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SCHOOL OF MATHEMATICS

VALUING MORTGAGE INSURANCE CONTRACTS USING AN OPTION BASED MODEL

SAC 420: PROJECT IN ACTUARIAL SCIENCE

A Project Submitted to the University of Nairobi in Partial Fulfilment of the Requirements for a Bachelor's Degree in Actuarial Science

Declaration

Declaration by the Candidates

This project is our original work and all materials which are not our own have been cited and acknowledged. The work has not been presented for award of degree in any other University.

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Dedication

We dedicate this project to our beloved families for their unending support, sacrifice and commitment to our education and well-being. May the Almighty God bless you and reward you abundantly.

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Abstract

In this study, we replicate a mortgage insurance contract as an option and then develop an option-based model for pricing the mortgage insurance contracts in an economy where we assume that the agents are risk neutral and that the price of the collateral follows a geometric Brownian motion. We extend the model to quantify the effects of legal inefficiencies in the pricing of mortgage insurance contracts. We demonstrate that these effects are quite significant in the pricing of mortgage insurance contracts. As an application of our model, we price a typical mortgage loan issued in the Kenyan market. We demonstrate that introducing a comprehensive legal system would reduce the time to repossession of the collateral from three to one year. This would translate to reduction in the actuarially fair price of the mortgage insurance premiums from 6 percent to 0.4 percent of the loan value. The conclusions of this paper are that mortgage insurers using other methods of valuation can adopt our valuation model as a reasonable check for the level of premiums charged to ensure they accurately price their mortgage insurance contracts.

1. Introduction

1.1 Background of the Study

Mortgage insurance is a financial agreement that insures the lender against loss in case the borrower defaults in payment of his or her mortgage or if the borrower dies. The Initial premium is usually required after the mortgage deal is closed. Subsequent premium payments depend on the chosen plan of payment. However, the most common method of payment in Kenya is monthly premiums. The lender of the mortgage remits the received premium payments to the insurer. Mortgage insurance is a beneficial for the homebuyers as it enables them to own homes by increasing their buying capacity. This is because mortgage insurance contracts enable the lenders to lower the initial down payment of the houses. This implies that accumulation of savings by borrowers in preparation for the purchase of their dream homes becomes easier. On the other hand, mortgage insurance is beneficial to lenders because it makes lenders more comfortable because of reduced risk of default making it possible for them to lend bigger loans to borrowers. Because of the aspect of reduced risk, mortgage insurance also leads to stable interest rates being charged by the borrowers. Mortgage insurance has been a key factor in overcoming the traditional barriers for financing home ownership.

1.2 Mortgage Industry in Kenya

Although the Kenyan mortgage market is ranked as the third most developed in Sub-Saharan Africa with an estimated mortgage assets equivalent to about 2.5% of the country's Gross Domestic Product (GDP), behind South Africa and Namibia (The World Bank, 2011), the market is still in the developmental stages. Therefore, a lot more needs to be done to enhance the growth of the industry.

The demand for housing in Kenya is immense due to urbanization and an increasing population. According to the World Bank, the current housing deficit in Kenya stands at about 200,000 units (World Bank, 2014). The housing demand in the country is met largely through self-construction, slum dwellings, and rental housing. Mortgages make up only a small percentage of this demand. The uptake of mortgage financing in the country is still very low. A research report by the World Bank identifies four main challenges facing the development of the mortgage

industry in the country. These include affordability, risk management, lack of sufficient long-term funds for lenders and high land prices in the country.

On affordability, a large percentage of the population cannot afford to seek mortgage financing mainly due to high interest rates charged by mortgage providers. For instance, a typical loan attracts an interest rate of 16.3 percent on average for an amount of KSHS 4 Million over a period of 15 years. This information implies that on average, only 3 percent of the entire population can access these products. Other reasons include low income and high and volatile inflation rates. Risk management is the second most significant challenge. There are weaknesses in the ability of the lenders to model and understand risks. Lack of effective risk sharing measures has forced the lenders to charge high-risk premiums. Insufficient data on mortgages, high level of informality as well as legal inefficiencies make mortgage insurance pricing in emerging economies a very complicated process. This paper focuses on the element of risk management.

