} << m_D \tag{3.6.6}$$

Having found that

$$\begin{aligned} m_1 &\equiv m_\nu \\ &= 1.97 eV \end{aligned} \tag{3.6.7}$$

and

$$\begin{aligned} m_D &\equiv m_e \\ &= 0.511 MeV \end{aligned} \tag{3.6.8}$$

then a unification scale Λ_{unif} can be found as follows: taking

$$M \equiv \Lambda_{unif} \tag{3.6.9}$$

and if

$$\begin{aligned} m_D &\equiv m_t \\ &= 174.3\,\mathrm{G}eV \end{aligned} \tag{3.6.10}$$

then

$$\begin{aligned} M &\equiv \Lambda_{unif} \\ &= 1.542 \times 10^{13} GeV \end{aligned} \tag{3.6.11}$$

3.7. A MASSIVE NEUTRINO OSCILLATION

It is suggested that a neutrino can change flavor if it has mass. In particular, results for this dynamical phenomenon can be found by considering two neutrino flavor states ν_e and ν_μ, whose lepton mixing matrix U is considered to be

$$U = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \tag{3.7.1}$$

The matrix (3.7.1) is applied to equation (2.8.1.8) to give

$$\begin{aligned} |\nu_e\rangle &= c|\nu_1\rangle + s|\nu_2\rangle \\ |\nu_\mu\rangle &= -s|\nu_1\rangle + c|\nu_2\rangle \end{aligned} \tag{3.7.2}$$

To determine the time evolution of the neutrino state, the evolving state is written as

$$|\nu_{\mu}(t=0)>=|\nu_{\mu}>$$

$$=-s|\nu_1>+c|\nu_2> \tag{3.7.3}$$

so that

$$|\nu_{\mu}(t)\rangle=-s|\nu_1\rangle e^{-i\frac{E_1}{\hbar}t}+c|\nu_2\rangle e^{-i\frac{E_2}{\hbar}t} \tag{3.7.4}$$

where E_1 and E_2 are the energy of the two neutrino mass eigenstates. They are given by

$$E_1=\sqrt{(pc)^2+(m_1c^2)^2}$$

$$E_2=\sqrt{(pc)^2+(m_2c^2)^2} \tag{3.7.5}$$

with

$$p_1=p_2=p$$

The approximate equations

$$E_1=\sqrt{p^2+m_1{}^2} \tag{3.7.6}$$

$$E_2=\sqrt{p^2+m_2{}^2} \tag{3.7.7}$$

together with the assumption that the neutrino is relativistic i.e.,

$$\gamma=\frac{E_{\nu}}{m_o}$$

$$=\frac{\sqrt{p^2+m_o{}^2}}{m_o}>>1;\ p>>m_o \tag{3.7.8}$$

makes equations (3.7.6) and (3.7.7) to become

$$E_1\approx p+\frac{m_1{}^2}{2p} \tag{3.7.9}$$

$$E_2\approx p+\frac{m_2{}^2}{2p} \tag{3.7.10}$$

The results (3.7.9) and (3.7.10) are arrived at, by applying the binomial expansion theorem that

$$(1+\vartheta)^n=1+n\vartheta+\frac{n(n-1)}{2!}\vartheta^2+\text{higher order terms} \tag{3.7.11}$$

to equation (3.7.6) and keeping only the first two terms. The expressions (3.7.9) are then substituted into equation (3.7.4) to get

$$\begin{aligned}|\nu_\mu(t)\rangle &= -s|\nu_1\rangle e^{-i\left(p+\frac{m_1^2}{2p}\right)t} + c|\nu_2\rangle e^{-i\left(p+\frac{m_2^2}{2p}\right)t} \\ &= e^{-i\left(p+\frac{m_1^2}{2p}\right)t}\left(-s|\nu_1\rangle + c|\nu_2\rangle e^{i\left(\frac{m_1^2-m_2^2}{2p}\right)t}\right)\end{aligned} \tag{3.7.12}$$

By defining

$$\Delta m^2 = m_1^{\ 2} - m_2^{\ 2} \tag{3.7.13}$$

$$t = \frac{L}{c} \tag{3.7.14}$$

$$e^{-i\chi} = e^{-i\left(p+\frac{m_1^{\ 2}}{2p}\right)t} \tag{3.7.15}$$

then, equation (3.7.12) becomes

$$|\nu_\mu(t)\rangle = e^{-i\chi}\left(-s|\nu_1\rangle + c|\nu_2\rangle e^{i\left(\frac{\Delta m^2}{2p}\right)L}\right) \tag{3.7.16}$$

