

# Analysis of Statically Determinate Structures

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**Nairobi University Press**

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## Preface

The material presented in this text is not new and the author does not claim ownership of originality. However, an attempt has been made to organise the material in a manner that will assist the student to progress systematically from the simpler and familiar to the more complex concepts and their applications.

A clear understanding of the basic conventional concepts in statical analysis of structures is essential for all civil engineering students. It is vital that a structural engineer is able to isolate relevant free bodies and to check the equilibria of applied force systems. This way, the engineer is able to visualize the blow of forces in a structure and especially in structural connections, and also to detect analytical errors and unstable forms in structural design.

At the beginning the student is reminded of the familiar Newton's Laws of Motion. In chapter two, equilibrium is discussed and the important concept of a free body is introduced followed by a presentation of idealised support types and a review of coplanar force systems. Chapter Three introduces the concept of structural determinacy. Structural forms, to be considered and analysed in the subsequent chapters, are defined. The cable and arch are dealt with in Chapter Four, the truss is considered in Chapter Five and the beam and frame in Chapter Six. In each case the basic statical equilibrium approach, of isolating a free body, identifying the system of forces applied to the body and solving for the unknown forces from the deduced equations of equilibrium is used to determine external reactions and internal reactants. Moreover, methods are developed for determining the deformations due to loading for the various structural forms. The final chapter deals with the prediction of the effect of moving loads by using influence lines.

The student is challenged to use his intuition in the selection of free bodies of interest and in developing an efficient strategy for solving unknown actions. Many worked examples are presented to clarify the application of basic principles and illustrate the methods of approach for a great variety of problems. A sufficient number of exercise problems with varying degrees of difficulty are given to challenge the student. It is hoped that as the student solves the problems, he will develop the valuable problem-solving proficiency that is so important in structural analysis and design.

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*Nairobi, September 1993*