Mortgage insurance (MI) for mortgage providers compensates the lender for losses incurred as a result of mortgage default or when the borrower is unable to recover the outstanding loan after foreclosure and sale of the house. MI for lenders is not well developed in the country because the mortgage industry is also not well developed. The common practice is where borrowers make their own arrangements with insurance companies to insure their mortgages. Major mortgage insurance providers in the country include British-American Investments Company, Kenyan Alliance Insurance Company Ltd, and Jubilee Insurance Company Ltd.

1.3 Loss Ratio

The loss ratio is an important element for insurance providers in mortgage insurance. It is the ratio of total losses to the insurer (Claims and expenses) to the total premiums received by the insurer. A lower loss ratio is usually desirable and indicates that the insurance company is in a strong financial position. On the contrary, a low loss ratio may be an indicator of financial problems by the insurer.

The Insurance Regulatory Authority (IRA) uses the loss ratio as one of the risk management policy-tool to ensure that insurance companies do not charge excessively high premiums or excessively low premiums that will result in high probability of ruin. Some of the measures

being implemented by the regulator to reduce the loss ratio in the insurance industry include advocating for more reinsurance, consumer education, and requiring insurance companies to set up actuarial functions or departments whose role will be to correctly price products and develop effective risk management measures. Undercutting by (especially) smaller insurers in a bid to stay in business has also been an issue of concern to the regulator hence reiterating the need to set up actuarial functions to enhance correct pricing and risk-based supervision.

1.4 Problem Statement

Banks and mortgage loan providers are exposed to potential losses originated from credit and prepayment risk embedded on mortgages. Both prepayment and credit risk are of opposing nature, not independent among them and generally lead to the termination of the mortgage contract. Consequently, they acquire mortgage insurance that generates a way to alleviate and diversify credit risk on their portfolios by ceding part of the credit risk to a third party.

The pricing of such contracts is a challenging task even when data is available. The task is generally more challenging in the case of emerging markets like Kenya, where, in the case of borrower default, the process of repossession of loan collateral may last a variable number of years and where data on payment behavior is either unavailable or of poor quality. Available valuation methods do not take into account the time to repossession. Since the collateral is not repossessed immediately how viable are these techniques?

Mortgage insurance differs from the other types of insurance contracts in several aspects. Because of these differences, it is difficult to use the conventional techniques of pricing these contracts. First, casualty insurance contracts cover only a single period and hence the historical performance of the policies can be reliably used to cover the subsequent periods. However, it is impossible to use this information on historical experience to determine the premiums for mortgage insurance contracts because mortgage insurance covers multiple periods.

Premiums for life mortgage insurances are determined at the date of inception of the contract unlike the other types of insurance policies. Secondly, mortgage insurance lasts for a definite period with a known date of termination. In addition, the risk involved decreases rather than increasing over the time because of the amortization schedule. This contradicts life insurance where risk increases with time. Thirdly, insurers can mitigate the risks of conventional insurance

policies through geographic diversification. In contrast, in mortgage insurance, prepayments and the rates of default are largely dependent on macroeconomic factors that include house-price growth rates, interest rates and household income among others hence a significant amount of systematic risk is involved in the mortgage insurance.

Finally, compulsory mortgage insurance covers the risks for the lender and excludes the borrower's risk. The borrower does not have any right to be temporarily uninsured or switch from one insurance company to another without termination of the existing loan. Because of these factors, the design and calculation of premiums for mortgage insurances requires extensive research with new approaches.

Due to complexity of pricing MI contracts, less than six insurance companies in Kenya are offering the product. This is a very shocking fact given that Kenya has 47 licensed insurance firms. This translates to less than 12.766% thus, how is fair competition represented in the market? There is thus a wakeup call for actuaries and other expertise to come up with a simple, accurate and relevant valuation technique to price MI contracts.

A more appropriate model for pricing should give more emphasis on minimizing default risk. Our proposed model replicates mortgage insurance contract as an option contract and comes up with a fair price using a simple option based technique.

