GEOMATHEMATICAL ANALYSIS OF GROWTH CENTRE PATTERNS: THE NAIROBI REGION

By

Gary L. Gaile

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INSTITUTE FOR DEVELOPMENT STUDIES
UNIVERSITY OF NAIROBI
P.O. BOX 30197
NAIROBI, KENYA

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ABSTRACT

This paper demonstrates the use of geomathematical techniques in testing growth center hypotheses. The hypotheses are: i) in an urban-rural region, there exists a "development surface emanating from a number of interrelated measures of prosperity and economic development; ii) The degree of prosperity and development described by this surface is inversely related to distance from a major growth center; iii) The surface is modified along directional sectors; iv) The degree of prosperity and development described by this surface is modified by the hierarchy of urbanism.

The statistical techniques of principal components analysis, trend surface analysis, computer cartography, spatial regression were applied to "development" data for Nairobi.

The analysis found that a development surface of considerable regularity exists, although the surface was not highly related the inverse of distance when analyzed concentrically; relatively high relations were discerned with the inverse of distance when analyzed along directional sectors. The hierarchy of urbanism was a significant variable affecting development a larger number of sectors, although its effect was major in only two sectors. Thus, a significant amount of regularity exists in the spatial pattern of development around Nairobi.

GEOMATHEMATICAL ANALYSIS OF GROWTH CENTER PATTERNS:
THE NAIROBI REGION*

Despite a refaluent amount of attention given to he growth center concept by academics (4,11,17), only recently has the concept been subjected to analyses employing rigorous scientific methodology. Friedmann's (6) explicit specification of definitions and postulates is a notable example. A signal effort is that of Casetti, King, and Odland (3) wherein a testing technique utilizing Casetti's "expansion method" (2) is applied to a case (Los Angeles and neighbouring cities) to demonstrate the applicability of formalizations of a theory developed in the study. Modifications and r refinements of this technique have recently been demonstrated (7).

Of specific interest is the development of a variety of geomathematical techniques which can be used in the analysis of center-periphery relationships intergral to the concept of "spread" (8). Spatially, spread may be defined as the complex set of processes whereby the absolute level of development of a peripheral area is increased due to spatial relationships with a core area. Although testing procedures for the process elements of the spread hypothesis are currently lacking, testing of the patterns which could be expected given that spread has occured have recently been refined. The most stimulating current works are those of Moseley (12) and this study will attempt to modify and emblish upon his methods in testing hypotheses of spread around Nairobi, Kenya.

Hypotheses.

This analysis will only test modified versions of the first two of Moseley's hypothese as they relate to the growth center concept. These are:

i) In an urban-rural[1] region, there exists a "development surface emanating from a number of interrelated measures of prosperity and economic development" (12, p. 61).

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1. Moseley uses only "rural" to describe his region. This is obviously not the case in this study's Nairobi context.

ii) The degree of properity and development described by this surface is inversely related to distance from a major growth center.

Moseleys's three remaining hypotheses were not incorporated into this study because it was felt that definite theoretical and technical difficulties were involved in his study[2], and, upon recognizance of these difficulties, it was determined that those hypotheses were not "both relevant and teatable" (12, p. 61) in this context.

Nonetheless, other hypotheses relevant to the growth center context were not stated by Moseley but are incorporated in this analysis. They are:

iii) The surface is modified along directional axes.

iv) The degree of prosperity and development described by this (development) surface is modified by the hierarchy of urbanism.[3]