To calculate the probability for a pure ν_μ state oscillating into ν_e state, the quantum mechanical amplitude describing neutrino transition is squared to give

$$P(\nu_\mu \to \nu_e) = \left|\langle \nu_e|\nu_\mu(t)\rangle\right|^2 \tag{3.7.17}$$

Using the definitions that

$$\langle \nu_e| = c\langle \nu_1| + s\langle \nu_2| \tag{3.7.18}$$

$$\langle \nu_i|\nu_j\rangle = \delta_{ij} \tag{3.7.19}$$

then the amplitude becomes

$$\langle \nu_e|\nu_\mu(t)\rangle = e^{-i\chi}\left(-sc + sce^{i\frac{\Delta m^2}{2p}L}\right) \tag{3.7.20}$$

Substituting equation (3.7.20) into equation (3.7.17), the transition probability is obtained as

$$\begin{aligned}P(\nu_\mu \to \nu_e) &= \left|\langle \nu_e|\nu_\mu(t)\rangle\right|^2 \\ &= e^{i\chi}e^{-i\chi}(sc)^2\left(-1+e^{i\frac{\Delta m^2}{2p}L}\right)\left(-1+e^{-i\frac{\Delta m^2}{2p}L}\right)\end{aligned} \tag{3.7.21}$$

The approximation that the neutrino is relativistic implies that

$$p_\nu = E_\nu , \tag{3.7.22}$$

Hence, equation (3.7.21) reduces to

$$P(\nu_\mu \to \nu_e) = \frac{1}{2}\sin^2 2\theta\left(1-\cos\frac{\Delta m^2}{2E_\nu}L\right) \tag{3.7.23}$$

The following trigonometric relation

$$\frac{1}{2}(1-\cos 2\theta) = \sin^2\theta \tag{3.7.24}$$

is applied in equation (3.7.23) to give

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E_\nu}L\right) \tag{3.7.25}$$

To make the argument of the second sin^2 term dimensionless, appropriate numbers of $\hbar's$ and $c's$ are introduced so that

$$\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right) \to \left(\frac{\Delta m^2 c^4}{4\hbar c}\frac{L}{E_\nu}\right) \tag{3.7.26}$$

and the variables assigned appropriate units of $\Delta m^2 c^4 (eV^2)$, L (meters) and $E_\nu (MeV)$.

Since $\hbar c = 197 eV\ nm$, the quantity in parenthesis reduces to

$$\begin{aligned}\left(\frac{\Delta m^2 c^4}{4\hbar c}\frac{L}{E_\nu}\right) &\to \left(\frac{\Delta m^2 c^4}{4\times 197 eV.nm}\frac{L}{E_\nu}\right)\left(\frac{10^{-6} MeV/eV}{10^{-9} m/nm}\right) \\ &= 1.27\Delta m^2 \frac{L}{E_\nu}\end{aligned} \tag{3.7.27}$$

Thus, from equations (3.7.25) up to (3.7.27), the transition probability becomes

$$P_{\nu_\mu\to\nu_e}(L,E) = \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2}{E_\nu}L\right) \tag{3.7.28}$$

where the term $\sin^2 2\theta$ which does not depend on distance describes the amplitude of the neutrino oscillations.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1. INTRODUCTION

In this chapter, the result of absolute neutrino mass investigations is discussed. In particular, particle mass generation result through the Higgs mechanism is discussed. Because of the chiral nature of neutrinos, it is seen that the standard mass generation mechanism does not immediately apply to the neutrino case in the framework of the standard electroweak model. This problem is resolved by suggesting that the absence of right-handed neutrinos is an experimental limitation of the standard electroweak model and, hence, neutrinos should also acquire mass through the Higgs mechanism. However, since the theory does not predict a the required numerical mass value for the neutrino, the equilibrium condition between neutrino weak interaction and the expansion rates in the early phase of the universe is used to calculate this value. When the value is used in the seesaw relation, an interesting grand unified energy scale is obtained. A result on the phenomenon of neutrino oscillation is also discussed. It is explicitly established that neutrino oscillation probability is proportional to mass squared difference, i.e., neutrino oscillations can only occur if neutrinos have hierarchical masses. Towards the end of the chapter, a result on the possibility of neutrino decays is also presented.