1.5 Objectives

Having clearly specified where the problem lies in mortgage insurance, we are seeking to come up with a favorable solution applicable in the Kenyan insurance market. The followings are the objectives we hope to achieve in our study

1. To replicate a mortgage insurance contract as an option contract
2. To develop an option-based model for pricing mortgage insurance contracts
3. To quantify the effects of legal inefficiency on the pricing of MI contracts and demonstrates that these effects can be quite significant.
4. To test sensitivities with respect to changes in parameter values

1.6 Significance of the Study

Globally, there has been an increased involvement of actuaries in general insurance and health assurance. The employment of actuarial expertise runs from enterprise risk management, pricing and asset management. Key role of an actuary is correct pricing of insurance products to come up with a viable yet competitive premium. As a result, companies with solid actuarial departments have shown huge success in terms of solvency and risk control.

The trend has extended to emerging economies in Africa, Latin America, Asia and the Indian subcontinent. Examples of emerging actuarial markets include Ghana, Kenya and India. However, for the success of actuarial valuation methods there should be a well-specified regulation and well-documented historical data and vital statistics.

In Kenya, most companies widely rely on valuation techniques and data used in the United Kingdom. Such data is highly questionable even when adjusted with a mathematical factor (formula). Micro insurance companies can shift the blame to lack of well-documented data and unaffordability of actuarial expertise.

Most insurance companies shy from offering mortgage insurance despite all the 52 commercial banks issuing mortgage products. This is solely due to the complexity of mortgages. Less than six insurance companies are currently offering mortgage insurance in Kenya despite the increasing volume of mortgage uptake in the country.

The main reason for the laxity to offer MI is due to the complexity of pricing the contracts to come up with a sustainable premium. The insurance regulatory authority (IRA) has passed a regulation that requires all insurance firms in Kenya to set up actuarial department. Most Kenyan firms are yet to consolidate such departments largely due to financial constraints. Years the Insurance Regulatory Act was passed; very few insurers have set up actuarial departments. Most have had to outsource actuarial services to other financial services companies such as ActServ and Alexander Forbes.

Using our simple option based model, all Kenyan insurance companies even those without resident actuaries can use their office experience data and historical data from existing insurers to accurately price mortgage insurance contracts. This will in turn grow the number of MI providers hence healthy competition and improved service. Rather than replace existing models, this will

also present another option for insurers looking for a reasonable check for their premiums. Our proposed valuation method is general and can be used in any market, it is particularly useful in emerging market economies where other existing methods may be either inappropriate or are too complex to implement due to inadequate relevant data.

2   Literature Review

2.1 Derivative as a Tool for Risk Hedging

The use of derivatives for mitigation of risk and uncertainty inherent in financial instruments has become an essential component in the global financial markets. Because of the similar of financial contracts with other type of contracts, there have been more studies on their applications to other types of contracts. In recent times, the most widely used derivatives are the options because they are not obligatory in nature. Options give the right but not the obligation to buy or sell the underlying asset at a specified price on a specified future date. Options have been widely used in pricing of mortgage insurance contracts because of this inherent property.

There is a vast amount of literature on the use of options in valuation of mortgage contracts. In 1973, Merton developed the contingent claims model that provides the motivation for the behavior of the borrower using the options theory. Most studies initially used this approach in the valuation of mortgages by focusing default and prepayment as individual risks. For example, Cunningham & Hendershott (1984) used the Black-Scholes option-pricing model to value the risk of default by considering the default risk as a put option sold by the FHA and purchased by the buyer of a home for the protection of risk of default to the lender. Quigley & Van Order (1990) and Schwartz & Torous (1992) studied the contingent claims model and provided empirical estimates of the option based prepayment models that were consistent with the model. Kau & Kennan (1995) developed the theoretical framework model for the pricing of mortgages as derivative assets in a stochastic economic environment by describing the default and prepayment as European compound put option and the American call option respectively. Kau, Keenan & Muller III (1995) emphasized on the importance of joining both the default and prepayment risk in the valuation of insurance contracts for mortgages. Deng, Quigley & Van Order (2000) expanded this study. They hypothesized that a borrower who decides to default on the payment of mortgage gives up his option to default the same loan in future but also gives the option to prepay the mortgage. Earlier in 1996, they had presented a model by considering default and prepayment jointly in proportional hazard framework by using the options based approach. In addition, they explained the importance of non-economic events that affected the prepayment and default behavior and the relevance of these events in the exercising of the

options for default or prepayment on mortgages in the rational way predicted by the theory of finance.

