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THE CHOICE OF ACTUARIAL FUNDING METHODS FOR FUNDED DEFINED BENEFIT PENSION SCHEMES

By

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DECLARATION

I hereby declare that the project work entitled: "**THE CHOICE OF ACTUARIAL FUNDING METHODS FOR FUNDED DEFINED BENEFIT PENSION SCHEMES**", is a record of an original work and the results embodied in this document have not been submitted to any other University or Institute for the award of any degree or diploma.

SUPERVISORS

This document has been presented to the University of Nairobi (School of Mathematics, Department of Actuarial Science), in partial fulfilment of the requirements for the award of the degree of Master of Science in Actuarial Science.

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LIST OF ABBREVIATIONS

SCR	Standard Contribution Rate
AL	Actuarial Liability
AAM	Attained Age Method
EAM	Entry Age Method
PUM	Projected Unit Method
CUM	Current Unit Method
PVFPB	Present value of future benefits accrual for all current active members based on projected final earnings
PVTPE	Present value of total projected earnings for all current active members throughout their expected future membership
PVTPB	Present value of total benefits based on projected final earnings for current active members
PVPSB	Present value of past service benefits based on projected final earnings for current active members
PVPSB_0	Present value of past service benefits based on current earnings for current active members

EXECUTIVE SUMMARY

Defined benefit pension schemes are pension schemes in which pension benefits payable at retirement are determined using a pre-defined formula contained in the schemes' trust deed and rules. The pension benefit formula is usually a multiple of a pension accrual factor, years of service and final salary as defined by the scheme rules.

The responsibility of the liability of these schemes solely lies with the sponsor who is required to set aside funds usually by way of regular contributions into a designated fund to meet the anticipated future benefit payments. It is therefore critical that the sponsor adopts an appropriate actuarial funding method that will result to the remittance of sufficient contributions in a manner not detrimental to normal business operations.

Four main actuarial methods of funding pension schemes have been developed to calculate an appropriate pattern of contributions to meet the expected future benefit payments. These methods are:

1. The Attained Age Method (AAM)
2. The Entry Age Method (EAM)
3. The Projected Unit Method (PUM)
4. The Current Unit Method (CUM)

The only difference in these funding methods is the timing of contributions but the overall long term cost of the scheme is the same.

The choice of any of the methods to form basis of funding a scheme takes into account the following major factors: stability, security, flexibility and realism.

This project examines and analyses the four funding methods, applies them to a model pension scheme and evaluates the results thus obtained to the extent to which they satisfy the factors of stability, security, flexibility and realism.

The project concludes by pointing out that the choice of actuarial funding methods should ideally maintain a balance amongst the need for stability, security, flexibility and realism. Therefore, one factor should not be used as the sole determinant (while ignoring the others) in determining the funding method to be adopted but rather carry out a trade-off amongst all the factors and adopt a method that is most suited in the circumstances.

The choice of a funding method should also maintain a balance of both the members' and sponsor's interests. While the members may need sufficient security, the sponsor may prefer considerable flexibility even when the scheme's funding level is below the statutory minimum.

CHAPTER 1

INTRODUCTION

1.1 Background

Defined benefit pension schemes are pension schemes in which pension benefits payable at retirement are determined using a pre-defined formula contained in the schemes' trust deed and rules. The pension benefit formula is usually a multiple of a pension accrual factor, years of service and final salary as defined by the scheme rules.

These schemes are called funded schemes when the employers opt to set aside funds systematically during the employees' working life with the funds expected to accumulate to meet the expected pension benefit at retirement. This is normally done by way of remittance of regular contributions to the scheme.

Four main actuarial methods of funding pension schemes have been developed to calculate an appropriate pattern of contributions to meet the expected future benefit payments. These methods are:

1. The Attained Age Method (AAM)
2. The Entry Age Method (EAM)
3. The Projected Unit Method (PUM)
4. The Current Unit Method (CUM)

Under the different funding methods, the timing of contributions is different but the contributions are sufficient to meet the overall long-term cost of the scheme.

1.2 Problem statement

The difference in the four funding methods is the timing of contributions but the fundamental long-term amount is the same. The timing of contributions ultimately affects the investment returns achieved and the level of funds accumulated at a point of time.

The choice of a funding method is governed by the following factors:

i. *Security:* this is the ability of a funding method to ensure that the scheme has sufficient assets to meet the scheme liabilities.

ii. *Stability:* this is the ability of a funding method to maintain relatively the same contribution pattern from time to time.

iii. *Realism:* this is achieved when a funding method relies on assumptions that are likely to be met in practice.

iv. *Flexibility:* this is achieved when a funding method allows the employers room to make best use of their finances.

The effect of the difference in timing of contributions has an impact on the extent to which the above factors that affect the choice of a funding method are satisfied. Each of the funding methods satisfies each of the factors differently, with some satisfying some factors more than others. Therefore, funding methods adopted by different schemes may differ depending on the factors under consideration or the factors intended to be achieved.

Therefore, there is need to analyse each of the funding methods and evaluate the extent to which each of them satisfy the factors above and recommend suitable funding methods for different forms of funded defined benefit pension schemes.

This is the basis of this study.

1.3 Objectives

1.3.1 Overall objective

The overall objective of the study will be to analyse the four funding methods and evaluate the extent to which each of them satisfy the qualities of security, stability, realism and flexibility, and recommend suitable funding methods for different forms of funded defined benefit pension schemes.

1.3.2 Specific objectives

The specific objectives of the study necessary for the achievement of the overall objective are:

i. To determine the standard contribution rate for each of the funding methods;

ii. To determine the actuarial liability for each of the funding methods;

iii. To determine the behaviour of the standard contribution rate and actuarial liability when valuation parameters are varied particularly age;

iv. To compare the standard contribution rates and actuarial liabilities obtained from the funding methods and establish the relationship between the funding methods; and

v. To evaluate the results obtained for the extent to which they satisfy the qualities of security, stability, realism and flexibility.

1.4 Justification of the study

This study is significant in that its findings and recommendations will enable prudent and objective choice of funding methods for defined benefit pension schemes taking into account the factors under consideration or the factors intended to be achieved or the form of the scheme.

CHAPTER 2

LITERATURE REVIEW

O'Regan and Weeder (1988) discusses actuarial methods of funding pension schemes and follows on from the report of the Working Party of the Pensions Standards Joint Committee on Terminology of Pension Funding Methods published in 1984.

The paper analyses the basic structure of the main funding methods, their behaviour and suitability under various conditions. The four main funding methods examined are:

i. The Current Unit Method

ii. The Projected Unit Method

iii. The Attained Age Method

iv. The Entry Age Method

The paper emphasizes that the cost of a pension scheme is not determined by the amount paid in by way of contribution rate but by the ultimate experience of the scheme. However, the ultimate experience and therefore the ultimate cost cannot be predicted with precision and the contribution rates established are only estimates. The long term contribution rate is not a cast iron figure but an estimate of the long term cost.

The paper also notes that not all the funding methods or strategies will give a sensible estimate of the contribution rate applicable to all scheme variants. One funding method will be most suited to a scheme of a certain type than the other. For example the rise in average age and past service caused by the introduction of one or a few dominant members into a small scheme will cause the standard contribution rate to rise significantly from year to year under the Current Unit Method but will result to a small effect under the Projected Unit Method and the Attained Age Method. Therefore, in such case, the Projected Unit and Attained Age methods are likely to result to a relatively stable contribution rate and thus more suited than the than Current Unit Method.
It is therefore up to the actuary's judgement to select and recommend a funding strategy that will produce suitable results neither too low, because of the danger of the scheme's

solvency, nor too high, because of the diversion of the company assets which could possibly be put into better use elsewhere.

The paper holds that there are a variety of reasons for valuing pension funds among them the need to establish an appropriate long-term contribution rate. In establishing the funding strategy suitable for determining a long-term contribution rate, actuaries will consider a funding strategy that achieves the following objectives:

i. The strategy should result to a fund that will be sufficient at any time to cover accrued benefits; and

ii. The strategy should result to the long-term contribution rate being fairly stable.

The paper summarizes the rationale and significant features of the funding methods as follows:

a. Current Unit Method

Under the current unit method the contribution rate should pay for benefits accruing in the coming year, based on earnings at the end of that year together with earnings inflation on previously accrued benefits.

The rationale of this method is that at any time the accumulated fund is sufficient to purchase past service benefits based on current earnings

The significant features are:

- The standard fund built up is generally the smallest of the four methods, given the same assumptions.
- A stable long-term contribution rate can be achieved for a large scheme with a reasonable prospect of age and past service stability. However, this is unlikely for small schemes where there is less prospect of age and past service stability.
- The method cannot be used to value benefits which are subject to revaluation.

b. Projected Unit Method

Under the projected unit method the contribution rate should be sufficient to purchase benefits based on projected final earnings, which will accrue over the next year.

The rationale of this method is to build up a fund sufficient to purchase past service benefits based on projected final earnings.

The significant features are:

- The standard fund built up is generally much larger than under the current unit method, and is the same as under the attained age method.
- The method is far less sensitive to changes in average age than the current unit method and therefore displays much greater stability.
- The fund built up is intended to be sufficient to purchase benefits on winding-up, based on earnings at projected date of exit, although its ability to do this will, of course, be subject to experience.

c. Attained Age Method

The rationale of the attained age method is to establish a contribution rate that will remain stable throughout the working lifetime of the current membership and assumes that there will be no new entrants in future. Consequently, the contribution rate allows for gradual ageing of the membership. At any point of time, the accumulated fund should be at least equal to the value of the accrued final earnings.

The significant features are:

- The contribution rate is higher than that required to maintain the funding level at 100% at earlier years. Other things being equal, the funding level rises gradually from the date the contribution rate is established. Once the membership has aged considerably the contribution rate becomes inadequate, the surplus is then drawn on to supplement the contribution rate, and the funding level will drop to 100%.
- The contribution rate will only remain stable if either the age structure remains stable or the valuation assumptions are met i.e. there are no new entrants and the periodic surplus is run off over the outstanding working life of the membership.
- The method is particularly suitable for schemes which are uncertain of new members, or for very small schemes which are prone to changes in average age.

d. The Entry Age Method

The rationale of the entry age method is that the contribution rate should be set at a level sufficient to purchase benefits for a new entrant.

The significant features are:

- The method is neutral as regards its attitude to future new entrants and if the new entrants enter at the assumed ages there will be neither strain nor release and the contribution rate will be adequate.
- The entry age method is the only one of the four methods where new entrants can create a capital strain or release and this occurs when new entrants occur at ages other than those assumed.
- In most cases, any realistic entry age assumption will be less than the average age of the members and as a result the standard fund under the entry age method is the largest of the four methods.
- On the closure to new entrants of a scheme funded on this method, the contribution rate should remain stable (if, of course, the assumptions are borne out in practice).
- The determination of the new entrant age assumption is generally difficult because the new entrant and withdrawal experience is unpredictable and dependent on the financial fortune of the employer, as well as the economy in general.

The paper also analyses the sensitivity of the contribution rates determined under the different funding methods to variations in membership profile which include age and past service; and actuarial assumptions which include interest rate, control period, earning increases, mortality, early retirement and withdrawals. The significant findings and conclusions made in this respect include:

- The contribution rates determined under the current unit, projected unit and attained age methods generally increase with age.
- The entry age method produces a constant contribution rate dependent on the assumed entry age.
- The level and stability of contribution rates determined under the projected unit and current unit methods increase with control period.

- The contribution rates determined under the four funding methods generally increase with increase in escalation rate of earnings and decrease in valuation interest rate, mortality and other forms of withdrawal. The converse is true.

Pugh (2006) outlines the regulatory framework within which occupational defined benefit pension plans are financed and addresses the challenges facing the funding of such plans.

The paper addresses the types of funding and actuarial costing methods that could be considered as best practice.

The paper outlines the challenges faced by regulatory authorities in establishing appropriate minimum funding requirements and maximum funding limitations and whether regulators should establish a precise set of actuarial assumptions (economic and demographic) to be used in actuarial costing or mandate a single actuarial funding method. The paper further highlights challenges related to the sharing of funding shortfalls and funding excesses (surpluses) between plan sponsors and plan members.

The paper holds that it is difficulty for regulators to justify mandating a single actuarial funding method. This is because employers in different industries or at different stages of their development will have correspondingly different funding objectives. Therefore all the actuarial funding methods namely: the Projected Unit Method, the Current Unit Method, the Attained Age Method and the Entry Age Method are sound and systematic and the use of either of the methods should not cause concern to a regulator.

The paper classifies the actuarial funding methods into two categories namely:

a. **Accrued benefits funding methods:**

   These methods focus on maintaining a certain level of funding. They are security driven in that they attempt to establish and maintain a sound relationship between the fund assets and accrued liabilities. The funding requirement is then the contribution required to achieve the funding objective.

This category comprise of two methods namely:

i. The Current Unit Method

ii. The Projected Unit Method

The Current Unit Method (CUM) calculates accrued liabilities of active employees without providing allowance for the effect of future salary increases while the Projected Unit Method (PUM) provides for future salary increases.

The objective of the CUM is to maintain a fund equal to the present value of accrued benefits based on current earnings while the PUM is based on projected earnings at retirement.

b. Prospective Funding Methods:

These methods define a certain level of contributions. They are contribution driven and their primary objective is stability of contributions. These contributions then define the targeted level of the fund at any point in time.

This category comprise of:

i. The Entry Age Method

ii. The Attained Age Method

The objective of the Attained Age Method (AAM) is to establish a stable contribution rate that will fund benefits accruing after valuation date based on future salaries.

The objective of the Entry Age Method (EAM) is to establish a stable contribution rate assuming all members joined the scheme at an assumed entry age and allowing for future salary increases.

This paper also notes that the funding method does not affect the true overall cost. The ultimate cost of any pension plan to the sponsor is given by:

Total benefits paid to plan beneficiaries – member contributions – investment income earned by the plan + expenses incurred in the operation of the plan and the fund

It can readily be seen that there are no actuarial calculations or actuarial estimates in the above formula. Nevertheless, because certain actuarial funding methods require higher employer contributions in the early years, which will hopefully result in greater investment income, the eventual employer cost is indirectly affected by the funding method. This is a timing issue and indeed actuarial funding valuations are all about 'timing' i.e. setting aside assets in an organized fashion to discharge the eventual obligation.

The paper notes that the Projected Unit Method (PUM) is arguably the most important actuarial funding method which has been widely adopted in many countries in absence of any particular legislative constraints. The justification of PUM's dominance is due to the following:

i. It is viewed as the most transparent method among the other methods. This is because the goal of PUM is to maintain the pension fund assets at such a level that with future investment income but without any future contributions, the fund will be able to pay all accrued benefits until the last plan beneficiary dies and recognizes future salary increases. Its definition of accrued liabilities is clear and readily comparable with the accumulating fund assets. Therefore it is easy to identify and understand favourable and unfavourable experience.

ii. It is the most preferred by major accounting bodies for the pension expensing requirements that are imposed on plan sponsors

The paper does not recommend setting of specific actuarial assumptions but urges the use of reasonable and appropriate assumptions that are independently realistic and perhaps with a margin of conservatism.

The paper notes that minimum funding requirements are becoming common so as to protect the plan members' benefits and ensure security of payment of such benefits. However, in times of economic downturn, this may lead to funding constraints on the part of the sponsor. Several jurisdictions have relaxed their minimum funding requirements and provided for longer amortization period to address shortfalls and ensure flexibility on the part of the sponsor.