4.2. FLATNESS PUZZLE

On the basis of the dynamical Friedman equation (3.3.1), a free neutrino gas equation (3.3.2) was used in the energy conservation law (3.3.3) to yield the resulting equation (3.3.8). For the curvature parameter $k = \pm 1$, equation (3.3.10) was obtained. This result was then used to generate the data for the various cosmological phases in the early universe as in table 3.3.1. From the data, it was observed that the cosmological energy density parameter Ω at any cosmological phase always tends to unity. This translates to the fact that the flatness puzzle is a natural phenomenon on the basis of inflation (3.3.31).

4.3. PARTICULATE NATURE OF THE UNIVERSE

To model the universe as an unstable particle, its creation as a spontaneous tunneling process that occurs by bubble nucleation was analyzed. This was done by considering the Euclidean equation (3.4.1) subject to the conditions (3.4.2) and (3.4.3). This yielded the action (3.4.33) that agreed with (2.4.19) which was obtained from the Bianchi identity (2.4.18) after applying Gauss' theorem. The calculations resulted to equation (3.4.39) which confirmed that the universe approaches the radiation-dominated phase in a neutrino-dominated and evolving model.

4.4. NEUTRINO MASS TERM

Neutrino mass generation was explicitly investigated using the standard Higgs mechanism. This was done by considering the free Dirac Lagrangian (2.2.3.1) which reduced to (2.2.3.2) in the massless neutrino case. When the projection operators (2.2.3.3) were applied on the massless Lagrangian (2.2.3.1), a gauge invariant result (2.2.3.5) was obtained. The established gauge invariance of (2.2.3.5) was interpreted to imply the isospinor doublet (2.2.3.6) be assigned a weak isospin charge (2.2.3.7), from which the neutrino had one-half as its third component; the electron and the other singlet particle were found to have one-half and zero components, respectively. These quantum numbers generated the transformation matrices of (2.4.3.18) and (2.2.3.19) for the electromagnetic and electroweak interactions, respectively. This led to the lepton Lagrangian (2.2.3.25), which, was found to be gauge invariant also. To determine the strengths of neutrino chiralities, the transformation matrix $SU(2)$ and the relation between the electric charge Q and the third component I given by the Gell-Mann-Nishijima relation (2.2.3.53) were applied. This helped to calculate the weak hypercharge, from which it was found that left-handed chiralities couple with half the strength of right-handed chiralities; i.e., the left-handed neutrino fields couple with half the strength of the right-handed fields to the hypercharge gauge field. To gauge $SU(2)$ interactions, three gauge potentials W_{μ}^{i} were introduced so that, by acting on the isospinor L, the ordinary derivative was replaced by the covariant derivative (2.2.3.27). Gauging $U(1)$ introduced a potential X_{μ} and a coupling constant $g^{'}$ so that, for L coupling with half the hypercharge strength of R, the corresponding covariant derivatives were (2.2.3.28). These were then put into the Lagrangian (2.2.3.11) and, when the gauge field kinetic terms were included, they generated a Lagrangian (2.2.3.29). To generate a mass term, an isospinor scalar field ϕ with quantum numbers (2.2.3.31) was introduced into the Lagrangian (2.2.3.29) and a perturbation introduced. This led to the result (2.2.3.46). The result of (2.2.3.46) was found to be important only in the massless neutrino case. To generate a mass term for the neutrino, the standard electroweak Lagrangian was written as in (3.5.1) and the gauge invariant mass term (3.5.2) added. This then generated the result (2.2.3.60) which was found to be $SU(2)$ invariant. The symmetry was then spontaneously broken by substituting the Higgs field (3.5.5) into the Lagrangian (3.5.2) to yield (3.5.6). This helped to generate a neutrino mass term (3.5.8). When the result (3.5.6) was compared with the standard mass term (3.4.7), it was recognized that the first term had precisely the form of a mass term. In terms of m_{ν_e} the Lagrangian (3.5.6) then reduced to (3.5.9). The result showed that, in addition to the mass term, the Lagrangian contained a term that described the coupling of the Higgs field to the neutrino with the coupling strength of $\frac{m_\nu}{\sigma}$. However, in the $SU(2)\times U(1)$ model, there is no coupling of the Higgs field to the neutrino since the theory is normally constructed with a left-handed neutrino only. The absence of the right-handed neutrino prevents one from adding the term (3.5.2) which will lead to both a mass term and an interaction term for the neutrino. However, since the coupling constant g_ν was arbitrary, the theory did not give the numerical mass value of the neutrino but it gave an attractive feature that through the standard Higgs mechanism, the model can also be modified to accommodate massive neutrinos.