Recently, Williamson et al. (2008) applied the Black-Scholes options model in the estimation of value of exotic options contracts on water in the Australian market that helped in enhancement of efficiency in the allocation of resources across the country. Manola & Urosevic (2010) studied the valuation of Mortgage backed securities by the use of option based model by paying a closer attention to the risk of default and demonstrated that option models could be used to value mortgage backed securities by taking into consideration the risk of default and prepayment. Hidetaka & Nakaoka (2006) expanded the use of options in the exploration of oil, gas and production projects by using the futures term structure approach. Their model tracks the underlying asset value and the volatility by using the real options technique. Case & Shiller (1996) emphasized on the importance of use of option contracts for hedging mortgage risk and risk of default.

These studies demonstrate the wide use of derivatives in mitigation of risk and provisions of alternative models for the valuation of mortgage contracts.

2.2 Does the Value of the Underlying Collateral Follow a Geometric Brownian Motion?

Many recent studies in finance and economics assume explicitly or implicitly that some quantities inherent with uncertainties that change over time follow the geometric Brownian motion process. The geometric Brownian motion, also referred to as the lognormal growth process has gained popularity as a valid model for modeling the growth of prices of stocks and other financial underlying assets over a time. GBM has been used in modeling growth of financial assets in many studies. Hull, (2000) refers to it as a model for stock prices because of its wide acceptance in modeling price of stocks. Under the GBM model, the Black-Scholes formula for the pricing of European put and call options provides a simple analytical evaluation of the asymmetric risks. The application of the GBM model in other processes other than modeling stock prices validates the assumption of this paper that the price of the underlying security in mortgage contracts follows the Geometric Brownian motion. In this study, possible future prices of houses have a lot of uncertainty inherent in them and hence their future prices move randomly justifying the assumption that their prices follow a GBM process.

Many studies have been carried out to support this assumption. The most recent examples have risen in the real option analysis where the value of the underlying asset has been assumed to evolve similarly with the stock prices. In this paper, the assumptions are made implicitly because the valuations of the mortage contracts are done by the use of the Black-Scholes formula. (Nembhard, Shi & Aktan, 2002) applied the cost of quality control charts by the use of real options where the price of the product and the sales volume were assumed to follow a GBM process. (Thorsen, 1998) applied the real options theory in making decisions of establishment of new forest stand by making assumptions that the future prices of round wood followed a GBM process. (Benninga & Tolkowsky, 2002) studied the application of the Black-Scholes model in allocation of capital for investment by assuming that the present value of future cash flows of machine studied followed a GBM process.

More closely, valuation of mortgages by assumption that the value of the underlying asset (House prices) follows a GBM has been carried out. Following the contingent claims model proposed by Merton (1973), the value of the underlying collateral (house prices) is assumed to follow a geometric Brownian motion. Kau et al. (1993, 1995) applied the GBM model to mortgages and observed that the price of houses followed a GBM motion. Ma, Kim & Lew (2007) analyzed the risk of the reverse mortgages in Korea insured by the government by considering the price of houses to follow the GBM process. Cunningham & Hendershott (1984), Hilliard et al. (1998), Yang et al. (1998) and Chinloy & Megbolugbe (1994) also stated that the price of houses follows the GBM process in their studies.

2.3 Actuarial Fair Value of the Insurance Contracts

In the modern world, use of valuation concepts from the field of financial mathematics has gained popularity because of the fact that cash flows that emanates from an insurance contract can be replicated by the use of derivative contracts. As a result, fair valuation of insurance contracts can be achieved by the use of derivative methods of valuation. Just like the financial options, the basic idea of the insurance contract is to transfer economic risk from one part to another against a specified form of payment. The buyer and the seller have to agree at the fair values of the contract at their onset. Thus, the price of insurance contract or an option refers to the premium paid while the value refers to the fair valuation of the underlying asset.