Maximum funding constraints have also been imposed particularly by tax authorities to prevent either the deliberate or accidental build-up of excessive assets within the pension fund.

The Actuarial Education (ActEd) Company (2011) consists of study notes and material for Pensions and Other Benefits prepared by the Actuarial Education (ActEd) Company for use by students sitting for the relevant professional examination; Specialist Technical 4 (ST4).

Chapter 19; Funding Methods, discusses the use of actuarial models for decision making purposes in pensions and other benefits and in particular, the use of these models for setting contributions.

The chapter defines the main actuarial funding methods namely:

i. The Attained Age Method
ii. The Entry Age Method
iii. The Projected Unit Method
iv. The Current Unit Method

The funding methods are classified into two main categories:

i. Prospective methods, which target a stable contribution rate.
ii. Accrued benefits methods, which fund for a target level of cover of benefits accrued to date i.e. target the Actuarial Liability.

The chapter goes further in deriving the algebraic expressions for the standard contribution rates and the actuarial liability for an individual active member whose only benefit will be a retirement benefit based on final earnings and uniform accrual; as well as establishing simple relationships between the funding methods.

The stated algebraic expressions for the standard contribution rate and the actuarial liability are:

i. Expressions for the standard contribution rate

$$SCR_{AAM} = \frac{\frac{(R-x)\times S}{A}\times\left(\frac{1+e}{1+i}\right)^{R-x}\times a_R^{'}}{S\times a_{\overline{R-x}|}}$$

$$SCR_{EAM} = \frac{\frac{(R-E)\times S}{A}\times\left(\frac{1+e}{1+i}\right)^{R-E}\times a_R^{'}}{S\times a_{\overline{R-E}|}}$$

$$SCR_{PUM} = \frac{\frac{1\times S}{A}\times\left(\frac{1+e}{1+i}\right)^{R-x}\times a_R^{'}}{S\times a_{\overline{1}|}}$$

$$SCR_{CUM} = \frac{\frac{1\times S\times(1+e)}{A}\times\left(\frac{1}{1+i}\right)^{R-x}\times a_R^{'}+CUAL\times e}{S\times a_{\overline{1}|}}$$

ii. Expressions for the actuarial liability

$$AL_{AAM} = \frac{(P+F)\times S}{A}\times\left(\frac{1+e}{1+i}\right)^{R-x}\times a_R^{'}-SCR_{AAM}\times S\times a_{\overline{R-x}|}$$

$$AL_{EAM} = \frac{(P+F)\times S}{A}\times\left(\frac{1+e}{1+i}\right)^{R-x}\times a_R^{'}-SCR_{EAM}\times S\times a_{\overline{R-x}|}$$

$$AL_{PUM} = \frac{P \times S}{A} \times \left(\frac{1+e}{1+i} \right)^{R-x} \times a_R^{'}$$

$$AL_{CUM} = \frac{P \times S}{A} \times \left(\frac{1}{1+i} \right)^{R-x} \times a_R^{'}$$

Where:

P = past service at the date of valuation

F = future service i.e. (R-x)

R = assumed retirement age

E = assumed entry age

S = salary earning at the date of valuation

A = rate of pension accrual

e = assumed annual earnings (or salary growth)

i = discount rate

$a_R^{'}$ = value of annuity payable from age R

$a_{\overline{R-x|}}$ = annuity to determine present value of future earnings

The chapter notes the following:

- Individual standard contribution rates generally increase with age under the attained age, projected unit and current unit methods.
- The standard contribution rate under the entry age method is constant and is dependent on the assumed entry age.
- The actuarial liabilities under the attained age method and projected unit method are equal.
- The current unit standard contribution rate is very low at younger ages and very high at older ages.
- Given that valuation assumptions remain constant, stability of contributions is achieved by maintaining stable membership profile.
- The ultimate choice of funding models is dependent on security, stability, flexibility and realism.

Aitken (1996) focuses on the mathematics of pension plans and presents an introduction on the calculation of funding costs and actuarial liabilities in pension plans. The book covers a background to pensions, different actuarial costing methods, quantifying experience gains and losses, retirement options and wider pension concepts.

The book describes in detail the actuarial costing methods namely: the Current Unit Method, the Projected Unit Method, the Entry Age Method and the Attained Age Method.

Scott (1999) presents an introduction to life assurance mathematics, derivation, application and concepts thereof. The book covers the following topics: Life tables, Assurances, Annuities, Premiums, Reserves, Profit testing, Joint Life functions, Contingencies, pension funds among others.

In particular, Chapter 19 deals exclusively with Pension Funds. This chapter makes an introduction to pension funds by stating that there are two broad types of pension plans namely:

i. Defined benefit plans
ii. Defined contribution plans

In defined benefit plans, pension benefits are set out in the rules of the scheme. The sponsor is responsible for ensuring that the scheme is sufficiently funded to meet the promised benefits as and when they arise.

On the other hand, in defined contribution plans, contributions of both the sponsor and the members are fixed and the pension benefit at retirement is determined by the level of accumulated contributions and investment income earned.

The chapter introduces valuation principles of defined benefit schemes taking into account service tables, salary scale functions and other modes of decrement. The chapter examines the determination of the mean present value of future benefits and future contributions as well as the corresponding reserves.

The reserve for each member calculated prospectively is given by:

Reserve = Mean present value of future benefits – Mean present value of future contributions

The contribution rate for each member expressed as a percentage of earnings is given by:

Contribution rate = Mean present value of future benefits divide by Mean present value of future earnings

Finally, the chapter discusses the valuation of pension benefits for the three main forms of defined benefit pension plans namely:

i. Fixed pension schemes
ii. Average salary schemes
iii. Final salary schemes

Blake (2006) provides an insight into the theory and practice of finance as relates to pension matters. The book discusses the various types of investment assets, corporate pension finance, the financial aspects of defined contribution pension plans, the financial aspects of defined benefit pension plans, the role of pension funds and pension fund management, pension fund performance measurement, risk management in pension funds among others.

CHAPTER 3

MODEL DERIVATION

3.1 Introduction

There are four main actuarial funding methods namely:

i. Attained Age Method

ii. Entry Age Method

iii. Projected Unit Method

iv. Current Unit Method

In this chapter, we derive the actuarial liability and the standard contribution rate under each of the methods for funded final salary defined benefit pension schemes.

Final salary defined benefit pension schemes are pension schemes in which the pension benefits payable to retirees depend on the final salary at the time of retirement. For example, a scheme may specify annual pension as: $^{1}\!/_{60} \times$ *final salary per year of service*.

The Actuarial Liability is the value of benefits earned to date as defined by the specific funding method.

The Standard Contribution Rate is the ideal contribution rate to cover the cost of future benefit accrual as defined by the specific funding method.

The general form of the models derived conform to those in The Actuarial Education (ActEd) Company (2011) study notes and material for Pensions and Other Benefits but have been modified for actuarial assumptions and benefits structure defined herein.

3.2 Valuation assumptions and notations

In deriving the models, financial/economic and demographic assumptions are required to project the benefits and earnings of members of the scheme.

For the purpose of this study, the assumptions used in deriving the models are:

i. The following are known or can be objectively determined:

i = valuation rate of interest per annum.

R = normal retirement age.

PAF = pension accrual rate.

S =salary at valuation date.

x = age of member at valuation date.

s_x = salary scale function at age x.

j = salary growth per annum.

l_x = number of persons who attain age x according to some mortality table.

a_R = value of annuity payable from age R.

$a_{\overline{R-x|}}$ = value of annuity to determine present value of future earnings.

P = past service

F = Future service i.e. (R-x)

ii. There are no other decrements other than mortality.

iii. There is no averaging of final salary.

iv. Contributions are paid continuously and salary growth is continuous.

v. Accrual of pension benefits is uniform.

In this chapter and subsequent chapters, the following abbreviations have been used:

SCR	Standard Contribution Rate
AL	Actuarial Liability
AAM	Attained Age Method
EAM	Entry Age Method
PUM	Projected Unit Method
CUM	Current Unit Method
PVFPB	Present value of future benefits accrual for all current active members

based on projected final earnings.

PVTPE Present value of total projected earnings for all current active members throughout their expected future membership.

PVTPB Present value of total benefits based on projected final earnings for current active members.

PVPSB Present value of past service benefits based on projected final earnings for current active members

$PVPSB_0$ Present value of past service benefits based on current earnings for current active members

3.3 Attained Age Method

The Attained Age Method targets a stable level of contribution; the Standard Contribution Rate.

The targeted stable level of contribution can be adjusted as appropriate when the experience does no follow the model or its parameters.

The Actuarial Liability is determined as the difference between the discounted value of the total expected benefits for the members and the discounted value of the future expected contributions.

3.3.1 Attained Age Standard Contribution Rate (SCR_{AAM})

Under the Attained Age Method, the standard contribution rate, expressed as a percentage of earnings is determined as:

The present value of all benefits which will accrue to present members after the valuation date (by reference to service after the valuation date and projected final earnings)

divided by

The present value of total projected earnings for all members throughout their expected future membership.

Mathematically, for a scheme with N active members, this is expressed as:

$$SCR_{AAM} = \frac{PVFPB}{PVTPE} \tag{3.1}$$

$$SCR_{AAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times \frac{\overline{S}_R}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i}}{S_i \int_0^{R-x_i} v^t \frac{l_{x_i+t}}{l_{x_i}} \frac{\overline{S}_{x_i+t}}{S_{x_i}} dt} \tag{3.2}$$

This is approximated by:

$$SCR_{AAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times \frac{S_{R_i - 1/2}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i}}{S_i \sum_{t=0}^{R-1-x_i} v^{t+1/2} \frac{l_{x_i+t+1/2}}{l_{x_i}} \frac{S_{x_i+t}}{S_{x_i}}} \tag{3.3}$$

Assuming that salary growth, j, is the only salary scale function, expression 3.2 becomes:

$$SCR_{AAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times (1+j)^{(R-x_i)} \times v^{(R-x_i)} \times a_{R_i}}{S_i \int_0^{R-x_i} v^t \frac{l_{x_i+t}}{l_{x_i}} (1+j)^{(R-x_i)} dt} \tag{3.4}$$

$$= \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i}}{S_i \int_0^{R-x_i} \left(\frac{1+j}{1+i}\right)^t \times \frac{l_{x_i+t}}{l_{x_i}} dt}$$

$$= \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i}}{S_i \, \overline{a}^{*}_{x_i:\overline{R-x_i|}}} \tag{3.5}$$

Where $\overline{a}^*$ is at the rate, $i^* = \dfrac{i-j}{1+j}$

3.3.2 Attained Age Actuarial Liability (AL_{AAM})

Under the Attained Age Method, the Actuarial Liability is expressed as:

The present value of total benefits based on projected final earnings for members in service

minus

The value of the SCR multiplied by the present value of total projected earnings for all members throughout their expected future membership

This is equivalent to:

The present value of all benefits accrued at the valuation date based on the projected final earnings for the members in service.

Mathematically, for a scheme with *N* active members, this is expressed as:

$$AL_{AAM} = PVTPB - SCR \times PVTPE \tag{3.6}$$

$$AL_{AAM} = \sum_{i}^{N} \frac{1}{PAF} \times (P+F)_i \times S_i \times \frac{\overline{S_{R_i}}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i} - SCR_{AAM} \times S_i \int_0^{R-x_i} v^t \frac{l_{x_i+t}}{l_{x_i}} \frac{\overline{S_{x_i+t}}}{S_{x_i}} dt \tag{3.7}$$

But,

$F_i = R - x_i$ And;

$$SCR_{AAM} = \sum_{i}^{N} \frac{\dfrac{1}{PAF} \times (R - x_i) \times S_i \times \dfrac{\overline{S}_R}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i}}{S_i \displaystyle\int_0^{R-x_i} v^t \frac{l_{x_i+t}}{l_{x_i}} \frac{\overline{S}_{x_i+t}}{S_{x_i}} dt}$$

Substituting these equations in expression 3.7 above and simplifying, we obtain:

$$AL_{AAM} = \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times \frac{\overline{S_{R_i}}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i} \tag{3.8}$$

And assuming that salary growth, *j*, is the only salary scale function, expression 3.8 becomes:

$$AL_{AAM} = \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i} \tag{3.9}$$

3.4 Entry Age Method

Just like the Attained Age Method, the Entry Age Method also targets a stable level of contribution; the Standard Contribution Rate.

Unlike in the AAM, under this method, when calculating the SCR, we need to use an entry age assumption, which forms the reference point of determining the members' expected period of membership in the scheme. The members' full expected period of membership will be based on the single assumed entry age.

The assumed entry age may be chosen as one of the actuarial assumptions or may be determined from inspection of the actual entry ages of members.

The Actuarial Liability is determined as the difference between the discounted value of the total expected benefits for the members and the discounted value of the future expected contributions.

3.4.1 Entry Age Standard Contribution Rate (SCR_{EAM})

Under the Entry Age Method, the standard contribution rate, expressed as a percentage of earnings is determined as:

The present value of all future benefits for members joining the scheme at the assumed entry age by reference to projected final earnings

divided by

The present value of total projected earnings for all members throughout their expected future membership.

Mathematically, for a scheme with N active members and assumed entry age, x_0 , this is expressed as:

$$SCR_{EAM} = \frac{PVFPB_{x_0}}{PVTPE_{x_0}} \tag{3.10}$$

$$SCR_{EAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_0) \times S_i \times \frac{\overline{S}_R}{S_{x_0}} \times v^{(R-x_0)} \times a_{R_i}}{S_i \int_0^{R-x_0} v^t \frac{l_{x_0+t}}{l_{x_0}} \frac{\overline{S}_{x_0+t}}{S_{x_0}} dt}$$

This is approximated by:

$$SCR_{EAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_0) \times S_i \times \frac{S_{R_i - 1/2}}{S_{x_0}} \times v^{(R-x_0)} \times a_{R_i}}{S_i \sum_{t=0}^{R-1-x_0} v^{t+1/2} \frac{l_{x_0+t+1/2}}{l_{x_0}} \frac{S_{x_0+t}}{S_{x_0}}} \tag{3.11}$$

Assuming that salary growth, j, is the only salary scale function, expression 3.11 becomes:

$$SCR_{EAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_0) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_0)} \times a_{R_i}}{S_i \int_0^{R-x_0} \left(\frac{1+j}{1+i}\right)^t \times \frac{l_{x_0+t}}{l_{x_0}} dt} \qquad (3.12)$$

This simplifies to:

$$SCR_{EAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_0) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_0)} \times a_{R_i}}{S_i \bar{a}^{*}_{x_0:\overline{R-x_0}|}} \qquad (3.13)$$

Where $\bar{a}^*$ is at the rate, $i^* = \frac{i-j}{1+j}$

3.4.2 Entry Age Actuarial Liability (AL_{EAM})

Under the Entry Age Method, the Actuarial Liability is expressed as:

The present value of total benefits based on projected final earnings for members in service

minus

The value of the SCR multiplied by the present value of total projected earnings for all members throughout their expected future membership

Mathematically, for a scheme with *N* active members, this is expressed as:

$$AL_{EAM} = PVTPB - SCR_{EAM} \times PVTPE \qquad (3.14)$$

$$AL_{EAM} = \sum_{i}^{N} \frac{1}{PAF} \times (P+F)_i \times S_i \times \frac{\overline{S_{R_i}}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i} - SCR_{EAM} \times S_i \int_0^{R-x_i} v^t \frac{l_{x_i+t}}{l_{x_i}} \frac{\overline{S_{x_i+t}}}{S_{x_i}} dt \tag{3.15}$$

And assuming that salary growth, *j*, is the only salary scale function, expression 3.15 becomes:

$$AL_{EAM} = \sum_{i}^{N} \frac{1}{PAF} \times (P+F)_i \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i} - SCR_{EAM} \times S_i \int_0^{R-x_i} \left(\frac{1+j}{1+i}\right)^t \frac{l_{x_i+t}}{l_{x_i}} dt \tag{3.16}$$

This simplifies to:

$$AL_{EAM} = \sum_{i}^{N} \frac{1}{PAF} \times (P+F)_i \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i} - SCR_{EAM} \times S_i \, \overline{a}^{*}_{x_i:\overline{R-x_i}|} \tag{3.17}$$

Where $\overline{a^*}$ is at the rate, $i^* = \dfrac{i-j}{1+j}$

3.5 Projected Unit Method

The Projected Unit Method targets a standard level of funding with the standard contribution rate being set to maintain this target from year to year. The target fund is determined taking into account future expected inflationary/earnings growth.