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2. Moseleys's third hypothesis involves subdividing of the periphery into "upward and downward transitional area". He uses the principal component scores of his first component as his index of development. He defines the subdivision of upward and downward transitional areas in terms of whether the trend surface value of these scores is positive or negative. It should be noted that of the eight variables which load highly on his component index, six are static and the other two are comparative-static migration figures. Since static variables obviously cannot be used to determine the direction of transition it can be said that their predominance on this component does not allow temporal inference regarding development. The argument which is made regarding the inclusion of the migration elements as evidence would of necessity have been prefaced by a value judgement regarding the developmental influence of migration - judgement which would be controversial given the present state of knowledge on migration. Further the use of the mean as the approximate point of division is arbitrary. Moseley realizes this problem to some extent and proceeds to characterize downward communes in terms of their negative values on some primarily comparative static variables, most of which load on components other than the first (development) component. By definition, however, these components are orthogonal to "development" and thus represent a dimension not explained by the development component. This issue is further confused if the relative versus absolute change argument on the definition of spread is introduced (8). No inference into the absolute change can be made from the mean of the surface. The whole surface may be rising in which case the mean would be identified (were it of temporal significance) for those areas rising at the average rate. Thus negative scores could still represent positive change. Due to the problems discussed above, the acceptance of Moseley's fourth hypothesis, which is predicated by our acceptance of his third hypothesis, is also seriously questioned. Further comment on this major technical difficulty and substantive difficulty will be made in the correlation and regression section of this paper.

3. This hypothesis is counter to Moseley's fifth hypothesis in that he tests only for non-influence below a certain level of the hierarchy. This author beleives a hypothesis which better accomodates the existant notions on the growth center concept is one which tests for influence of urbanism such that this influence is a declining function of position in the heirachy.

The Index of Development and the Development Surface

The delimitation used for the Nairobi "region" is that developed by the Nairobi Urban Study Group (NUSG) (13). The area is approximately 120 km by 95 km in extent and is gridded into 233 cells each measuring 36 km. Data collected by NUSG for each cell are used in this study. It is felt that the use of equi-era data cells greatly enhances the inferential power of such a regional analysis.

Thirty-two variables per cell were coded from raw data and maps. Of these variables, those considered to be most relevant to development were included in a principal components analysis[4] to determine a dimension or component useful an an "index of development".[5] The variables selected and the principal component loadings are shown in Tables 1 and 2.

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4. The principal components program was executed using a Biomed X72 program on the University of California, Los Angeles Campus Computing Network IBM 360/91 computer facitlity.

5. It should be noted that principal components analysis is primarily useful as a static technique and is less useful in dynamic situations (ecepting those where rates are used as variables or when analysis is in S-mode) since components change over time in structure, thus changing the meaning of the component scores. They are, nonetheless, used in this study to maintain a complimentarity with the Mosley study. It is felt that principal components analysis is not optimal as an indexing technique even in a static study. In a dynamic analysis an index would be better developed using some fixed relationship between fewer major variables.