4.5. NEUTRINO MASS BOUNDS

Results of section 4.4 on neutrino-mass-generation mechanism are found to give a mass term to the neutrino but not the-much-sought-after numerical value. To explicitly determine this value, neutrino interactions at very high energies were considered. However, at the moment these high energies are not available within the realm of particle accelerators. They are suggested to have existed at the very early universe where neutrinos and photons were very dominant. Nevertheless, extrapolation into the very early universe was found to generate a big bang singularity which was regarded as one of the cosmological puzzles. The ensuing calculations performed managed to show that, indeed, the universe did not begin from zero volume at zero time; instead, it emerged quantum mechanically with a finite size from a non-classical space-time state (3.6.2.16) and began to evolve exponentially. This exponential behavior was established by rewriting the Friedman equation (3.4.1) in the form (3.4.15) and then solved for the three cases of the curvature parameter *k*. This gave rise to the result (3.4.18). To find out why the initial non-classical space-time state began to expand, it was suggested that the inflaton field was initially kept in equilibrium by massive neutrino interactions. As neutrinos begun to decay and became less massive, the transition amplitude/tunneling probability also grew as calculated in equation (3.3.33). This result established, firmly, that the entropy in a neutrino-dominated and expanding universe always increases. To establish the exact numerical mass value required for equilibrium to be maintained, neutrino distribution functions in the early universe were considered. In particular, three cases were established; in the first case, the temperature of the present neutrino background was found to correspond to the result (2.7.1.69). This value was applied in equation (2.7.1.11) to yield equation (2.7.1.72). This was found to be far much lower than the energy density required for a critically expanding universe. The second case was for massive neutrinos that decouple when still relativistic. In this case, the number density of neutrinos was found to correspond to (2.7.3.3). This was then used in the energy density relation of massive neutrinos (3.5.12) that combined with (3.5.15) to yield (3.5.16). This bound was found to be valid for a weakly interacting massive neutrino that decouples while still relativistic and the value of m depended on the value of the total number of degrees q at the time of decoupling. However, neutrinos with masses less than $1\,MeV$ were found to decouple at temperature $T_D \approx 1\,MeV$, when total number of degrees of freedom is 10.75. Hence, for a massive Majorana neutrino, the constraint mass equation (3.5.16) was found to correspond to (3.5.18) which yielded the result (3.5.23), whereas, for a Dirac massive neutrino, the constraint equation corresponded to (3.5.20) which yielded (3.5.24).

For weakly interacting massive neutrinos that decouple when they are non-relativistic, the value of N was modified to the form (2.7.3.6) so that the number density depended strongly on m. In this case, the decoupling temperature T_D was calculated by invoking the equilibrium condition between the weak interaction rates and the expansion rate of the universe. The average value of neutrino reaction rates was found to take the form (2.7.3.9) and, when the cross section σ_o was taken in the form (2.7.3.10), result (2.7.3.13) was obtained. Combining the expansion rate result (2.7.2.14) with the reaction rates (2.7.3.13) yielded the neutrino-Boltzmann transport result (2.7.3.16). To determine the actual mass, the exponential part of equation (2.7.3.16) was solved by the numerical methods of successive approximation technique to yield (3.5.37). This was then substituted into equation (2.7.3.17) and (2.7.3.18) to give (3.5.39). For a neutrino and its antineutrino, this corresponded to (3.5.40). The constraint equation that $\Omega h^2 \leq 0.1296$ then gave the mass bound (3.5.41) and, for three flavors of neutrinos, the result reduced to (3.5.42). When the masses $1.97eV$ and $174.3GeV$ -for the electron neutrino and the top quark leptons-, respectively, were inserted into the calculated seesaw relation (3.6.6), a unification energy-scale result (3.6.11) of $\sim 1.542 \times 10^{13}\,GeV$ was obtained.