The resultant standard contribution rate can be adjusted as appropriate when the experience does not follow the model or its parameter values.

The Actuarial Liability is determined as the discounted value of benefits that have accrued over the past period of membership taking into account any future expected inflationary growth of the on-going benefits up to retirement age.

3.5.1 Projected Unit Standard Contribution Rate (SCR_{PUM})

Under the Projected Unit Method, the standard contribution rate, expressed as a percentage of earnings is determined as:

The present value of all benefits that will accrue in the year following the valuation date by reference to service in that year and projected final earnings

divided by

The present value of all members' earnings in that year.

Mathematically, for a scheme with N active members this is expressed as:

$$SCR_{PUM} = \frac{PVFPB(1)}{PVTPE(1)} \tag{3.18}$$

$$SCR_{PUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \frac{\overline{S}_R}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i}}{S_i \int_0^1 v^t \frac{l_{x_i+t}}{l_{x_i}} \frac{\overline{S}_{x_i+t}}{S_{x_i}} dt} \tag{3.19}$$

This is approximated by:

$$SCR_{PUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \frac{S_{R_i - 1/2}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i}}{S_i v^{1/2} \frac{l_{x_i + 1/2}}{l_{x_i}}} \tag{3.20}$$

Assuming that salary growth, *j*, is the only salary scale function, expression 3.19 becomes:

$$SCR_{PUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \left(\frac{1+j}{1+i} \right)^{(R-x_i)} \times a_{R_i}}{S_i \int_0^1 \left(\frac{1+j}{1+i} \right)^t \times \frac{l_{x_i+t}}{l_{x_i}} \, dt}$$

This simplifies to:

$$SCR_{PUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \left(\frac{1+j}{1+i} \right)^{(R-x_i)} \times a_{R_i}}{S_i \overline{a}^{*}_{x_i:\overline{1|}}} \tag{3.21}$$

Where $\overline{a}^{*}$ is at the rate, $i^{*} = \frac{i-j}{1+j}$

3.5.2 Projected Unit Actuarial Liability (AL_{PUM})

Under the Projected Unit Method, the Actuarial Liability is given as:

The present value of all benefits accrued at the valuation date based on projected final earnings for members in service.

Mathematically, for a scheme with *N* active members, this is expressed as:

$$AL_{PUM} = PVPSB \tag{3.22}$$

$$AL_{PUM} = \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times \frac{\overline{S_{R_i}}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i} \tag{3.23}$$

Assuming that salary growth, j, is the only salary scale function, expression 3.23 becomes:

$$AL_{PUM} = \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i} \tag{3.24}$$

3.6 Current Unit Method

Just like the Projected Unit Method, the Current Unit Method targets a standard level of funding with the standard contribution rate being set to maintain this target from year to year.

However, the CUM differs with the PUM in that the target fund is determined without making allowance for inflationary/earnings growth between the date the target fund should be held and the date payment starts i.e. retirement date.

The resultant standard contribution rate can be adjusted as appropriate when the experience does not follow the model or its parameter values.

The Actuarial Liability is determined as the discounted value of benefits that have accrued over the past period of membership based on current earnings for members in service.

3.6.1 Current Unit Standard Contribution Rate (SCR_{CUM})

Under the Current Unit Method, the standard contribution rate, expressed as a percentage of earnings is determined as:

The sum of:

a. The present value of all benefits that will accrue in the year following the valuation date by reference to service in that year and projected earnings at the end of that year and;

b. The present value of all benefits accrued at the valuation date in respect of members in service multiplied by the projected percentage increase in earnings over the next year

Divided by:

The present value of all members' earnings in that year.

Mathematically, for a scheme with *N* active members, this is expressed as:

$$SCR_{CUM} = \frac{PVFPB_1(1) + \Delta AL_{CUM_1}}{PVTPE(1)} \tag{3.25}$$

$$SCR_{CUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \frac{\overline{S}_{x_i+1}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i} + AL_{CUM} \times \left(\frac{\overline{S_{x_i+1}}}{S_{x_i}} - 1 \right)}{S_i \int_0^1 v^t \frac{l_{x_i+t}}{l_{x_i}} \frac{\overline{S}_{x_i+t}}{S_{x_i}} dt} \tag{3.26}$$

This is approximated by:

$$SCR_{CUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \frac{S_{x_i+\frac{1}{2}}}{S_{x_i}} \times v^{(R-x_i)} \times a_{R_i} + AL_{CUM} \times \left(\frac{S_{x_i+\frac{1}{2}}}{S_{x_i}} - 1 \right)}{S_i v^{\frac{1}{2}} \frac{l_{x_i+\frac{1}{2}}}{l_{x_i}}} \tag{3.27}$$

Assuming that salary growth, *j*, is the only salary scale function, expression 3.26 becomes:

$$SCR_{CUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times (1+j) \times v^{(R-x_i)} \times a_{R_i} + AL_{CUM} \times (1 + j - 1)}{S_i \int_0^1 \left(\frac{1+j}{1+i} \right)^t \frac{l_{x_i+t}}{l_{x_i}} dt} \tag{3.28}$$

$$SCR_{CUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times (1+j) \times v^{(R-x_i)} \times a_{R_i} + AL_{CUM} \times j}{S_i \overline{a}^{*}_{x_i:\overline{1}|}} \tag{3.29}$$

Where $\overline{a}^{*}$ is at the rate, $i^{*} = \frac{i-j}{1+j}$

3.6.2 Current Unit Actuarial Liability (AL_{CUM})

Under the Current Unit Method, the Actuarial Liability is given as:

The present value of all benefits accrued at the valuation date based on current earnings for members in service.

Mathematically, for a scheme with *N* active members, this is expressed as:

$$AL_{CUM} = PVPSB_0 \tag{3.30}$$

$$AL_{CUM} = \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times v^{(R-x_i)} \times a_{R_i} \tag{3.31}$$

General Remarks:

The models for the Standard Contribution Rate (SCR) and Actuarial Liability (AL) derived in this chapter are applicable to funded final salary defined benefit pension schemes taking into account the assumptions listed under Section 3.2. However, the models can be modified for other forms of defined benefit pension schemes namely; the fixed pension schemes and the average salary schemes, by modifying the calculation of pension benefits based on the underlying formulae and assumptions.

It is important to note that the general forms of the models for the Standard Contribution Rate (SCR) and the Actuarial Liability (AL) under the different forms of defined benefit pension schemes are the same; the difference is in the calculation of underlying benefits.

The next chapter analyses and discusses results obtained by applying the models to a final salary model pension scheme.

CHAPTER 4

MODEL APPLICATION AND ANALYSIS

4.1 Introduction

For the purposes of demonstrating the application of the four models discussed and analysing the results thereof, we shall consider a model defined benefit pension scheme with membership profile illustrated in ***appendix I*** and which calculates annual pension benefit on retirement using the formula:

$$\frac{1}{40} \times \mathit{Final\ Annual\ Salary} \times \mathit{Years\ of\ Service}$$

The data for the model scheme consists of 30 members randomly generated.

We will assume that the scheme provides the pension benefit as the only benefit. However, in the real world, schemes will have varied benefits which if incorporated in our current study will lead to complex and time consuming calculations and yet give rise to the same findings and conclusions.

We will study and compare the four methods assuming the valuation assumptions used remain the same. The essential assumptions used are:

- Valuation rate of interest, i : 10%
- Salary growth, j : 5%
- Normal Retirement Age : 60 years
- Mortality : A1949-52 ultimate (*Appendix II*)
- Contributions and pension benefits are paid continuously
- Earnings growth is continuous

4.2 Determination of Standard Contribution Rates (SCRs)

The standard contribution rates for the model scheme, under each of the four methods, are obtained using the following formulae:

$$SCR_{AAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times \overline{a}_{R_i}}{S_i \overline{a}^{*}_{x_i:\overline{R-x_i}|}} \qquad (4.1)$$

$$SCR_{EAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_0) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_0)} \times \overline{a}_{Ri}}{S_i \overline{a}^{*}_{x_0:\overline{R-x_0}|}} \qquad (4.2)$$

$$SCR_{PUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times \overline{a}_{R_i}}{S_i \overline{a}^{*}_{x_i:\overline{1}|}} \qquad (4.3)$$

$$SCR_{CUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times (1+j) \times v^{(R-x_i)} \times \overline{a}_{R_i} + AL_{CUM} \times j}{S_i \overline{a}^{*}_{x_i:\overline{1}|}} \qquad (4.4)$$

Where:

$i = 0.10$, $j = 0.05$, $i^* = \frac{i-j}{1+j} = 0.047619047$, $R = 60$, $x_0 = 20$, $PAF = 40$

$\overline{a}_{R_i} = \sum_{t=0}^{100-R_i-1} v^{t+\frac{1}{2}} \frac{l_{R_i+t+\frac{1}{2}}}{l_{R_i}} @ i$, *A1949-52 ultimate* and $\overline{a}^{*}_{x_i:\overline{n}} = \sum_{t=0}^{n} v^{t+\frac{1}{2}} \frac{l_{x_i+t+\frac{1}{2}}}{l_{x_i}} @ i^*$,

A1949-52 ultimate.

The other variables S_i, x_i and P_i are obtained from data of the model scheme.

Applying the above formulae on the data of the model scheme we obtain the following results:

Funding Method	SCR expressed as percentage of earnings
AAM	12.39%
EAM	6.62%
PUM	11.71%
CUM	16.74%

Source: Model Pension Scheme (Appendix III)

4.3 Comparison of actual SCRs with simple average SCRs

The values obtained in **Section 4.2** above are the actual standard contribution rates of the model scheme under each of the funding methods.

However, it is easy to obtain a simple average of the sum of the individual standard contribution rates of the members and establish how they compare with the actual standard contribution rates obtained.

The simple average SCRs are given by the formula:

$$\overline{SCR} = \frac{\sum_{i=1}^{N} SCR_i}{N} \qquad (4.5)$$

Where;

SCR_i is the SCR of the i^{th} member and N is the total membership which in this case is 30.

The results are tabulated below:

Funding Method	SCR expressed as percentage of earnings	
	Actual	**Simple Average**
AAM	12.39%	12.89%
EAM	6.62%	6.62%
PUM	11.71%	9.71%
CUM	16.74%	11.74%

Source: Model Pension Scheme (Appendix III-A)

Remarks

Except for the EAM, we observe that the actual SCRs are not straight averages of the individual SCRs of members but are weighted by members' earnings.

The actual SCR_{EAM} is equal to the simple average SCR_{EAM} because the assumed entry age x_0 is applied uniformly to all members and consequently gives the same SCR_{EAM} for all members.

The graph of actual SCRs and simple average SCRs is shown below:

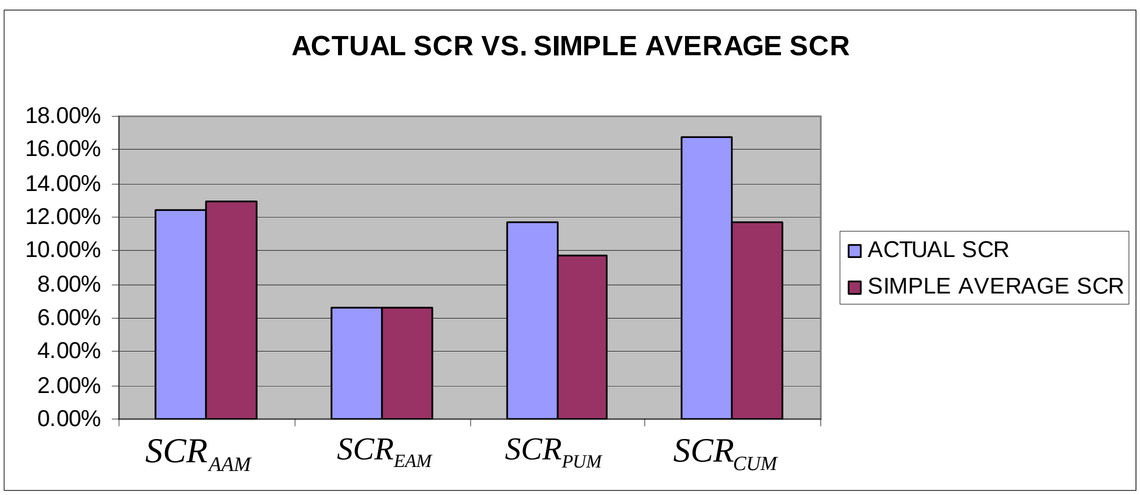

Figure 4.1 Actual Standard Contribution Rates versus Simple Average Contribution rates

4.4 Variation of SCRs with Age

4.4.1 Individual members' SCRs

The graph below shows variation of individual SCRs with age under each of the funding methods based on data of the model pension scheme.

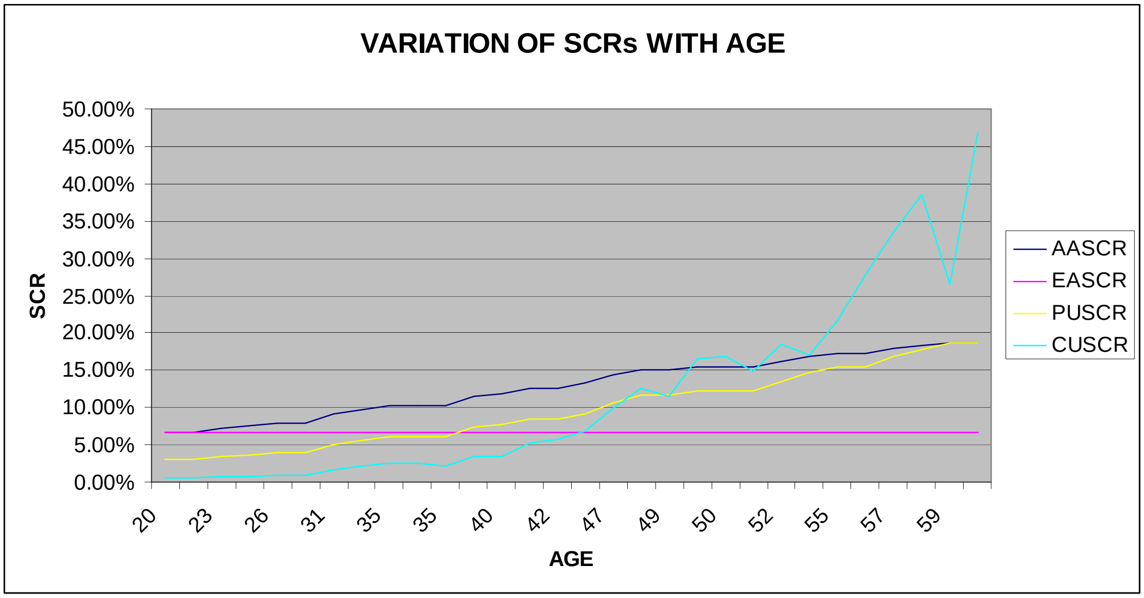

Figure 4.2 Variation of Standard Contribution Rates with Age

Source: Model Pension Scheme (Appendix III-A)

Remarks

From the graphs it is observed that the SCRs for individual members under the AAM, PUM and CUM are increasing with age.