TABLE 1

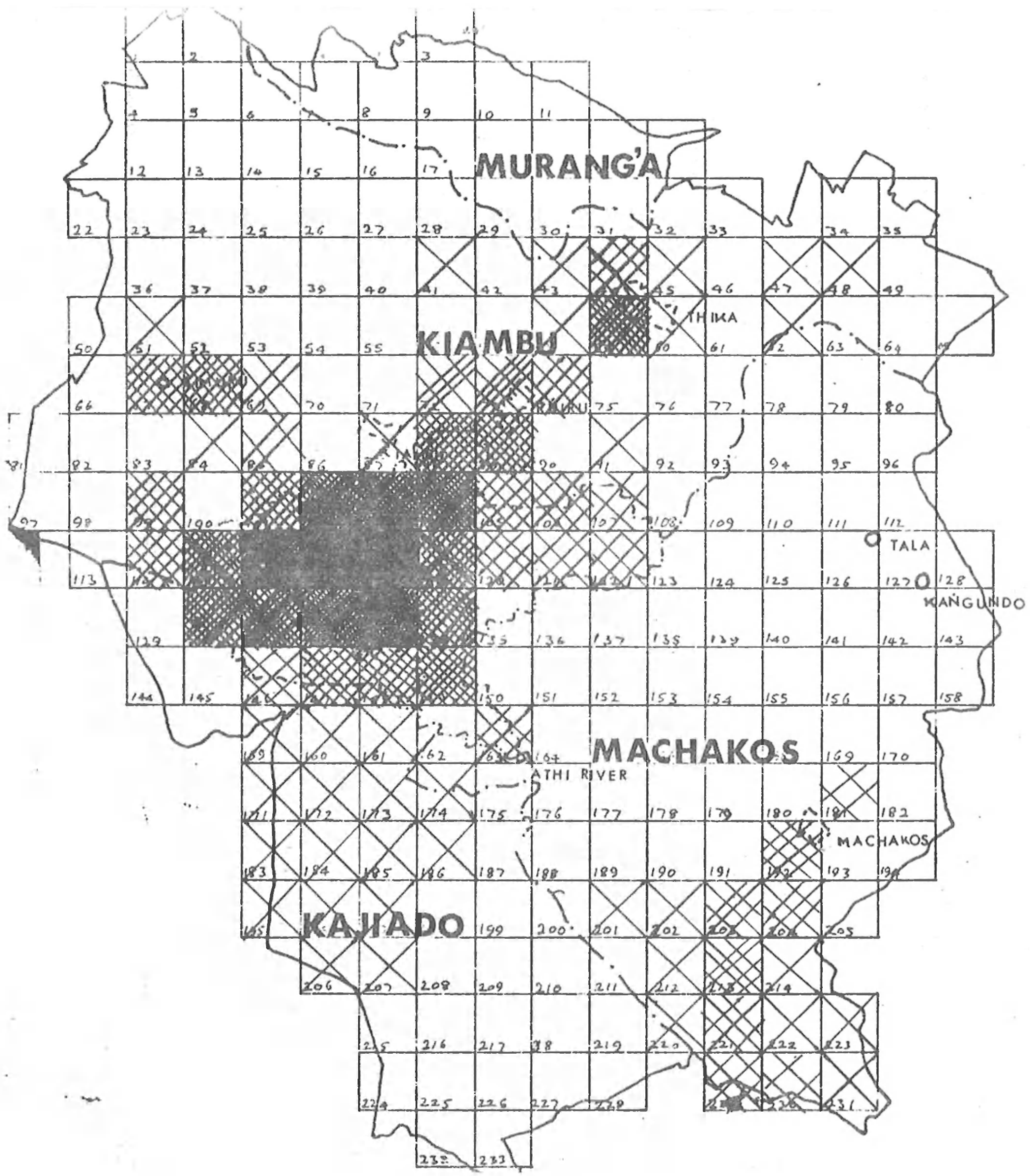
Variables included in the principal components analysis

1. Change in population density 1962/1969.
2. Sex Ratio (Men /Women), 1969
3. Men 15-39 as % of total male population, 1969
4. Women 15-39 as % of total female population, 1969
5. Road distance in km: all weather road, bound surface
6. Road distance in km: all weather road, loose surface
7. Road distance in km: dry weather road
8. Infrastructure rating
9. Primary school pupils per 1000 population
10. Secondary school pupils per 1000 population

Table 2

Highest loadings on the principal components

Variable	Component 1	Component 2
Change pop. density	.89641	
Sex-ratio		.87116
% Men 15-39		.80139
% Women 15-39	.61641	
Road distance, bound	.91865	
Road distance, loose	.83096	
Road dist., dry weather		-.49797
Infrastructure rating	.92350	
Primary pupils		-.81438
Secondary pupils		-.57319
Eigen values	4.15	3.27
% of total variance	41.5	32.7 CUM.% = 74.2



NAIROBI
STUDY REGION
6000 DATA CELL GRID

Index of Development

Figure 1

Positive Values Only

(One diagonal line = 0.1 factor score)

Grids are 36 km^2

The first component is identified as the "index of development". A more accurate label might be the "index of growth and capacity for development". The variables which loaded highest on this component are the infrastructure rating, the change in population density, all weather road distances in the cell, and per cent of women in the labor force age group. This component accounted for 41.5% of the total variance within the included variables. Figure 1 is a map of this index in the study area and is also the base map for locations on other computer maps in the text. The other component will not be explicitly considered in this study.