4.6. NEUTRINO OSCILLATION PROBABILITY AND DECAY

Here, the result on the effect of the neutrino mass on the phenomenon of flavor oscillation is discussed. In particular the result on the probability of neutrino oscillations is discussed. The effect was established by considering a case of two neutrino flavours ν_e and ν_μ with the lepton mixing matrix (3.7.1). The matrix then generated the neutrino eigenstate equation (2.8.1.8) whose evolution equation took the form of equation (3.7.4). The approximate equations (3.7.6) were then applied to equation (2.8.1.10) together with the binomial expansion theorem to yield (3.7.12). Definitions (3.7.13) were used in (3.7.2) to yield equation (3.7.16). To calculate the probability for one neutrino state, say ν_μ, oscillating into another neutrino state, say ν_e, the quantum mechanical amplitude describing the transition was squared to obtain (3.7.17). Using the definitions (3.7.18), the amplitude was then calculated as in equation (3.7.20). This was then squared to get the transition probability as (3.7.23). Using the trigonometric relation (2.8.2.17), equation (3.7.23) reduced to (3.7.25). In the more experimentally acceptable units, this yielded (3.7.28). In equation (3.7.28), the second sine factor was found to oscillate with distance L (or time) traveled by neutrinos; this showed that the oscillation phase is proportional to the energy difference of the mass eigenstates and to the distance L. To model neutrino decay, an assembly of massive neutrinos each having a probability λ of decaying per unit time was considered. The number decaying per unit time was modeled into the form of equation (2.8.2.2.29) which was integrated to yield (2.8.2.2.30). More appropriately, the time dependence of the neutrino wave function was expressed as (2.8.2.2.32) and a small imaginary energy part was added to the energy E. This generated the result (2.8.2.2.35) that agreed with the decay law (2.8.2.2.30) on taking Γ as $\lambda\hbar$. It was then considered that the wave function of a decaying neutrino state should take the form (2.8.2.2.37). To express this as a function of energy, a Fourier transformation was made, i.e., a neutrino wave function $\psi(t)$ was expressed as an integral (2.8.2.2.38), which was inverted to yield (2.8.2.2.39). On substituting $\hbar\omega$ for E, the probability density $P(E)$ of finding a neutrino with energy E was found to be proportional to the square of $g(\omega)$ which yielded (2.8.2.2.42). The condition (2.8.2.2.43) gave the constant of integration in equation (2.8.2.2.44). This was substituted into (2.8.2.2.42) to yield the uncertainty relation (2.8.2.2.45).

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1. CONCLUSIONS

A related beta decay process was studied in the context of a neutrino-dominated and evolving universe. At first, the decay process was found to be inadequate in as much as the physical laws are concerned. In particular, the two-body decay process failed to account for the experimentally observed facts on the basis of black-body radiation. This necessitated the introduction of a hypothetical particle X, so that the process became a three-body decay process. To describe physical reality, the appropriate quantum numbers were calculated for this difficult-to-detect and controversial particle from the viewpoint of conservation laws. It was firmly established that the neutrino is a lepton that participates in various weak interaction reactions, including those taking place in the sun.

Having modeled the evolving universe as a decay process, the neutrino-dominated and evolving universe was found to be described by the dynamical Friedman equation. The equation was used to generate data for the important phases of the early universe. The flatness problem was resolved on the basis of this data and seen to be a natural phenomenon. This was found to imply that the universe must remain flat on the large scale. The fine-tuning was vindicated by inflation which ensuingly resulted.

To make meaningful investigations on the physical decay processes in the early universe, it became mandatory that the big bang singularity be addressed first. Bubble nucleation indicated that the physical interpretation of the instanton solution as a probability of barrier penetration is a remedy for the cosmological puzzle that the size of the universe should vanish at zero time. The dependence of the probability amplitude on the mass of the neutrino was an indication that entropy always increases. This was consistently found to be truly a phase transition.