Of course, the SCRs for individual members under the EAM are the same and constant for all members due to use of common assumed entry age.

The observation implies that at successive ages, the following ratio holds for individual members' SCRs determined using the AAM, PUM and CUM methods:

$$\frac{SCR(x+1)}{SCR(x)} > 1 \tag{4.6}$$

This can be shown mathematically as follows:

For the AAM;

$$\frac{SCR_{AAM}(x+1)}{SCR_{AAM}(x)} = \frac{\frac{1}{PAF}\times(R-1-x)\times S_2\times\left(\frac{1+j}{1+i}\right)^{(R-1-x)}\times\overline{a_R}}{S_2\,\overline{a^*_{x+1:\overline{R-1-x}|}}} \div \tag{4.7}$$

$$\frac{\frac{1}{PAF}\times(R-x)\times S_1\times\left(\frac{1+j}{1+i}\right)^{(R-x)}\times\overline{a_R}}{S_1\,\overline{a^*_{x:\overline{R-x}|}}}$$

$$= \frac{R-x-1}{R-x}\times\frac{\overline{a^*_{x:\overline{R-x}|}}}{\overline{a^*_{x+1:\overline{R-1-x}|}}}\times\frac{1+i}{1+j}$$

$$>1 \ \text{ if } \ i > j$$

For the PUM;

$$\frac{SCR_{PUM}(x+1)}{SCR_{PUM}(x)} = \frac{\frac{1}{PAF}\times S_2\times\left(\frac{1+j}{1+i}\right)^{(R-1-x)}\times\overline{a_R}}{S_2\,\overline{a^*_{x+1:\overline{1}|}}} \div \frac{\frac{1}{PAF}\times S_1\times\left(\frac{1+j}{1+i}\right)^{(R-x)}\times\overline{a_R}}{S_1\,\overline{a^*_{x:\overline{1}|}}} \tag{4.8}$$

$$= \frac{\overline{a^*_{x:\overline{1}|}}}{\overline{a^*_{x+1:\overline{1}|}}}\times\frac{1+i}{1+j}$$

$$>1 \ \text{ if } \ i > j$$

Similar results can be shown for the CUM.

4.4.2 Overall scheme SCRs

If we re-order the membership data of the model scheme on basis of age and re-classify it into five sets of six members each, and consider each set to represent a separate scheme denoted as A, B, C, D and E, we obtain the following results:

Scheme	**Average age**	**SCR_{AAM}**	**SCR_{PUM}**	**SCR_{CUM}**
A	23.17	7.44%	3.62%	0.73%
B	34.67	10.31%	6.12%	2.50%
C	44.00	13.32%	9.61%	7.90%
D	50.83	15.61%	12.66%	15.83%
E	57.17	17.66%	17.00%	33.34%

Source: Model Pension Scheme (Appendix III-B)

Below is the graphical representation of the results:

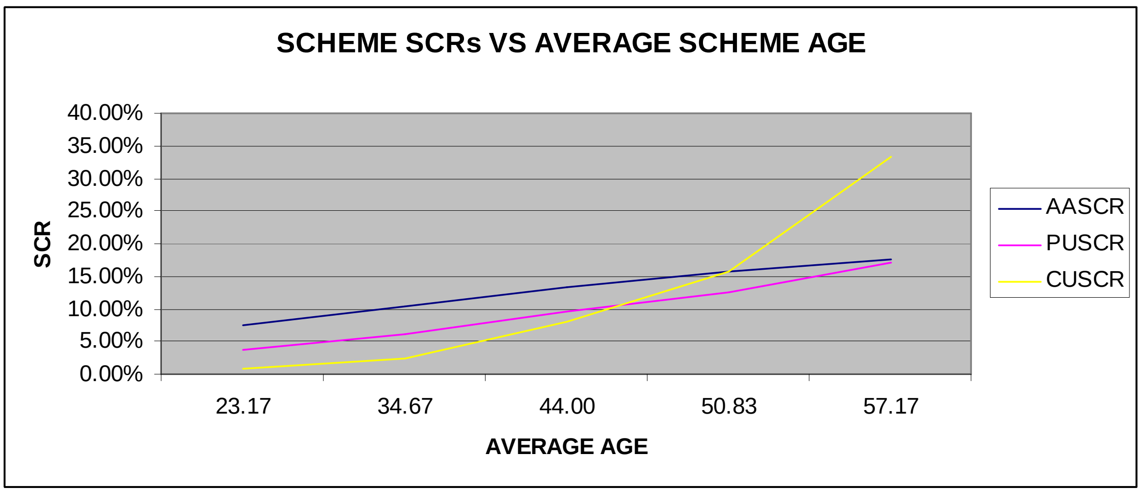

Figure 4.3 Scheme Standard Contribution Rates versus Average Scheme Age

Remarks

From the results obtained, it is observed that the scheme SCRs determined using the AAM, PUM and CUM is higher for schemes with higher average age than those with lower average age.

The results are consistent with the behaviour of individual members' SRCs vis-à-vis age.

In general, the individual SCRs and overall scheme SCRs increase with individual member's age and average scheme age respectively.

4.5 Comparison of SCRs of the different funding methods

The graphs of individual members' SCRs vis-à-vis age for AAM, EAM, PUM and CUM is plotted below:

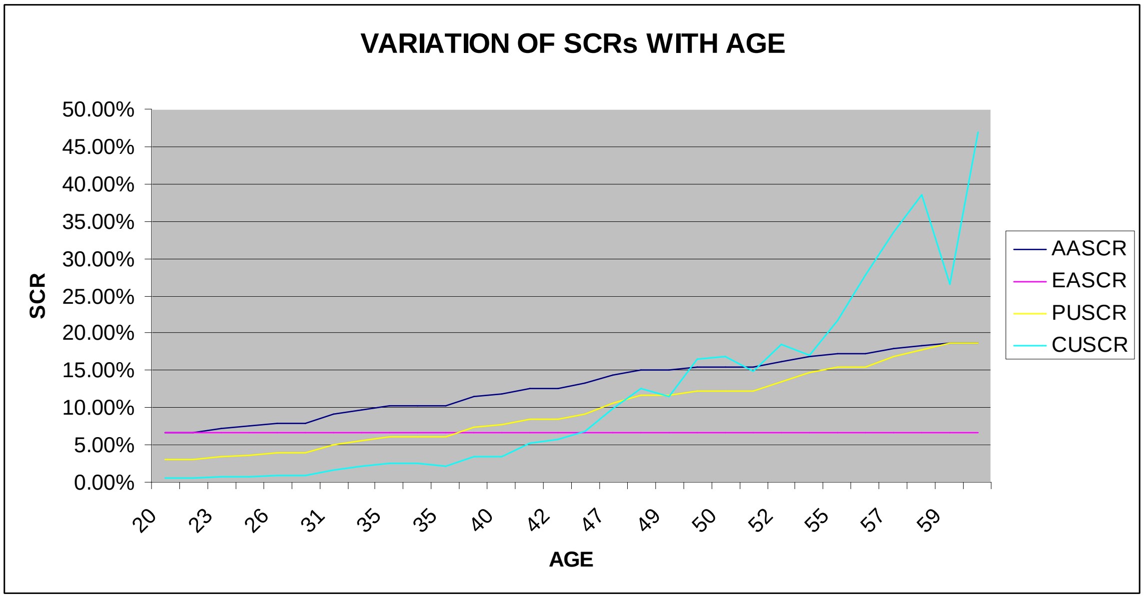

Figure 4.4 Variation of Standard Contribution Rates with Age

Source: Model Pension Scheme (Appendix III-A)

From the graphs, the following relationships are observed:

4.5.1 $\mathrm{SCR}_{\mathrm{AAM}}$ vs. $\mathrm{SCR}_{\mathrm{EAM}}$

The $\mathrm{SCR}_{\mathrm{AAM}}$ and the $\mathrm{SCR}_{\mathrm{EAM}}$ start at the same level, and then the $\mathrm{SCR}_{\mathrm{AAM}}$ increases progressively with age while the $\mathrm{SCR}_{\mathrm{EAM}}$ remains constant throughout.

From the previous results of **4.3.1**, it was shown that except for the EAM, SCRs of AAM, PUM and CUM increase with age.

It is not always true that the SCR_{AAM} and the SCR_{EAM} will start at the same level. This is because the SCR_{EAM} depends on the choice of the assumed entry age which in this case happened to be the age of the youngest scheme member. And when the assumed entry age equals the exact age of a member, then the SCR of the member under AAM and EAM is the same. This is evident from the SCR formulae (for i^{th} individual member) as shown below:

$$SCR_{AAM_i} = \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times \overline{a}_{R_i}}{S_i \, \overline{a}^{*}_{x_i:\overline{R-x_i|}}} \tag{4.9}$$

$$SCR_{EAM_i} = \frac{\frac{1}{PAF} \times (R - x_0) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_0)} \times \overline{a}_{R_i}}{S_i \, \overline{a}^{*}_{x_0:\overline{R-x_0|}}} \tag{4.10}$$

And if $x_0 = x_i$ then it is clear that $SCR_{AAM_i} = SCR_{EAM_i}$ (4.11)

In addition, the following results hold:

If $x_0 < x_i$, then $SCR_{AAM_i} > SCR_{EAM_i}$ (4.12)

and

If $x_0 > x_i$, then $SCR_{AAM_i} < SCR_{EAM_i}$ (4.13)

Comparable results hold for overall scheme SCRs when the assumed entry age x_0 is compared with the average age of scheme members $\overline{x}$ i.e.

If $x_0 = \overline{x}$ then $SCR_{AAM} \cong SCR_{EAM}$ (4.16)

If $x_0 < \bar{x}$, then $SCR_{AAM} > SCR_{EAM}$ (4.15)

And

If $x_0 > \bar{x}$, then $SCR_{AAM} < SCR_{EAM}$ (4.16)

4.5.2 SCR_{AAM} vs. SCR_{PUM}

From the graph it is observed that the SCR_{AAM} is higher than the SCR_{PUM} at all ages until at or near retirement age where the two meet.

This relationship can be proved mathematically by determining the ratio of the formulae of the two SCRs at age *x* for a member. This is as follows:

$$\frac{SCR_{AAM}}{SCR_{PUM}} = \frac{\frac{1}{PAF}\times(R-x)\times S \times\left(\frac{1+j}{1+i)}\right)^{(R-x)}\times\overline{a_R}}{\overline{a^*_{x:\overline{R-x|}}}} \div \frac{\frac{1}{PAF}\times S\times\left(\frac{1+j}{1+i)}\right)^{(R-x)}\times\overline{a_R}}{S\overline{a^*_{x:\overline{1|}}}}$$

$$= \frac{(R-x)\times\overline{a^*_{x:\overline{1|}}}}{\overline{a^*_{x:\overline{R-x|}}}} > 1 \text{ For } (R-x)>1 \tag{4.17}$$

And

$$= \frac{(R-x)\times\overline{a^*_{x:\overline{1|}}}}{\overline{a^*_{x:\overline{R-x|}}}} = 1 \text{ For } (R-x)=1 \tag{4.18}$$

From the simplified formula above it is observed that the SCR_{AAM} equals SCR_{PUM} at one year before retirement i.e. when $(R-x)=1$

4.5.3 SCR_{AAM} vs. SCR_{CUM}

From the graph it is observed that the SCR_{CUM} starts off at a very low level but ends up at a very high level when compared to the SCR_{AAM}.

At younger ages, this observation is consistent with normal expectation since SCR_{AAM} considers future earnings growth till retirement while SCR_{CUM} considers earnings growth in the next year and therefore SCR_{CUM} should be lower than SCR_{AAM}.

However, the reverse effect in older ages is attributed to the element of the SCR_{CUM} arising from the revaluation of the AL_{CUM}. At younger ages, the AL_{CUM} is small due to few or no past service years and the fact that the accrued benefits are discounted over a longer period. However, the AL_{CUM} increases rapidly with age due to increase in past service years and discounting accrued benefits over a shorter period as retirement age approaches. Therefore, at younger ages, the SCR_{CUM} component arising from revaluation of the AL_{CUM} is very low while at older ages it is very high.

Consequently, at lower ages characterized by few or no past service years, the impact of the revaluation of the AL_{CUM} on the SCR_{CUM} is not significant and hence $SCR_{CUM} <$ SCR_{AAM}. However, at older ages characterized by many past service years, the impact of the revaluation of the AL_{CUM} on the SCR_{CUM} is very significant such that $SCR_{CUM} >$ SCR_{AAM}.

This result can also be proved mathematically by finding the ratio of the two SCRs at age *x* for a member. This is as follows:

$$\frac{SCR_{CUM}}{SCR_{AAM}} = \frac{\frac{1}{PAF} \times S \times \left(\frac{1}{1+i)}\right)^{(R-x)} \times (1+j) \times \overline{a_R} + AL_{CUM} \times j}{\overline{a^*_{x:\overline{1}|}}} \div \frac{\frac{1}{PAF} \times (R-x) \times S \times \left(\frac{1+j}{1+i}\right)^{(R-x)} \times \overline{a_R}}{S\overline{a^*_{x:\overline{R-x}|}}} \tag{4.19}$$

Substituting AL_{CUM} with:

$$AL_{CUM} = \frac{1}{PAF} \times P \times S \times \left(\frac{1}{1+i}\right)^{(R-x)} \times \overline{a}_R$$

And simplifying we obtain the simplified formula as:

$$\frac{SCR_{CUM}}{SCR_{AAM}} = \left[\frac{1}{(1+j)^{R-x-1}} + \frac{Pj}{(R-x)(1+j)^{R-x}}\right] \times \frac{\overline{a^*}_{x:\overline{R-x|}}}{(R-x)\overline{a^*}_{x:\overline{1|}}} \tag{4.20}$$

From the formula we note that the ratio increases with age x and past service years P. At one year before retirement the ratio simplifies to:

$$\frac{SCR_{CUM}}{SCR_{AAM}} = \left[1 + \frac{Pj}{1+j}\right] \tag{4.21}$$

We can test check the formula by applying it to a member of the model scheme aged 59 with 32 past service years. From the table attached in appendix 1, the member's SCR_{CUM}= 46.90% and SCR_{AAM}= 18.58%. This gives a ratio of 2.52.

Applying the simplified formula above we get:

$$\frac{SCR_{CUM}}{SCR_{AAM}} = \left[1 + \frac{32 \times 0.05}{1.05}\right] = \frac{2.65}{1.05} = 2.52$$

This is equal to the result obtained by directly dividing the computed SCRs.