The Correlation and Regression Analyses

A Although Moseley pursued a proper course in using correlation and regression analyses to test his hypotheses, his methodology was not well adapted to the problem at hand. For example,

Letting Z be the "index of development", in order to prove hypothesis (ii), it must be shown that:

$$\frac{\partial Z}{\partial d} < 0$$

where "d" is distance from the growth center. One possible test of this relationship is the linear regression equation:

$$Z = \alpha + \beta d$$

If β is negative and the correlation is significant, the hypothesis that the level of development is not a function of distance from the growth center can be rejected.

Moseley uses the log of distance versus his index "Z". It is felt this use of logs does not contribute to understanding unless the dependent variable side of the equation "Z" is also log-transofrmed (such that β becomes the exponent of the distance decay curve). A further unacknowledged difficulty arises from the necessary skewness of the distance distribution wherein many more cases are further away from the center than are near (thus violating a normality assumption of regression).

In the present analysis of Nairobi, the test for hypothesis (ii) resulted in the rejection of the hypothesis in that β values were positive. For this reason and the technical difficulty of non-normality, the concentric, i.e., omnidirectional, hypothesis was not proven in this case.

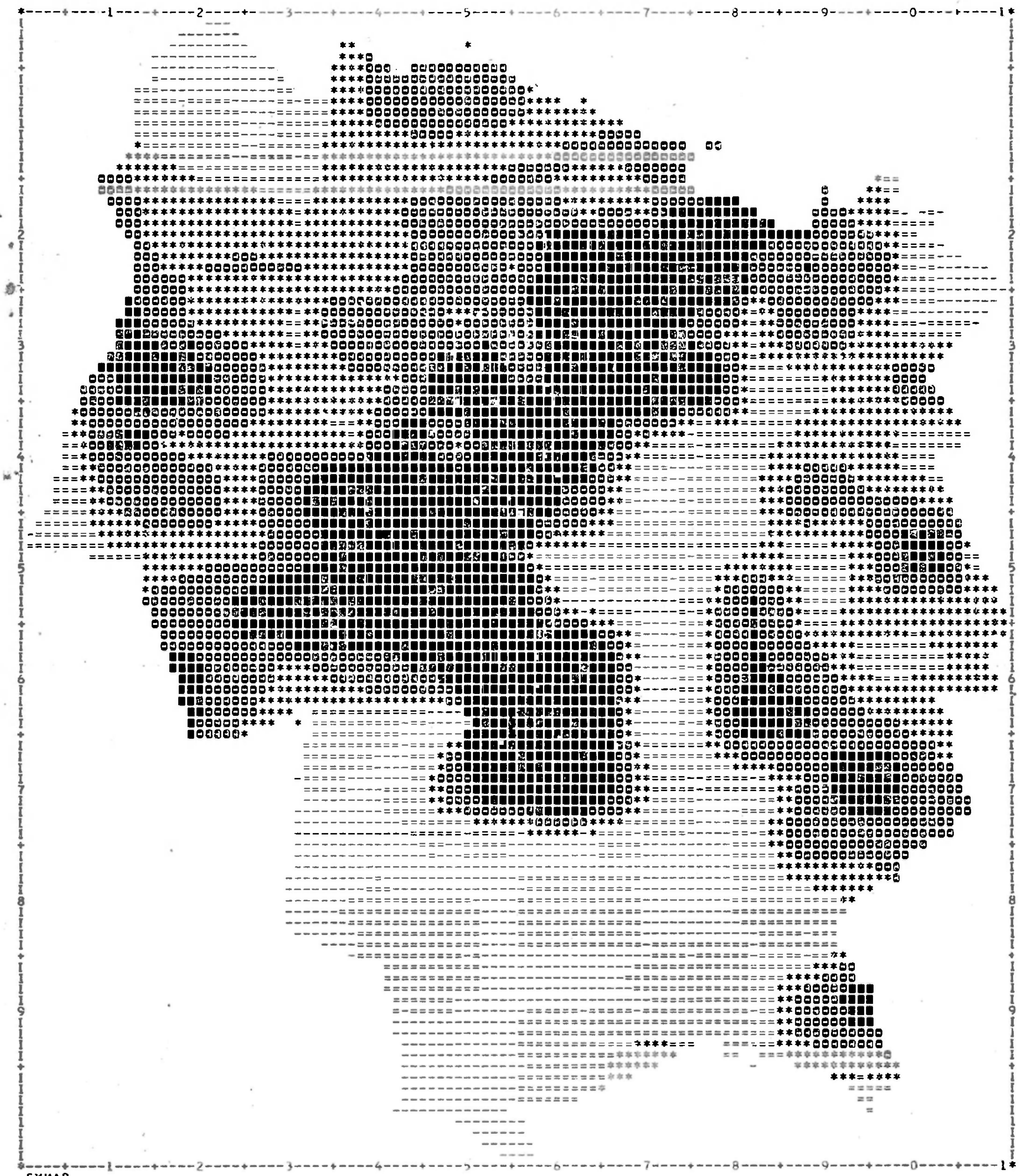
It is argued that Moseley's division of the region into three concentric rings does not adequately test the relationships posited by Friedmann (5). This is especially observable in the middle ring where a lack of significance is used inferentially to prove the existence of a trough of development, thus seemingly inputing some meaning to a relationship with an r^2 of 0.02. This analysis, thus, does not seek to duplicate this aspect of Moseley's study but rather attempts to investigate other approaches towards testing hypothesis (ii) regarding the inverse relationship between the index of development and distance.