Further, it was established that a neutrino has a finite mass and it can acquire this mass through the Higgs mechanism. This was found to be true by studying a standard electroweak Lagrangian which was modified by adding a mass term to it. A scalar field that was introduced generated both a mass term and an interaction term for a neutrino. To determine the numerical mass value, the neutrino Boltzmann Transport equation that governs the neutrino phase-space distribution functions in an early expanding universe was set up and then solved in the context of the flat model. The masses found established that weakly interacting massive neutrinos close the universe and ultimately contribute to the unseen matter component that is appropriately needed to control the expansion of the universe. Essentially, due to the slight heating of neutrinos by electron-positron annihilations, the calculated neutrino temperature was found to be 1.946 K, slightly lower than that of photons 2.726 K, so that much of the energy density of the universe is due to massive neutrinos. These massive neutrinos were found to possess masses corresponding to $1.97\,eV$ (or $3.94\,eV$) for relic Dirac (Majorana) neutrinos. Very massive weakly interacting neutrinos of mass $1.57\,GeV$ were also found to close the universe. These results were found to be unique from those of the previous investigations. In particular, Bergkvist (1972) managed to obtain 60 eV using electrostatic and magnetic spectrometric methods; Lubimov et al., (1980) obtained the range $14eV \le m_{\bar{\nu}_e} \le 46eV$ using a toroidal spectrometer and tritium molecule; Robertson et al., (1991) obtained 9.3 eV using gaseous tritium molecule while Weinheimer et al., (1993) obtained 7.2 eV using a frozen tritium source. On the cosmological front, Marx and Szalay (1976) obtained 130 eV by numerically integrating the Friedman equation while Bernstein and Feinberg (1981) obtained 15 eV when they used 10 gigayears as the age of the universe and 0.4 as the value of Hubble's constant. The results further indicated that cosmology has a mass gap for neutrino masses in the range between, above $1.97\,eV$ and $1.57\,GeV$ as opposed to the $SU(2)\times U(1)$ model of particle physics. For instance, the present upper limits for muon neutrinos and tau neutrinos from particle physics data group ($2.8\,eV, 190\,KeV$ and $18.2\,MeV$) allow these masses to be in this mass gap (Yao, 2006). This shows that cosmology strongly constrains the value of the muon and tauon neutrino masses. In particular, the presence of GeV masses calculated in this study requires the existence of a fourth family of leptons in addition to electron, muon and tauon families. It has also been found that the mass scale of heavy neutrinos can arise from lepton-number violation, beyond the standard electroweak model interactions. This mass scale is small if the energy scale Λ is big. If one would wish to have neutrino mass in the eV range say 1.97 eV, then the calculated energy scale from the seesaw relation should correspond to $\Lambda \approx 1.542\times 10^{13}\,GeV$ when the mass of 174.3 GeV is chosen for the top quark. This corresponds to an interesting grand unification energy scale which could naturally explain the smallness of the neutrino mass that signals the presence of new physics at very large energy scales beyond the standard electroweak model. In addition, it has been found that massive neutrinos will propagate with different velocities rather than with the uniform velocity, say c. Consequently the time of arrival of higher energy neutrinos from any given source will precede that of their lower energy counterparts in the neutrino detectors. This leads to the phenomenon of neutrino oscillation. In particular, it was found that a neutrino oscillation is highly related to mass and an oscillation in matter is different from oscillations in a vacuum. More so, when both the mixing angle and the mass differences are positive, the neutrino oscillations are enhanced, whereas in the case where the mass difference is negative the situation is opposite. This means that, in the absence of the charge-parity violating phase in the mixing matrix, the probabilities of neutrino oscillations are different from those of antineutrino oscillations (matter can induce the charge-parity violating effects). Thus matter effects may mimic the charge-parity violation and this may make it difficult to disentangle the genuine charge-parity violation from the macroscopic one in the long baseline neutrino oscillation experiments. This issue demands further investigations. In particular, many of the experiments on neutrino oscillations, like Fermilab's Long Baseline Neutrino Experiment and MINOS, will be used to put the evidence for neutrino mass on a more solid footing. This is because neutrinos produced by accelerators or in nuclear reactors ('man made') will be preferred as they can be controlled unlike atmospheric or primordial neutrinos. However, from the theory of neutrino oscillations, it has been found that neutrinos can oscillate over appreciably long distances thereby motivating the cascading of long-baseline experiments. This will pave way for collaborative research. On the basis of these results, it is clear that a neutrino has a finite mass of $1.97\,eV$ (or $3.94\,eV$) for a light case and *1.57 GeV* for the heavy one; can acquire mass through the a quasi-Higgs mechanism and that weakly interacting eV massive neutrinos are the hot dark matter particles that dominate the larger scales of the universe, while the GeV massive ones dominate the smaller scales. The $SU(2)\times U(1)$ model of Particle Physics that allows only massless neutrinos is incomplete. To incorporate neutrino mass into the model and to explain, convincingly, why it is so small may require major changes to the model. Besides, the quantum nature of the neutrino, i.e., whether it is a Dirac or Majorana particle is not given any attention in this work. These aspects remain open puzzles for the future to resolve.

5.2. RECOMMENDATIONS

- A comprehensive and systematic study of the effect of neutrino mass on the standard electroweak gauge model is suggested. This may help in shedding more light on the possibility of establishing the exact quantum nature of the neutrino.
- The instanton solution as clearly revealed that a neutrino-dominated and evolving universe is truly a phase transition. Detailed studies are suggested to properly identify the nature of the phase transition in the grand unified models that still remains a puzzle.
- Additional detailed investigations on neutrino mass in the context of the formation of structures in the universe are also suggested. This may help in determining the exact nature of the neutrino mass that is required for the formation of structures in the universe.

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