From the simplified formula above, we can deduce that a member remaining with one year to retirement and no past service $(P = 0)$ will have the SCR_{CUM} equal to the SCR_{AAM} as would be expected.

From the graph it is also noted that SCR_{CUM} does not progressively increase with age alone but also depends on the level of past service. The graph will progressively increase with age provided past service is also increasing, otherwise the graph becomes irregular. This is illustrated below:

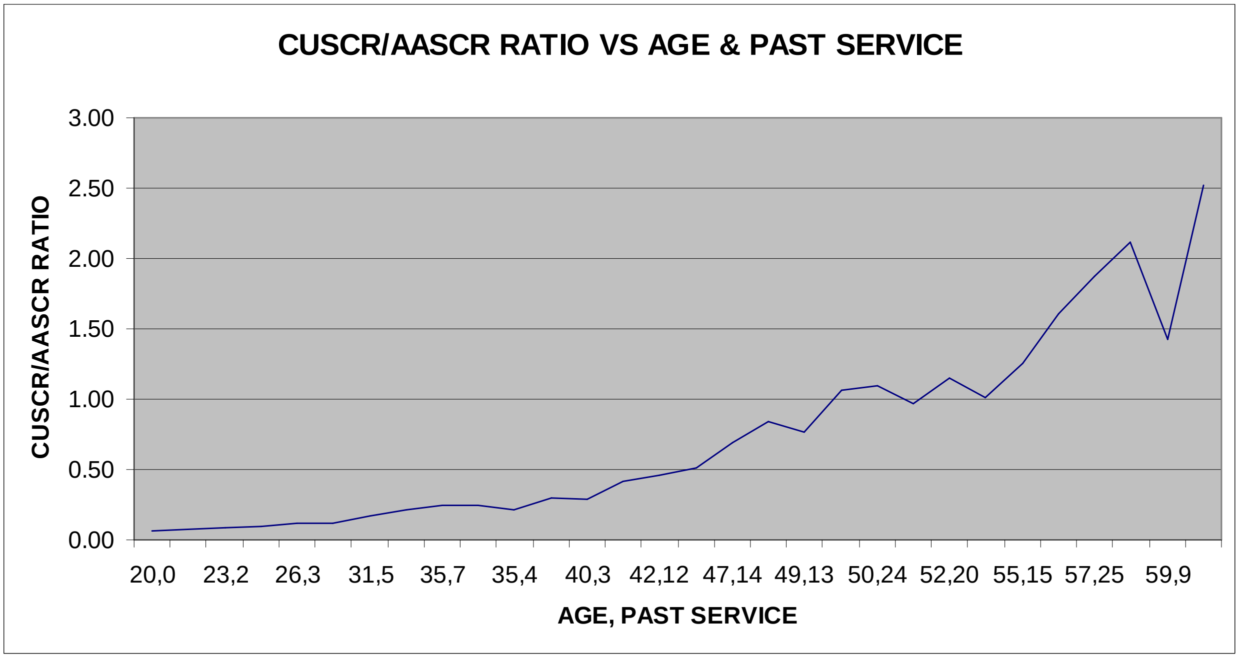

Figure 4.5 Variation of CUSCR/AASCR Ratio with Age and Past Service

Source: Model Pension Scheme (Appendix V)

Remarks

From the graph above it is observed that the graph increases smoothly provided subsequent ages and past service years are higher than preceding values.

However, the graph takes irregular shape if the subsequent past service years corresponding to the high age values are lower than the preceding values corresponding to low age values. This scenario is likely when older members join an existing scheme with younger members.

4.5.4 SCR_{PUM} vs. SCR_{CUM}

The graph shows that the SCR_{CUM} starts off at a very low level and end up at a very high level compared to the $SCR_{PUM.}$

We recall that the SCR_{CUM} is the sum of two elements namely: the accrual bit for calculating benefits accrued over the next year, and the revaluation bit for calculating the level of increase of past service accrued benefits due to increase in earnings in the next year.

The formula for the accrual bit is given by:

$$\frac{\frac{1}{PAF}\times S \times\left(\frac{1}{1+i}\right)^{(R-x)}\times(1+j)\times\overline{a_R}}{\overline{a^*_{x:\overline{1|}}}}$$

The formula for the revaluation bit is given by:

$$\frac{1}{PAF}\times P\times S\times\left(\frac{1}{1+i}\right)^{(R-x)}\times\overline{a_R}\times j$$

When we compare the SCR_{PUM} with the accrual bit of SCR_{CUM} we note that the SCR_{PUM} will always be higher due to the link with projected earnings till retirement. However, this is only true at younger ages and the reverse happens at older ages.

At younger ages, the revaluation bit is small due to small AL_{CUM} occasioned by few past service years, non-consideration of future earnings growth and discounting over longer period. Therefore, the revaluation bit does not have significant effect on the value of SCR_{CUM}. Consequently the SCR_{CUM} is lower than the SCR_{PUM} at younger ages.

At older ages, the revaluation bit gets larger as accrued benefits increase and significantly increases the value of $SCR_{CUM.}$ Near retirement age, the revaluation bit is by far the dominant part of the total SCR_{CUM} and results to SCR_{CUM} that is much higher than the $SCR_{PUM.}$

This result can be explained by calculating the ratio of the two SCRs. Applying the same procedures used in the preceding sections, the simplified ratio is obtained as:

$$\frac{SCR_{CUM}}{SCR_{PUM}}=\left[\frac{1+j(1+P)}{(1+j)^{R-x}}\right] \tag{4.22}$$

Remarks

From the ratio, it can be deduced that at younger ages, $\frac{SCR_{CUM}}{SCR_{PUM}}<1$ and at older ages $\frac{SCR_{CUM}}{SCR_{PUM}}>1$ provided P is increasing with age.

At one year to retirement and no past service years, the formula simplifies to:

$$\frac{SCR_{CUM}}{SCR_{PUM}}=\left[\frac{1+j}{1+j}\right]=1 \tag{4.23}$$

This means that an employee joining a scheme at one year before retirement will have the same SCR under the CUM and PUM.

4.5.5 SCR_{EAM} vs. SCR_{PUM} and SCR_{CUM}

From the graph it is observed that the SCR_{EAM} remains constant throughout all ages while the SCR_{PUM} and SCR_{CUM} start off low and increase progressively with age.

These results are obvious from explanations given in the preceding sections i.e. the SCR_{EAM} is based on a single assumed entry age while the SCR_{PUM} and SCR_{CUM} are based on age of individual members.

General Remark

The differences noted in the standard contribution rates determined using the four actuarial funding methods affect the timing of the contributions also referred as the 'pace of funding'.

However, the standard contributions rates, regardless of the underlying method, will ultimately result to the same required fund value at the future date of expected pension payment.

4.6 Determination of actuarial liability

The actuarial liabilities for the model scheme, under each of the four methods, are obtained using the following formulae:

$$AL_{AAM} = \sum_{i}^{N} \frac{1}{PAF} \times (P_i + R - x_i) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i} \quad minus \quad \sum_{i}^{N} SCR_i \times S_i \times \overline{a^{*}_{\overline{x_i:R-x_i|}}} \qquad (4.24)$$

$$= \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i}$$

$$AL_{EAM} = \sum_{i}^{N} \frac{1}{PAF} \times (P_i + R - x_i) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i} \quad minus \quad \sum_{i}^{N} SCR_i \times S_i \times \overline{a^{*}_{\overline{x_i:R-x_i|}}} \qquad (4.25)$$

$$AL_{PUM} = \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times a_{R_i} \qquad (4.26)$$

$$AL_{CUM} = \sum_{i}^{N} \frac{1}{PAF} \times P_i \times S_i \times \left(\frac{1}{1+i} \right)^{(R-x_i)} \times a_{R_i} \tag{4.27}$$

Where:

$i = 0.10$, $j = 0.05$, $i^* = \frac{i-j}{1+j} = 0.047619047$, $R = 60$, $x_0 = 20$, $PAF = 40$

$\overline{a_{R_i}} = \sum_{t=0}^{100-R_i-1} v^{t+\frac{1}{2}} \frac{l_{R_i+t+\frac{1}{2}}}{l_{R_i}} @i$, *A1949-52 ultimate* and $\overline{a^*_{x_i:\overline{R-x_i|}}} = \sum_{t=0}^{R-x_i} v^{t+\frac{1}{2}} \frac{l_{x_i+t+\frac{1}{2}}}{l_{x_i}} @i^*$, *A1949-52 ultimate*.

From the formulae above, we notice that the formulae for the actuarial liability under the Attained Age Method (4.24) and the Projected Unit Method (4.26) are similar. Consequently, we conclude that;

$$AL_{AAM} = AL_{PUM} \tag{4.28}$$

Applying the above formulae on the data of the model scheme we obtain the following results:

FUNDING METHOD	**STANDARD CONTRIBUTION RATE (%)**	**ACTUARIAL LIABILITY (KSHS)**
AAM	12.39%	44,790,405.68
PUM	11.71%	44,790,405.68
EAM	6.62%	54,968,737.04
CUM	16.74%	34,402,387.03

Source: Model Pension Scheme (Appendix IV)

Remarks

From the above results we notice an inverse relationship between the standard contribution rates computed for the different funding methods and the corresponding actuarial liability i.e. when the standard contribution rate is high, the actuarial liability is low and vice versa.

It is also important to note that the actuarial liabilities determined under the AAM and PUM are equal although the standard contribution rates are differing (even if by a small margin). The SCR_{AAM} (12.39%) is greater than SCR_{PUM} (11.71%) and the status quo is expected to remain for all average ages and will be equal at average age of one year to retirement age. Considering that the SCR_{PUM} is the amount required year by year to keep the fund equal to the $AL_{PUM,}$ then paying the $SCR_{AAM} > SCR_{PUM}$ will result to a surplus against the AL definition.

4.7 Variation of Actuarial Liability with Age

The graph below shows the behaviour of the actuarial liability for each of the funding methods at different ages and past service.

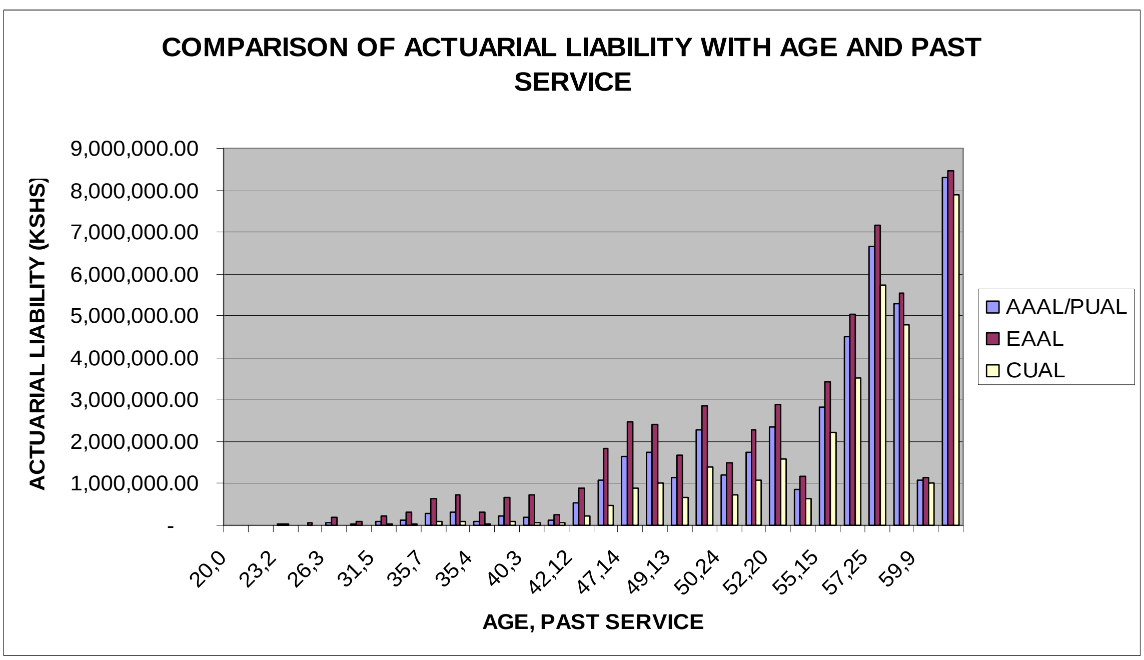

Figure 4.6 Comparison of Actuarial Liability with Age and Past Service
Source: Model Pension Scheme (Appendix IV)

Remarks

From the graph, we note that the actuarial liability increases with age and past service.

This is consistent with what is expected since the old are expected to have been in service for longer periods than the young.

4.8 Comparison of Actuarial Liability across Funding Methods

From the table and the graph above, we observe that:

i. The Actuarial Liability for AAM and PUM are equal
ii. The EAM gives the highest Actuarial Liability
iii. The CUM gives the lowest Actuarial Liability

This observation can be arranged in descending order as follows:

$$AL_{EAM} > AL_{AAM} = AL_{PUM} > AL_{CUM} \qquad (4.29)$$

This result is attributed to the inverse relationship between the standard contribution rates and the corresponding actuarial liabilities. i.e.

i. When the prevailing standard contribution rate is low, more funds should be set aside now to meet future benefit payments since the current contributions may not be enough.

ii. When the prevailing standard contribution rate is high, the funds to be set aside now to build the required fund are relatively less as the high contribution rate will provide more funds for future benefits outgo.

Also, from the individual definitions of the actuarial liabilities we note that:

i. The definition for the actuarial liability under the AAM and PUM is the same and hence the equality i.e. the present value of all benefits accrued at valuation date based on projected final earnings for the members in service.

ii. The EAM provides a constant standard contribution rate for all members based on an assumed entry age. Where the entry age assumed is less than the average age of scheme members, the resulting SCR will be lower than that expected under the AAM and PUM. In this circumstance, the EAM will give the highest Actuarial Liability i.e. more funds will be required to be set aside now since the low contribution rate may not provide enough funds for the expected future outgo.

iii. The actuarial liability under the CUM does not take into account escalation of members' future earnings and hence the lowest Actuarial Liability.

4.9 Assessment of Funding Methods

4.9.1 Assessment for Stability

Stability is the ability of a funding method to maintain relatively the same contribution pattern from time to time.

The behaviour of a standard contribution rate when applied to a scheme will depend on the long term actuarial assumptions (demographic and financial/economic assumptions) and the profile of the membership in terms of age, sex and salary distribution; and in some cases past service distribution.

Assuming that the long-term actuarial assumptions remain constant, below is an assessment of the funding methods for stability, security, flexibility and realism.

i. Stability of SCR_{PUM}

The formula of the SCR_{PUM} is given by:

$$SCR_{PUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times \overline{a}_{R_i}}{S_i \, \overline{a}^{*}_{x_i:\overline{1}|}}$$

From the formula, we note that the SCR_{PUM} will remain stable from year to year if the membership profile is stable in terms of age, sex and salary distribution.

This therefore implies that for an ongoing scheme the SCR_{PUM} will remain stable from year to year if new entrants replace members who leave, die or retire such that the overall membership profile remains unchanged.

However, if there are no new entrants, the SCR_{PUM} is likely to increase from year to year due to the expected increase in the average age of members.

ii. Stability of SCR_{CUM}

The formula of the SCR_{CUM} is given by:

$$SCR_{CUM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times S_i \times (1+j) \times v^{(R-x_i)} \times \overline{a}_{R_i} + AL_{CUM} \times j}{S_i \overline{a}^{*}_{x_i:\overline{1}|}}$$

From the formula, we construe that the SCR_{CUM} of a scheme will remain stable from year to year provided that the age, sex, salary and past service distribution of the membership remains stable.