Hypothesis (iii) raises the question of the importance of directional axes. Previous work by this author (7) has shown that directionality in one case was an important component in the explanation of core-periphery patterns of development. The importance of directionality follows from notions of "axes of development" (14) along transport routes and from sectoral theories of urban structure. It is further substantiated by notions of non-concentric barriers to the diffusion of development. Thus this analysis proceeds to test an unspecified process which is postulated to contain both concentric and sectoral elements, i.e., the index of development is postulated to be an inverse function of distance from the growth center and that the specification of this function varies with respect to direction.

To facilitate this approach, a radial pattern was focussed on Nairobi's Central Business District (CBD) with twelve regular sectors identified at 30° intervals. Each of the 233 data cells was assigned to a sector depending on the location of the data cell's center. Each sector also contained the cell representing the values of the CBD cell. The resulting equations varied but were all significant. The correlation coefficient values are presented for each radial in Table 3.

Thus the pattern which was shown to be indeterminate when a concentric omnidirectional hypothesis was considered has been shown to be strongly in force when the pattern is disected along directional sectors. In future analyses it would be useful to subject these sectors to an analysis of variance to determine if the variation of the residuals between sectors is significantly greater than the variation within sectors. More sophisticated inferential tests (10) for dealing with directional statistics are currently being adapted to this problem by this author.

Figure 2



SYMAP

6.525 SECONDS FOR MAP

THIS DATA IS MAPPED FOR AN EQUI-AREA GRIDDED REGION AROUND NAIROBI
AN INDEX OF COMMERCIAL IMPORTANCE IS THE VARIABLE MAPPED
NO SEARCH RADIUS OR INTERPOLATION MODIFICATIONS HAVE BEEN MADE

TABLE 3

Correlation coefficients of the relationship between the index of development and distance along radial sectors.

Sector No.	R
1 (north)	-0.70676
2	-0.67025
3	-0.63673
4 (East)	-0.59984
5	-0.48752
6	-0.65452
7 (South)	-0.77056
8	-0.92449
9	-0.93984
10 (West)	-0.92163
11	-0.76925
12	-0.69778

In order to test for the hierarchical effects of diffusion on the pattern of development surfaces as formalized in hypothesis (iv), another variable is introduced into the analysis. This variable is the index of commercial importance.[6] This index, which assigns values to cells on the basis of number and quality of urban functions in the cell, is consistent with functional categorizations or the urban hierarchy and thus is a relatively accurate surrogate index of that hierarchy. Figure 2 is a map of the index for the region.