The past service stability requirement is due to the need for the incorporation of the component for revaluation of actuarial liability when determining the SCR.

This implies that for an ongoing scheme, new members join in a manner such that the age, sex, salary and past service distribution of active membership remains constant.

iii. Stability of SCR_{AAM}

The formula of the SCR_{AAM} is given by:

$$SCR_{AAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_i) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_i)} \times \overline{a}_{R_i}}{S_i \overline{a}^{*}_{x_i:\overline{R-x_i}|}}$$

From the formula, we deduce that the SCR_{AAM} of a scheme will remain stable from year to year if the age, sex and salary distribution of the membership remains stable.

The SCR_{AAM} will also remain stable from year to year for a closed scheme (i.e. scheme closed to new entrants) if the calculation is performed once at the date of closure.

iv. Stability of SCR_{EAM}

The formula of the SCR_{EAM} is given by:

$$SCR_{EAM} = \sum_{i}^{N} \frac{\frac{1}{PAF} \times (R - x_0) \times S_i \times \left(\frac{1+j}{1+i}\right)^{(R-x_0)} \times \overline{a}_{R\,i}}{S_i \overline{a}^{*}_{x_0:\overline{R-x_0}|}}$$

From the formula, we infer that the SCR_{EAM} will remain stable from year to year if and only if the entry age assumptions remain unchanged.

General remark

In the real world, SCRs rarely remain stable because actual experience, especially financial and economic experience, rarely if ever follows the assumed parameters and actuarial assumptions.

4.8.2 Assessment for Security

The security of a funding method is defined as the ability of a funding method to ensure that there will be sufficient assets to meet the scheme's liabilities.

All other things being equal and assuming money is actually put into the fund, the funding method that provides the largest target fund/actuarial liability will provide the greatest security.

From the results of the model scheme analysed herein, we noted that for a given set of parameter (with $i > j$);

i. The Actuarial Liability for the Entry Age Method will exceed that for the Attained Age Method, provided that the assumed entry age is lower than the weighted average age of the membership;

ii. The Actuarial Liability for the Attained Age Method is equal to that of the Projected Unit Method;

iii. The Actuarial Liability for the Projected Unit Method will exceed that for the Current Unit Method.

Based on these results we can arrange the Actuarial Liabilities arising from the four funding methods in descending order (i.e. from largest to smallest) as follows:

$$AL_{EAM} > AL_{AAM} = AL_{PUM} > AL_{CUM}$$

Therefore, based on these results, we can conclude that the Entry Age Method provides the greatest security and the Current Unit Method provides the least security.

4.8.3 Assessment for Flexibility

A funding method is said to be flexible if it allows the employer to make the best use of his finances (i.e. maximise returns).

This is achieved when the funding method allows flexibility in employer contributions. Flexibility in employer contributions is desirable because it is likely to motivate the employer to continue with sponsorship of the scheme.

If flexibility results in the employer being able to achieve a better overall return on his money, then the long term cost to the employer of providing the scheme benefits will be lower. This might ensure that the sponsorship of the scheme continues or might result in benefits being improved.

Flexibility is best achieved by using a method that targets a good level of security whilst not running a large risk of breaching any statutory maximum level. Having a good level of security is important because if an employer needs to temporarily reduce contributions he or she is paying to the scheme, this should be possible without reducing the funding level below a satisfactory level.

Therefore, funding methods that ensure the greatest security provide the greatest flexibility and vice versa. In this regard, using a realistic set of parameters, the Current Unit Method is unlikely to enable flexibility due to the low level of security and hence high contribution requirement.

On the other end, the Entry Age Method provides the greatest security and therefore the greatest expected flexibility. However, the method has the greatest risk of surplus funds which may surpass statutory maxima and this may lead to enforced actions. If legislation exists to stop schemes from holding excessive assets (in relation to liabilities), the employer may be forced to dispose of the surplus funds rather than returning the extra money in the scheme to allow reduced contributions in future. In this case, the use of Projected Unit Method and the Attained Age Method is less likely to result in breach of statutory minima or maxima. The flexibility may therefore be greater than under either of the other two methods.

An employer could also aim to build up assets well in excess of the Actuarial Liability implied by the different funding methods and could use the extra reserve to fund contribution flexibility later.

In practice, except in cases that are tightly regulated or where scheme rules are very specific, the choice of parameters can make any of the methods very flexible. The apparent willingness of the sponsor and trustees to run significant deficits in their funding plans for extended periods of time means that the sponsors are less constrained than the funding method may sound. Thus, although a funding plan is being targeted, there may be a great deal of flexibility about how and when the target fund will be met.

4.8.4 Assessment for Realism

A funding method is realistic if its underlying assumptions are likely to be met in practice, otherwise it is unrealistic.

All the funding methods are unrealistic to a greater or lesser extent because some of the implicit assumptions are not borne out in practice except where the approach used for setting assumptions deliberately counteracts market movements.

In particular, the interest rates used to underpin discount rates and the values of the assets relative to the liabilities vary significantly.

In assessing the extent to which a funding method is realistic, we consider the extent to which the underlying assumptions conform to actual experience.

For example the Projected Unit Method would not give a realistic assessment of ongoing cost for a closed scheme. This is because the cost of accrual increases with age (if $i > j$).

The SCR_{PUM} calculates an average contribution rate applicable to the cost of accrual of the members' benefits over the year following the valuation date. In a closed scheme the age of the membership will rise from year to year and hence the SCR will rise at each successive valuation. Therefore, at any valuation date, the calculated contribution rate using the PUM method will understate the cost of future accrual since it is only valid in the following year which will apparently be the cheapest year. In this case of a closed scheme, the Attained Age Method will be more realistic and therefore preferred.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.0 Discussion

The overall objective of the study was to analyse the four main actuarial methods of funding pension schemes namely: Attained Age Method (AAM), Projected Unit Method (PUM), Entry Age Method (EAM), Current Unit Method (CUM); and evaluating the extent to which each of them satisfied the qualities of security, stability, realism and flexibility and recommend suitable funding methods for different forms of funded defined benefit pension schemes.

The analysis involved:

i. Determining the standard contribution rate for each of the funding methods.
ii. Determining the actuarial liability for each of the funding methods.
iii. Establishing the behaviour of the standard contribution rates and actuarial liabilities when valuation parameters (particularly age) are varied.
iv. Comparing the standard contribution rates and actuarial liabilities across the funding methods and establishing the relationship amongst them.
v. Evaluating the results obtained for the extent to which they satisfied the qualities of security, stability, flexibility and realism.

It is worth noting that the difference in the four funding methods is the timing of the contributions but the fundamental long-term cost is the same.

Based on the results obtained from the analysis of the ***Model Pension Scheme*** data as detailed in ***Section 4***, the following conclusions and recommendations have been made:

5.1 Conclusions

1. Except for the EAM, actual SCRs are not straight simple averages of individual members' SCRs but are weighted by members' earnings.

2. Individual SCRs and overall scheme SCRs increase with individual members' age and average scheme age respectively.

3. SCR_{EAM} is constant throughout all ages and how it compares with SCRs of other funding methods is dependent on the assumed entry age.

SCR_{AAM} is always higher than SCR_{PUM} at all ages until one year to retirement age.

SCR_{CUM} starts very low (at younger ages) compared to SCR_{AAM} and SCR_{PUM} but ends up very high (at older ages).

At younger ages we have:

$SCR_{AAM} > SCR_{PUM} > SCR_{CUM}$

At older ages we have:

$SCR_{CUM} > SCR_{AAM} > SCR_{PUM}$

4. An inverse relationship exists between the SCRs computed for the different funding methods and the corresponding actuarial liabilities i.e. when the SCR is high, the actuarial liability is low and vice versa. This gives rise to what is called the "High/ Low" rule which states that if a method has a higher Actuarial Liability than another, then in the long-term the SCR must be lower i.e. when large funds are held now, smaller contributions will be received in future.

5. The Actuarial Liability increases with age and past service i.e. the cost of accrual increases with age and past service. This implies that the actuarial liability attributable to older members is greater than that attributable to younger members.

6. The Actuarial Liability for AAM equals that of PUM.

The EAM gives the highest Actuarial Liability while the CUM gives the lowest Actuarial Liability.

These results when arranged in descending order are as follows:

$AL_{EAM} > AL_{AAM} = AL_{PUM} > AL_{CUM}$

7. Stability of a funding method refers to the ability of the funding method to maintain the same contribution pattern from year to year. This is dependent on the underlying long term actuarial assumptions (demographic, financial and economic assumptions) remaining consistent with actual experience and membership profile (i.e. age, sex, salary and past service) remaining unchanged.

Because experience, especially financial and economic experience rarely follows the parameters set, the SCRs will rarely be stable.

However, a scheme that aims to achieve relative stability should adopt objective and realistic methods of determining and setting assumptions that are likely to follow actual experience.

8. Security of a funding method refers to the ability of a funding method to ensure that there will be sufficient assets to meet the scheme's liabilities.

The funding method that provides the largest actuarial liability will provide the greatest security and vice versa.

All things being constant, EAM will provide the greatest security and CUM the least security provided that the assumed entry age is lower than the weighted average age of the membership.

9. Flexibility of a funding method is achieved when the funding method gives the employer room to make the best use of his finances and ultimately enable him maximise returns.

A funding method that is flexible will allow the employer to vary contributions (usually by reducing) at some point in time without leading to undesirable effects of under-funding to the extent of even going below the set statutory minima (if any).

Flexibility is best achieved by using a method that targets a good level of security whilst not running a large risk of breaching the set statutory maximum level (if any).

The EAM provides the greatest security and hence expected flexibility; but faces the risk of accumulation of excess surplus funds that may breach some set statutory maxima (if any) which may lead to enforced actions of disposal.

The CUM provides the least security and therefore has the least flexibility due to the low target fund and any reduction in the contributions is likely to breach the set statutory minimum which exists in most cases.

The PUM and the AAM provide moderate security and hence moderate flexibility and are less likely to breach the set statutory maxima or minima.

An employer could also enhance flexibility of the funding methods by building up assets well in excess of the actuarial liability implied by the funding methods and could use the extra reserve to fund contribution flexibility later.

10. Realism of a funding method is achieved when the underlying assumptions of the funding method are likely to be met in practice.

All funding methods are unrealistic to some extent because some of the implicit assumptions are rarely borne in practise except where the approach used for setting assumptions deliberately take into account market conditions.

5.2 Recommendations

1. Ongoing schemes that intend to ensure stability of contributions in the long term should ensure that a balance is maintained between entrants and exits such that the overall membership profile (i.e. age, sex, salary and past service distribution) remains relatively constant from year to year.

Closed schemes that intend to ensure stability of contributions in the long term should determine the Standard Contribution Rate using the Attained Age Method and this rate should be calculated once at the time of closure.

2. The level of security of a funding method adopted in funding a scheme is determined by the extent in which the resultant accumulated scheme assets match the accrued scheme liabilities.

The accumulated assets at a point of time are supposed to match the accrued liabilities commonly referred as the actuarial liability. Therefore, the actuarial liability determined based on any of the four funding methods provides guidance to the amount of funds to be set aside to fully meet scheme benefits accrued in respect of service up to that point of time.

Therefore, schemes that intend to enjoy a great level of security (in terms of funding level) should adopt the funding method that establishes the largest actuarial liability at a given point of time.

In terms of the level of security, the funding methods are ranked as follows in descending order:

i. Entry Age Method
ii. Attained Age Method and Projected Unit Method
iii. Current Unit Method

That is:

$$\mathbf{AL_{EAM} > AL_{AAM} = AL_{PUM} > AL_{CUM}}$$

3. The extent of flexibility in a scheme particularly in terms of payment of contributions is determined by the level of security. A scheme with a high level of security has high flexibility and vice versa.

Therefore, where the sponsor expects to have some flexibility in payment of contributions in future, the scheme should adopt a funding method that ensures considerable security through considerable build up of assets at present. This basically means adoption of a funding method that establishes the largest actuarial liability.

Alternatively, the sponsor may choose to build excess assets over and above those implied by the actuarial liabilities determined by the various funding methods so as to provide cushion in future years when there may be forced reduction in contributions say, due to financial difficulties. However, the present build up of excess assets should not breach statutory maximum limits.

Therefore, in terms of ensuring flexibility into the future, the ranking of the funding methods in descending order is:

i. Entry Age Method
ii. Attained Age Method and Projected Unit Method
iii. Current Unit Method

4. Realism in funding methods is achieved through the use of actuarial assumptions which comprise financial, economic and demographic assumptions, that will likely be met and be reflective of the actual experience.

Since future occurrences are shrouded in uncertainty, it is difficult to establish actuarial assumptions that will exactly match the actual experience. However, the use of statistical and analytical tools can to a great extent provided an objective way of determining and setting reliable estimates of various actuarial parameters that can be relied on and which are not likely to significantly deviate from the actual experience.

Therefore, schemes should adopt objective techniques of establishing actuarial assumptions by taking into account prevailing market conditions and demographic experience.

In conclusion, the choice of actuarial funding methods should ideally maintain a balance amongst the need for stability, security, flexibility and realism. Therefore, one factor should not be used as the sole determinant (while ignoring the others) in determining the funding method to be adopted but rather carry out a trade-off amongst all the factors and adopt a method that is most suited in the circumstances.

The choice of a funding method should also maintain a balance of both the members' and sponsor's interests. While the members may need sufficient security, the sponsor may prefer considerable flexibility even when the scheme's funding level is below the statutory minimum. It is for this reason that in many countries, the sponsors are allowed a reasonable period of time (e.g. 6 years, for Kenya) to settle actuarial deficits.

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APPENDIX I: MEMBERSHIP DATA OF MODEL PENSION SCHEME

COUNT	AGE	CURRENT MONTHLY SALARY	CURRENT ANNUALISED SALARY	PAST SERVICE YEARS
1	20	15,000	180,000	0
2	20	20,000	240,000	1
3	23	20,000	240,000	2
4	24	25,000	300,000	1
5	26	45,000	540,000	3
6	26	25,000	300,000	3
7	31	30,000	360,000	5
8	33	30,000	360,000	7
9	35	55,000	660,000	7
10	35	65,000	780,000	7
11	35	35,000	420,000	4
12	39	55,000	660,000	5
13	40	70,000	840,000	3
14	42	15,000	180,000	9
15	42	45,000	540,000	12
16	44	90,000	1,080,000	11
17	47	95,000	1,140,000	14
18	49	80,000	960,000	16
19	49	65,000	780,000	13
20	50	70,000	840,000	23
21	50	35,000	420,000	24
22	50	65,000	780,000	19
23	52	75,000	900,000	20
24	54	50,000	600,000	10
25	55	105,000	1,260,000	15
26	55	100,000	1,200,000	25
27	57	135,000	1,620,000	25
28	58	95,000	1,140,000	27
29	59	55,000	660,000	9
30	59	120,000	1,440,000	32

APPENDIX II: A1949-52 ULTIMATE

At i = 10%

$Age\ \ x$	q_x	p_x	l_x	$D_x = v^x l_x$	$D_{x+\frac{1}{2}} = v^{x+\frac{1}{2}} l_{x+\frac{1}{2}}$	$\overline{N} = \sum D_{x+\frac{1}{2}}$	$\overline{a} = \overline{N} / D_x$
10	0.00111	0.99889	100,000	38,554.3	36,739.71	398,143.65	10.3268
11	0.00111	0.99889	99,889	35,010.5	33,362.66	361,403.95	10.3227
12	0.00111	0.99889	99,778	31,792.4	30,296.03	328,041.28	10.3182
13	0.00111	0.99889	99,667	28,870.1	27,511.27	297,745.26	10.3133
14	0.00111	0.99889	99,557	26,216.4	24,982.48	270,233.99	10.3078
15	0.00111	0.99889	99,446	23,806.6	22,686.14	245,251.50	10.3018
16	0.00111	0.99889	99,336	21,618.4	20,600.87	222,565.36	10.2952
17	0.00111	0.99889	99,226	19,631.3	18,707.28	201,964.49	10.2879
18	0.00111	0.99889	99,115	17,826.8	16,987.74	183,257.22	10.2799
19	0.00111	0.99889	99,005	16,188.2	15,426.26	166,269.48	10.2710
20	0.00111	0.99889	98,896	14,700.2	14,008.30	150,843.22	10.2613
21	0.00111	0.99889	98,786	13,349.0	12,720.68	136,834.92	10.2506
22	0.00111	0.99889	98,676	12,122.0	11,551.42	124,114.23	10.2388
23	0.00112	0.99888	98,567	11,007.7	10,489.58	112,562.81	10.2258
24	0.00112	0.99888	98,456	9,995.8	9,525.31	102,073.23	10.2116
25	0.00112	0.99888	98,346	9,076.9	8,649.67	92,547.92	10.1959
26	0.00113	0.99887	98,236	8,242.5	7,854.49	83,898.25	10.1787
27	0.00113	0.99887	98,125	7,484.7	7,132.38	76,043.76	10.1599
28	0.00114	0.99886	98,014	6,796.6	6,476.62	68,911.39	10.1391
29	0.00115	0.99885	97,902	6,171.7	5,881.09	62,434.77	10.1163
30	0.00116	0.99884	97,790	5,604.2	5,340.27	56,553.67	10.0913
31	0.00118	0.99882	97,676	5,088.8	4,849.11	51,213.40	10.0639
32	0.00120	0.99880	97,561	4,620.7	4,403.04	46,364.28	10.0340
33	0.00123	0.99877	97,444	4,195.6	3,997.90	41,961.24	10.0012
34	0.00127	0.99873	97,324	3,809.5	3,629.91	37,963.34	9.9654
35	0.00132	0.99868	97,200	3,458.8	3,295.65	34,333.43	9.9264
36	0.00139	0.99861	97,072	3,140.2	2,991.98	31,037.78	9.8840
37	0.00147	0.99853	96,937	2,850.8	2,716.10	28,045.80	9.8380
38	0.00158	0.99842	96,795	2,587.8	2,465.41	25,329.70	9.7882
39	0.00171	0.99829	96,642	2,348.8	2,237.60	22,864.29	9.7344
40	0.00188	0.99812	96,476	2,131.6	2,030.53	20,626.70	9.6764
41	0.00208	0.99792	96,295	1,934.2	1,842.28	18,596.17	9.6143
42	0.00231	0.99769	96,095	1,754.7	1,671.12	16,753.89	9.5479
43	0.00259	0.99741	95,873	1,591.5	1,515.48	15,082.76	9.4770
44	0.00292	0.99708	95,624	1,443.1	1,373.92	13,567.28	9.4016
45	0.00330	0.99670	95,345	1,308.1	1,245.13	12,193.37	9.3217
46	0.00372	0.99628	95,031	1,185.2	1,127.96	10,948.24	9.2373
47	0.00420	0.99580	94,677	1,073.5	1,021.36	9,820.28	9.1482
48	0.00474	0.99526	94,279	971.8	924.36	8,798.91	9.0544
49	0.00534	0.99466	93,833	879.2	836.09	7,874.55	8.9560
50	0.00599	0.99401	93,331	795.0	755.78	7,038.46	8.8529
51	0.00671	0.99329	92,772	718.4	682.71	6,282.68	8.7449
52	0.00750	0.99250	92,150	648.7	616.24	5,599.97	8.6320
53	0.00837	0.99163	91,459	585.3	555.77	4,983.74	8.5142
54	0.00931	0.99069	90,693	527.7	500.78	4,427.97	8.3914
55	0.01035	0.98965	89,849	475.2	450.78	3,927.19	8.2635
56	0.01148	0.98852	88,919	427.6	405.33	3,476.40	8.1307
57	0.01272	0.98728	87,898	384.2	364.02	3,071.08	7.9927
58	0.01408	0.98592	86,780	344.9	326.50	2,707.05	7.8497
59	0.01557	0.98443	85,558	309.1	292.42	2,380.55	7.7017
60	0.01720	0.98280	84,226	276.6	261.48	2,088.14	7.5487
61	0.01899	0.98101	82,777	247.1	233.41	1,826.66	7.3909
62	0.02096	0.97904	81,205	220.4	207.95	1,593.25	7.2284
63	0.02312	0.97688	79,503	196.2	184.88	1,385.29	7.0615
64	0.02549	0.97451	77,665	174.2	163.99	1,200.41	6.8902
65	0.02810	0.97190	75,686	154.3	145.09	1,036.42	6.7150
66	0.03095	0.96905	73,559	136.4	128.01	891.32	6.5361
67	0.03409	0.96591	71,282	120.1	112.59	763.31	6.3537
68	0.03753	0.96247	68,852	105.5	98.69	650.72	6.1685
69	0.04130	0.95870	66,268	92.3	86.19	552.02	5.9806
70	0.04543	0.95457	63,531	80.4	74.96	465.84	5.7907

APPENDIX II: A1949-52 ULTIMATE

At i = 10%

$Age\ \ x$	q_x	p_x	l_x	$D_x = v^x l_x$	$D_{x+1/2} = v^{x+1/2} l_{x+1/2}$	$\bar{N} = \sum D_{x+1/2}$	$\bar{a} = \bar{N}/D_x$
71	0.04995	0.95005	60,645	69.8	64.90	390.88	5.5992
72	0.05489	0.94511	57,616	60.3	55.91	325.98	5.4065
73	0.06028	0.93972	54,453	51.8	47.90	270.07	5.2133
74	0.06616	0.93384	51,171	44.3	40.80	222.16	5.0201
75	0.07257	0.92743	47,785	37.6	34.52	181.36	4.8273
76	0.07953	0.92047	44,318	31.7	29.00	146.84	4.6357
77	0.08709	0.91291	40,793	26.5	24.17	117.84	4.4458
78	0.09528	0.90472	37,240	22.0	19.98	93.67	4.2580
79	0.10414	0.89586	33,692	18.1	16.35	73.69	4.0731
80	0.11369	0.88631	30,183	14.7	13.25	57.34	3.8915
81	0.12397	0.87603	26,752	11.9	10.62	44.09	3.7136
82	0.13500	0.86500	23,435	9.5	8.41	33.47	3.5400
83	0.14681	0.85319	20,272	7.4	6.57	25.07	3.3711
84	0.15942	0.84058	17,296	5.8	5.06	18.50	3.2073
85	0.17282	0.82718	14,538	4.4	3.84	13.44	3.0489
86	0.18704	0.81296	12,026	3.3	2.86	9.60	2.8961
87	0.20205	0.79795	9,776	2.4	2.10	6.73	2.7491
88	0.21785	0.78215	7,801	1.8	1.51	4.63	2.6082
89	0.23440	0.76560	6,102	1.3	1.06	3.12	2.4732
90	0.25168	0.74832	4,671	0.9	0.73	2.06	2.3441
91	0.26963	0.73037	3,496	0.6	0.49	1.33	2.2206
92	0.28819	0.71181	2,553	0.4	0.32	0.83	2.1020
93	0.30730	0.69270	1,817	0.3	0.21	0.51	1.9873
94	0.32688	0.67312	1,259	0.2	0.13	0.30	1.8743
95	0.34683	0.65317	847	0.1	0.08	0.17	1.7595
96	0.36706	0.63294	553	0.1	0.05	0.10	1.6359
97	0.38747	0.61253	350	0.0	0.03	0.05	1.4902
98	0.40795	0.59205	215	0.0	0.01	0.02	1.2956
99	0.42840	0.57160	127	0.0	0.01	0.01	0.9970
100	1.00000	0.00000	73	0.0	0.00	0.00	0.4767

APPENDIX II: A1949-52 ULTIMATE

At $i^* = \dfrac{i-j}{1+j} = \dfrac{0.10-0.05}{1+0.05} = 0.04762 = 4.762\%$

$Age\ \ x$	q_x	p_x	l_x	$D_x = v^x l_x$	$D_{x+\frac{1}{2}} = v^{x+\frac{1}{2}} l_{x+\frac{1}{2}}$	$\overline{N} = \sum D_{x+\frac{1}{2}}$	$\bar{a} = \overline{N} \big/ D_x$
10	0.00111	0.99889	100,000	62,800.9	61,322.99	1,246,682.50	19.8513
11	0.00111	0.99889	99,889	59,879.8	58,470.61	1,185,359.50	19.7956
12	0.00111	0.99889	99,778	57,094.6	55,750.90	1,126,888.89	19.7372
13	0.00111	0.99889	99,667	54,438.9	53,157.70	1,071,137.99	19.6760
14	0.00111	0.99889	99,557	51,906.7	50,685.12	1,017,980.29	19.6117
15	0.00111	0.99889	99,446	49,492.3	48,327.55	967,295.17	19.5444
16	0.00111	0.99889	99,336	47,190.2	46,079.63	918,967.63	19.4737
17	0.00111	0.99889	99,226	44,995.2	43,936.28	872,887.99	19.3996
18	0.00111	0.99889	99,115	42,902.3	41,892.63	828,951.71	19.3219
19	0.00111	0.99889	99,005	40,906.7	39,944.03	787,059.09	19.2403
20	0.00111	0.99889	98,896	39,004.0	38,086.07	747,115.06	19.1548
21	0.00111	0.99889	98,786	37,189.7	36,314.53	709,028.99	19.0652
22	0.00111	0.99889	98,676	35,459.9	34,625.39	672,714.46	18.9711
23	0.00112	0.99888	98,567	33,810.5	33,014.66	638,089.07	18.8725
24	0.00112	0.99888	98,456	32,237.5	31,478.70	605,074.41	18.7693
25	0.00112	0.99888	98,346	30,737.7	30,014.19	573,595.71	18.6610
26	0.00113	0.99887	98,236	29,307.7	28,617.68	543,581.52	18.5474
27	0.00113	0.99887	98,125	27,943.9	27,286.01	514,963.84	18.4285
28	0.00114	0.99886	98,014	26,643.6	26,016.17	487,677.83	18.3038
29	0.00115	0.99885	97,902	25,403.5	24,805.19	461,661.66	18.1731
30	0.00116	0.99884	97,790	24,220.9	23,650.33	436,856.47	18.0363
31	0.00118	0.99882	97,676	23,093.2	22,548.90	413,206.14	17.8930
32	0.00120	0.99880	97,561	22,017.5	21,498.34	390,657.24	17.7431
33	0.00123	0.99877	97,444	20,991.4	20,496.21	369,158.90	17.5862
34	0.00127	0.99873	97,324	20,012.6	19,540.11	348,662.70	17.4221
35	0.00132	0.99868	97,200	19,078.7	18,627.77	329,122.59	17.2508
36	0.00139	0.99861	97,072	18,187.5	17,756.96	310,494.82	17.0719
37	0.00147	0.99853	96,937	17,336.6	16,925.58	292,737.87	16.8855
38	0.00158	0.99842	96,795	16,524.3	16,131.60	275,812.28	16.6913
39	0.00171	0.99829	96,642	15,748.3	15,373.02	259,680.68	16.4895
40	0.00188	0.99812	96,476	15,006.7	14,647.91	244,307.66	16.2799
41	0.00208	0.99792	96,295	14,297.7	13,954.41	229,659.76	16.0627
42	0.00231	0.99769	96,095	13,619.4	13,290.88	215,705.35	15.8381
43	0.00259	0.99741	95,873	12,970.3	12,655.67	202,414.47	15.6060
44	0.00292	0.99708	95,624	12,348.7	12,047.13	189,758.80	15.3667
45	0.00330	0.99670	95,345	11,752.9	11,463.78	177,711.67	15.1206
46	0.00372	0.99628	95,031	11,181.7	10,904.29	166,247.89	14.8679
47	0.00420	0.99580	94,677	10,633.7	10,367.43	155,343.60	14.6086
48	0.00474	0.99526	94,279	10,107.7	9,851.95	144,976.18	14.3431
49	0.00534	0.99466	93,833	9,602.6	9,356.74	135,124.23	14.0717
50	0.00599	0.99401	93,331	9,117.1	8,880.85	125,767.48	13.7946
51	0.00671	0.99329	92,772	8,650.6	8,423.35	116,886.63	13.5120
52	0.00750	0.99250	92,150	8,202.0	7,983.36	108,463.28	13.2240
53	0.00837	0.99163	91,459	7,770.4	7,560.02	100,479.93	12.9310
54	0.00931	0.99069	90,693	7,355.2	7,152.60	92,919.91	12.6333
55	0.01035	0.98965	89,849	6,955.5	6,760.39	85,767.30	12.3309
56	0.01148	0.98852	88,919	6,570.6	6,382.68	79,006.91	12.0243
57	0.01272	0.98728	87,898	6,199.9	6,018.86	72,624.23	11.7137
58	0.01408	0.98592	86,780	5,842.8	5,668.31	66,605.37	11.3995
59	0.01557	0.98443	85,558	5,498.7	5,330.48	60,937.06	11.0820
60	0.01720	0.98280	84,226	5,167.1	5,004.85	55,606.58	10.7617
61	0.01899	0.98101	82,777	4,847.4	4,690.95	50,601.73	10.4390
62	0.02096	0.97904	81,205	4,539.2	4,388.32	45,910.78	10.1144
63	0.02312	0.97688	79,503	4,242.0	4,096.58	41,522.46	9.7884
64	0.02549	0.97451	77,665	3,955.6	3,815.38	37,425.88	9.4615
65	0.02810	0.97190	75,686	3,679.5	3,544.43	33,610.50	9.1344