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6. The Index of Commercial Importance is a functional definition of the hierarchy with accessability modification and is measured by a rating system as follows:

shop	- 1 point
bank	- 30 points full time; 15 points part time
hotel	- 2 points if less than 20 beds; 10 points if more
bar/cafe	- 1 point
petrol station	- 1 point if petrol only; 2 points if service is available
market days	- 2 points for each day of the week

proximity effect: assuming proximity to be important, the point figures for each cell were divided by eight and distributed to the eight adjacent cells.

The linear regression model along directional sectors was expanded to a multiple regression model including as the second independent variable the Index of Commercial Importance (ICI) such that:

$$Z = \alpha' + \beta_1 d + \beta_2 ICI$$

where β_1 should be negative and β_2 positive.

The correlation coefficients resultant after the addition of this second independent variable to include the hierarchical element's effect on the development pattern are shown in Table 4.

TABLE 4

Correlation coefficients of the relationship between the index of development and distance and the level of the urban hierarchy as analyzed along radial sectors.

Sector No.	R	% Improvement over Table 3
1 (North)	0.76226	7.86
2	0.68468*	2.16
3	0.63847*	0.27
4 (East)	0.68217	13.73
5	0.51767*	6.19
6	0.65834	0.60
7 (South)	0.77096*	0.05
8	0.92632*	0.20
9	0.96259	2.42
10 (West)	0.92195	0.03
11	0.76925**	**
12	0.84720	21.42

* Sign of regression coefficient is opposite
** Hierarchical variable excluded as insignificant

For half of the sectors analyzed the inclusion of the hierarchical variable must be rejected as one which would add to the explanation of the development surface. For the other sectors, however, the inclusion of this variable resulted in a positive effect (especially for sectors 4 and 12). The interpretation of this phenomena must first view the super-dominant influence of the primate city of Nairobi on its hinterland. In this case it is probable that hierarchical effects are much less important in that proximate locales conduct most interaction directly with the primate city. The significance of the variable of hierarchical status on sectors 4 and 12 is expected due to the influences of developmental buildup around the cities of Machakos and Limuru, respectively.

SYMAP

4.178 SECONDS FOR MAP

TIME = 8.5

THIS MAP IS A THIRD DEGREE POLYNOMIAL TREND SURFACE OF FACTOR SCORES OF A FACTOR LABELLED 'DEVELOPMENTAL GROWTH AND CAPACITY'. DATA IS FOR 233 EQUI-AREA GRIDDED CELLS IN THE NAIROBI REGION.

DATA VALUE EXTREMES ARE -1.77 0.94

SYMAP

7.759 SECONDS FOR MAP

TIME = 3.5

THIS MAP IS RESIDUALS FROM A 3RD DEGREE TREND SURF. OF FACTOR SCORES OF A FACTOR LABELLED 'DEVELOPMENTAL GROWTH AND CAPACITY'. DATA IS FOR 233 EQUI-AREA GRIDDED CELLS IN THE NAIROBI REGION.

DATA VALUE EXTREMES ARE -0.87 9.25

Trend Surface Analysis

Another method of spatially analyzing growth center hypotheses is the use of trend surface analysis. The analysis was first applied to this problem by Robinson and Salih (15) and theirs have been the major refinements of the technique in this context (16). The technique has been used both to historically identify the locations of poles and centers and to analyze the existence of patterns resultant from spread and backwash effects.