APPENDIX II: A1949-52 ULTIMATE

At $i^* = \dfrac{i-j}{1+j} = \dfrac{0.10-0.05}{1+0.05} = 0.04762 = 4.762\%$

$Age\ \ x$	q_x	p_x	l_x	$D_x = v^x l_x$	$D_{x+\frac{1}{2}} = v^{x+\frac{1}{2}} l_{x+\frac{1}{2}}$	$\overline{N} = \sum D_{x+\frac{1}{2}}$	$\overline{a} = \overline{N} / D_x$
66	0.03095	0.96905	73,559	3,413.6	3,283.50	30,066.07	8.8078
67	0.03409	0.96591	71,282	3,157.6	3,032.40	26,782.58	8.4820
68	0.03753	0.96247	68,852	2,911.3	2,790.99	23,750.18	8.1579
69	0.04130	0.95870	66,268	2,674.7	2,559.22	20,959.18	7.8362
70	0.04543	0.95457	63,531	2,447.7	2,337.06	18,399.96	7.5174
71	0.04995	0.95005	60,645	2,230.3	2,124.56	16,062.90	7.2023
72	0.05489	0.94511	57,616	2,022.5	1,921.81	13,938.34	6.8915
73	0.06028	0.93972	54,453	1,824.6	1,728.96	12,016.53	6.5857
74	0.06616	0.93384	51,171	1,636.7	1,546.18	10,287.57	6.2855
75	0.07257	0.92743	47,785	1,459.0	1,373.69	8,741.39	5.9915
76	0.07953	0.92047	44,318	1,291.6	1,211.70	7,367.70	5.7044
77	0.08709	0.91291	40,793	1,134.8	1,060.45	6,156.00	5.4247
78	0.09528	0.90472	37,240	988.9	920.13	5,095.56	5.1528
79	0.10414	0.89586	33,692	854.0	790.92	4,175.43	4.8892
80	0.11369	0.88631	30,183	730.3	672.94	3,384.50	4.6344
81	0.12397	0.87603	26,752	617.8	566.22	2,711.56	4.3887
82	0.13500	0.86500	23,435	516.6	470.70	2,145.33	4.1524
83	0.14681	0.85319	20,272	426.6	386.19	1,674.64	3.9257
84	0.15942	0.84058	17,296	347.4	312.37	1,288.45	3.7087
85	0.17282	0.82718	14,538	278.8	248.81	976.08	3.5015
86	0.18704	0.81296	12,026	220.1	194.93	727.26	3.3042
87	0.20205	0.79795	9,776	170.8	150.01	532.33	3.1167
88	0.21785	0.78215	7,801	130.1	113.26	382.32	2.9388
89	0.23440	0.76560	6,102	97.1	83.77	269.06	2.7701
90	0.25168	0.74832	4,671	71.0	60.62	185.28	2.6103
91	0.26963	0.73037	3,496	50.7	42.86	124.66	2.4587
92	0.28819	0.71181	2,553	35.3	29.56	81.80	2.3142
93	0.30730	0.69270	1,817	24.0	19.86	52.24	2.1752
94	0.32688	0.67312	1,259	15.9	12.98	32.38	2.0392
95	0.34683	0.65317	847	10.2	8.24	19.40	1.9017
96	0.36706	0.63294	553	6.4	5.07	11.16	1.7548
97	0.38747	0.61253	350	3.8	3.03	6.09	1.5842
98	0.40795	0.59205	215	2.2	1.75	3.06	1.3622
99	0.42840	0.57160	127	1.3	0.98	1.31	1.0343
100	1.00000	0.00000	73	0.7	0.34	0.34	0.4885

APPENDIX III: STANDARD CONTRIBUTION RATES

A. STANDARD CONTRIBUTION RATES OF OVERALL SCHEME MEMBERSHIP

COUNT	AGE	CURRENT MONTHLY SALARY	CURRENT ANNUALISED SALARY	PAST SERVICE YEARS	SCR_{AAM}	SCR_{EAM}	SCR_{PUM}	SCR_{CUM}
1	20	15,000.00	180,000.00	0	6.62%	6.62%	3.01%	0.45%
2	20	20,000.00	240,000.00	1	6.62%	6.62%	3.01%	0.47%
3	23	20,000.00	240,000.00	2	7.25%	6.62%	3.46%	0.65%
4	24	25,000.00	300,000.00	1	7.47%	6.62%	3.62%	0.69%
5	26	45,000.00	540,000.00	3	7.92%	6.62%	3.97%	0.91%
6	26	25,000.00	300,000.00	3	7.92%	6.62%	3.97%	0.91%
7	31	30,000.00	360,000.00	5	9.17%	6.62%	5.02%	1.58%
8	33	30,000.00	360,000.00	7	9.71%	6.62%	5.50%	2.06%
9	35	55,000.00	660,000.00	7	10.29%	6.62%	6.04%	2.50%
10	35	65,000.00	780,000.00	7	10.29%	6.62%	6.04%	2.50%
11	35	35,000.00	420,000.00	4	10.29%	6.62%	6.04%	2.23%
12	39	55,000.00	660,000.00	5	11.51%	6.62%	7.28%	3.40%
13	40	70,000.00	840,000.00	3	11.84%	6.62%	7.63%	3.45%
14	42	15,000.00	180,000.00	9	12.51%	6.62%	8.37%	5.22%
15	42	45,000.00	540,000.00	12	12.51%	6.62%	8.37%	5.74%
16	44	90,000.00	1,080,000.00	11	13.20%	6.62%	9.19%	6.74%
17	47	95,000.00	1,140,000.00	14	14.29%	6.62%	10.57%	9.81%
18	49	80,000.00	960,000.00	16	15.03%	6.62%	11.61%	12.56%
19	49	65,000.00	780,000.00	13	15.03%	6.62%	11.61%	11.54%
20	50	70,000.00	840,000.00	23	15.40%	6.62%	12.17%	16.43%
21	50	35,000.00	420,000.00	24	15.40%	6.62%	12.17%	16.81%
22	50	65,000.00	780,000.00	19	15.40%	6.62%	12.17%	14.94%
23	52	75,000.00	900,000.00	20	16.15%	6.62%	13.36%	18.54%
24	54	50,000.00	600,000.00	10	16.88%	6.62%	14.68%	16.98%
25	55	105,000.00	1,260,000.00	15	17.24%	6.62%	15.39%	21.70%
26	55	100,000.00	1,200,000.00	25	17.24%	6.62%	15.39%	27.73%
27	57	135,000.00	1,620,000.00	25	17.94%	6.62%	16.91%	33.59%
28	58	95,000.00	1,140,000.00	27	18.27%	6.62%	17.72%	38.58%
29	59	55,000.00	660,000.00	9	18.58%	6.62%	18.58%	26.55%
30	59	120,000.00	1,440,000.00	32	18.58%	6.62%	18.58%	46.90%
SIMPLE AVERAGE	**41.97**				**12.89%**	**6.62%**	**9.71%**	**11.74%**
WEIGHTED AVERAGE (ACTUAL)	**41.97**				**12.39%**	**6.62%**	**11.71%**	**16.74%**

APPENDIX III: STANDARD CONTRIBUTION RATES

B. STANDARD CONTRIBUTION RATES OF SEGMENTED SCHEME MEMBERSHIP (GROUPS OF 5)

SUB-SCHEME	COUNT	AGE	CURRENT MONTHLY SALARY	CURRENT ANNUALISED SALARY	PAST SERVICE YEARS	SCR_{AAM}	SCR_{EAM}	SCR_{PUM}	SCR_{CUM}
A	1	20	15,000.00	180,000.00	0	6.62%	6.62%	3.01%	0.45%
	2	20	20,000.00	240,000.00	1	6.62%	6.62%	3.01%	0.47%
	3	23	20,000.00	240,000.00	2	7.25%	6.62%	3.46%	0.65%
	4	24	25,000.00	300,000.00	1	7.47%	6.62%	3.62%	0.69%
	5	26	45,000.00	540,000.00	3	7.92%	6.62%	3.97%	0.91%
	6	26	25,000.00	300,000.00	3	7.92%	6.62%	3.97%	0.91%
	SIMPLE AVERAGE	23.17				7.30%	6.62%	3.51%	0.68%
	WEIGHTED AVERAGE	23.17				7.44%	6.62%	3.62%	0.73%
B	7	31	30,000.00	360,000.00	5	9.17%	6.62%	5.02%	1.58%
	8	33	30,000.00	360,000.00	7	9.71%	6.62%	5.50%	2.06%
	9	35	55,000.00	660,000.00	7	10.29%	6.62%	6.04%	2.50%
	10	35	65,000.00	780,000.00	7	10.29%	6.62%	6.04%	2.50%
	11	35	35,000.00	420,000.00	4	10.29%	6.62%	6.04%	2.23%
	12	39	55,000.00	660,000.00	5	11.51%	6.62%	7.28%	3.40%
	SIMPLE AVERAGE	34.67				10.21%	6.62%	5.99%	2.38%
	WEIGHTED AVERAGE	34.67				10.31%	6.62%	6.12%	2.50%
C	13	40	70,000.00	840,000.00	3	11.84%	6.62%	7.63%	3.45%
	14	42	15,000.00	180,000.00	9	12.51%	6.62%	8.37%	5.22%
	15	42	45,000.00	540,000.00	12	12.51%	6.62%	8.37%	5.74%
	16	44	90,000.00	1,080,000.00	11	13.20%	6.62%	9.19%	6.74%
	17	47	95,000.00	1,140,000.00	14	14.29%	6.62%	10.57%	9.81%
	18	49	80,000.00	960,000.00	16	15.03%	6.62%	11.61%	12.56%
	SIMPLE AVERAGE	44.00				13.23%	6.62%	9.29%	7.25%
	WEIGHTED AVERAGE	44.00				13.32%	6.62%	9.61%	7.90%
D	19	49	65,000.00	780,000.00	13	15.03%	6.62%	11.61%	11.54%
	20	50	70,000.00	840,000.00	23	15.40%	6.62%	12.17%	16.43%
	21	50	35,000.00	420,000.00	24	15.40%	6.62%	12.17%	16.81%
	22	50	65,000.00	780,000.00	19	15.40%	6.62%	12.17%	14.94%
	23	52	75,000.00	900,000.00	20	16.15%	6.62%	13.36%	18.54%
	24	54	50,000.00	600,000.00	10	16.88%	6.62%	14.68%	16.98%
	SIMPLE AVERAGE	50.83				15.71%	6.62%	12.69%	15.87%
	WEIGHTED AVERAGE	50.83				15.61%	6.62%	12.66%	15.83%
E	25	55	105,000.00	1,260,000.00	15	17.24%	6.62%	15.39%	21.70%
	26	55	100,000.00	1,200,000.00	25	17.24%	6.62%	15.39%	27.73%
	27	57	135,000.00	1,620,000.00	25	17.94%	6.62%	16.91%	33.59%
	28	58	95,000.00	1,140,000.00	27	18.27%	6.62%	17.72%	38.58%
	29	59	55,000.00	660,000.00	9	18.58%	6.62%	18.58%	26.55%
	30	59	120,000.00	1,440,000.00	32	18.58%	6.62%	18.58%	46.90%
	SIMPLE AVERAGE	57.17				17.98%	6.62%	17.10%	32.51%
	WEIGHTED AVERAGE	57.17				17.66%	6.62%	17.00%	33.34%

APPENDIX IV: ACTUARIAL LIABILITIES

COUNT	AGE	CURRENT MONTHLY SALARY	CURRENT ANNUALISED SALARY	PAST SERVICE YEARS	AL_{AAM}	AL_{EAM}	AL_{PUM}	AL_{CUM}
1	20	15,000.00	180,000.00	0	-	-	-	-
2	20	20,000.00	240,000.00	1	7,045.13	7,045.13	7,045.13	1,000.73
3	23	20,000.00	240,000.00	2	16,200.53	42,073.68	16,200.53	2,663.94
4	24	25,000.00	300,000.00	1	10,607.49	53,827.14	10,607.49	1,831.46
5	26	45,000.00	540,000.00	3	62,865.62	179,873.84	62,865.62	11,966.77
6	26	25,000.00	300,000.00	3	34,925.34	99,929.91	34,925.34	6,648.21
7	31	30,000.00	360,000.00	5	88,142.95	230,168.97	88,142.95	21,414.01
8	33	30,000.00	360,000.00	7	135,432.35	301,675.37	135,432.35	36,275.33
9	35	55,000.00	660,000.00	7	272,502.58	619,073.59	272,502.58	80,470.77
10	35	65,000.00	780,000.00	7	322,048.50	731,632.43	322,048.50	95,101.81
11	35	35,000.00	420,000.00	4	99,091.85	319,637.04	99,091.85	29,262.10
12	39	55,000.00	660,000.00	5	234,453.18	652,723.20	234,453.18	84,155.18
13	40	70,000.00	840,000.00	3	187,562.54	738,431.83	187,562.54	70,690.35
14	42	15,000.00	180,000.00	9	132,332.75	256,861.39	132,332.75	54,986.99
15	42	45,000.00	540,000.00	12	529,331.02	902,916.94	529,331.02	219,947.97
16	44	90,000.00	1,080,000.00	11	1,065,063.62	1,837,193.48	1,065,063.62	487,917.92
17	47	95,000.00	1,140,000.00	14	1,645,137.32	2,464,615.45	1,645,137.32	872,451.45
18	49	80,000.00	960,000.00	16	1,737,669.81	2,405,820.09	1,737,669.81	1,015,979.55
19	49	65,000.00	780,000.00	13	1,147,133.59	1,690,005.69	1,147,133.59	670,705.25
20	50	70,000.00	840,000.00	23	2,289,741.99	2,857,162.57	2,289,741.99	1,405,702.95
21	50	35,000.00	420,000.00	24	1,194,647.99	1,478,358.28	1,194,647.99	733,410.24
22	50	65,000.00	780,000.00	19	1,756,416.99	2,283,307.53	1,756,416.99	1,078,287.67
23	52	75,000.00	900,000.00	20	2,341,308.83	2,893,706.98	2,341,308.83	1,584,689.98
24	54	50,000.00	600,000.00	10	856,533.24	1,168,861.66	856,533.24	639,158.29
25	55	105,000.00	1,260,000.00	15	2,826,559.69	3,406,890.95	2,826,559.69	2,214,683.48
26	55	100,000.00	1,200,000.00	25	4,486,602.68	5,039,299.12	4,486,602.68	3,515,370.60
27	57	135,000.00	1,620,000.00	25	6,647,497.03	7,150,702.27	6,647,497.03	5,742,357.87
28	58	95,000.00	1,140,000.00	27	5,292,673.83	5,542,597.17	5,292,673.83	4,800,611.18
29	59	55,000.00	660,000.00	9	1,070,030.97	1,146,549.51	1,070,030.97	1,019,077.11
30	59	120,000.00	1,440,000.00	32	8,300,846.28	8,467,795.83	8,300,846.28	7,905,567.89
TOTAL ACTUARIAL LIABILITIES					**44,790,405.68**	**54,968,737.04**	**44,790,405.68**	**34,402,387.03**

APPENDIX V: SCR_{AAM} / SCR_{CUM} RATIO

AGE, PAST SERVICE	SCR_{AAM}	SCR_{CUM}	RATIO
20,0	6.62%	0.45%	0.07
20,1	6.62%	0.47%	0.07
23,2	7.25%	0.65%	0.09
24,1	7.47%	0.69%	0.09
26,3	7.92%	0.91%	0.11
26,3	7.92%	0.91%	0.11
31,5	9.17%	1.58%	0.17
33,7	9.71%	2.06%	0.21
35,7	10.29%	2.50%	0.24
35,7	10.29%	2.50%	0.24
35,4	10.29%	2.23%	0.22
39,5	11.51%	3.40%	0.29
40,3	11.84%	3.45%	0.29
42,9	12.51%	5.22%	0.42
42,12	12.51%	5.74%	0.46
44,11	13.20%	6.74%	0.51
47,14	14.29%	9.81%	0.69
49,16	15.03%	12.56%	0.84
49,13	15.03%	11.54%	0.77
50,23	15.40%	16.43%	1.07
50,24	15.40%	16.81%	1.09
50,19	15.40%	14.94%	0.97
52,20	16.15%	18.54%	1.15
54,10	16.88%	16.98%	1.01
55,15	17.24%	21.70%	1.26
55,25	17.24%	27.73%	1.61
57,25	17.94%	33.59%	1.87
58,27	18.27%	38.58%	2.11
59,9	18.58%	26.55%	1.43
59,32	18.58%	46.90%	2.52