Trend surface analysis uses the methods of multiple regression analysis to describe large-scale, systematic changes of a variable in space. If systematic variations in the mathematical expectation of a variable exist over the spatial plane of the study area, than a "trend" exists which can be described by deterministic polynomial functions of the x,y-coordinates of the plane. The residuals from the trend may contain further systematic variability (often at a spatial scale different from that of the overall trend). The degree of the trend surface polynomial is a controlled variable. For a full discussion of trend surface techniques and methods of determining optimal polynomia order see (1,9).

As Moseley realized, a "cubic" trend surface (third order polynomial surface) is the minimum order necessary to adequately display the expected concentric pattern. Figure 3 is the result of a "cubic" trend surface applied to the "index of development" ("Z") for the 233-equi-area gridded data cells of the Nairobi region. As is visually evident, the pattern is almost concentric with the distortions along directional lines creating somewhat of an elliptical pattern. The ridges in the surface toward the northwest and southeast represent corriders of transportation and "development". The resultant polynomial equation and statistics relevant to the surface are shown in Table 5.

TABLE 5

The Polynomial equation and statistics relevant to the trend surface shown in figure 3.

$$Z = -0.507\ (E\ 01) + 0.124\ (E\ -01)\ X + 0.691\ (E\ -02)\ Y - 0.123\ (E{-04})\ X^2 - 0.315\ (E\ -05)\ XY - 0.449\ (E\ -05)\ Y^2 + 0.424\ (E\ -08)\ X^3 - 0.191\ (E\ -08)\ X^2Y + 0.379\ (E\ -08)\ XY^2 - 0.134\ (E\ -09)\ Y^3$$

Standard Deviation	0.88
Variation Explained by Surface	0.507 (E 02)
Variation Not Explained by Surface	0.18129 (E.03
Total Variation	0.232 (E 03)
Coefficient of Determination	0.219
Coefficient of Correlation	0.468

Figure 4 is a map of the residuals from the trend surface. The notable features include the high residuals at the city center itself. This is due to the extremely high "index of development" at the city center which can be expected in the case of a primate city. The extremity of this value is a major factor in diminishing the value of the correlation coefficient of the surface (0.468). The residual surface also shows peaks at Thika in the northeast. The differential values along the Thika-Nairobi axis indicate that the corridor displayed in figure 1 is quite disjoint in its development. The northwest-axis is along the main highway to the west with a peaking towards the corner. It is evident that low values along some of the hill country depressed the value of the trend surface in these areas. Low residual values are found where expected in the National Park directly south and in the hill country directly north.

The trend surface does indicate that the pattern of development around Nairobi is remarkably regular, although more elliptical than concentric. The residuals conform to what is known about local deviations in the area in that urban centers and transport axes display irregular patterns and that empty areas in hills and parks have low residuals. The urban hierarchical dimension is visually at least less important than was originally thought in that few urban areas appreciably affect the surface.

Conclusions

The work of Moseley is singled out and his hypotheses are modified and embelished upon. Two of the hypothese are included in a study of Nairobi, Kenya and two additional hypotheses are specified. The data are analyzed via the statistical techniques of principal components analysis, regression and correlation analysis with spatial modifications, and trend surface analysis.

Hypothesis (i), that a development surface could be specified, is accepted after a principal components analysis of selected variables yields a recognizable "index of development" (Figs. 1 and 3).

Hypothesis (ii), that the index varies inversely with distance from the growth center cannot be accepted, although a spatial modification of related hypothesis (iii) allows this relationship to be made more specific. When analyzed along directional axes (hypothesis iii), the development index was found to be highly inversely correlated with distance from the growth center and the degree of correlation as well as the slope of the development surface varied with direction.

Hypothesis (iv), that the hierarchy of urbanism affected the development surface, was less adequately substantiated, being unacceptable in half of the sectors cases analyzed. In some cases, however, the urban hierarchy was significant as a factor influencing the surface and in other cases it was found to be a discernable positive residual from the trend surface analysis. The lack of potency of this hypothesis is perhaps explainable by the great dominance effect a primate city has over the entire region in relationship to the minimal effects of spread from centers within its region that are several orders of magnitude different in the urban hierarchy.

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