Just like in the options contract, the value of the collateral (house prices) is the underlying asset and has uncertain future values. In addition, the underlying stochastic processes of the underlying asset in a derivative and mortgage contracts are similar. The future values of the stocks and houses occur randomly through up and down movement from the initial value. Deng et al. (2000) in their application of the option market paradigm to the solution of insurance problems points out the other similarities inherent between options and insurance contracts. They stated that, insurance contracts and options are derivatives (As shown above) where the insurance prepayment depends on the insured loss while the terminal price of the option is evaluated by the underlying securities.

2.4 Legal Inefficiency Faced in Repossession of Collateral

Valuation of mortgage insurance contracts in developing economies is major problem because of lack of data and legal inefficiencies inherent in the economic systems. Recently, studies on valuation of mortgage insurance contracts have expanded to include the significance of costs incurred due to delay in possession of collateral in case of default. Bozovic, Karapandza & Urosevic (2009) state that decreasing the time to repossession from four years to two years can reduce the actuarially fair value of the Mortgage insurance contract by 55 percent of its original price. Arunada (2003) states that the need for risk sharing mechanisms between lenders and insurers faces major hurdles in developing economies because of existence of inefficient legal systems and unclear property rights. Bardhan, Karapandza & Urosevic, (2006) states the challenge for repossession of the collateral in emerging markets is more difficult where data on default is unavailable and repossession of the collateral can last for several years. Jaffee & Renaud, (1997) discusses the potential economic benefits of well-functioning mortgage systems for emerging economies and the reasons for underdevelopment of mortgage markets in these economies.

2.5 The Black-Scholes Model

Black-Scholes model received a major motivation in 1973 with the publication of the pioneer paper, "The Pricing of Options and Corporate Liabilities by Black and Scholes". The major assumptions that were riskless free rate of interest and the fact the underlying asset follows a GBM process. Merton, (1973) stated that the influence of the model is not limited to financial options or derivatives. The model has been widely criticized because of its shortcomings. Black

& Scholes, (1973) tested the model with data from OTC market and their results were lower than the actual market values. Geske, Roll & Shastri, (1983) found that the differences were created by imperfections of the OTC (Over the Counter) market. Merton, (1973) extended the model by showing that its basic form remained the same if the structure of the payment was increased or lifted, option was exercised prior to maturity and interest rate stochastic. Thorp, (1973) showed that the model was still applicable even with limitations on the usage of proceeds from short selling. In addition, Cox & Ross, (1979) showed that the hedge mechanism used in Black-Scholes model was not applicable if the returns from ordinary stocks did not follow a stochastic process. In addition, Mac-Beth & Merville, (1979), Latane & Rendleman, (1976) solved the Black-Scholes formula in the form of the implied variance rates. They collected data from 1975-1976 and found bias in the strike price that was opposite of the strike price reported by the model. Galai, (1977) studied the model by the use of data from the Chicago Board Option exchange and found that the excess daily returns on the use of the hedged portfolio was significantly different from zero and additionally found that transactions costs of one percent eliminated the excess positive returns.

3   Methodology

3.1 The Black-Scholes Model

In their paper "The pricing of options and corporate liabilities", Fischer Black and Myron Scholes came up with a theorem for the valuation of European put option contract on non-dividend paying stokes.

The model proposes that;

$$P(S_t,t) = Ke^{-r(T-t)}N(-d_2) - S_tN(-d_1)$$

Where; $d_1 = \dfrac{\ln\left(\frac{S_t}{K}\right)+(r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$P(S_t,t)$ = Price of the European put option at time t.

K = The strike price

r = The annualized risk free rate of interest

$T - t$ =The current annualized time to maturity.

σ = The annualize standard deviation of the share price (the underlying asset)

S_t = The current share price (the underlying asset price)

N = The cumulative distribution function for a standard normal distribution

ln = The natural logarithm.

e = Exponential function

3.1.1   Assumptions of the Black-Scholes Model

The Black Scholes model for valuing put options makes use of certain assumptions. These include assumptions that;

1) The stock price (the price of the underlying asset) follows a Geometric Brownian Motion.

2) No transaction costs and commissions charged. This means that without any transaction costs or taxes trading can take place continuously.

3) Assets are perfectly divisible and short selling is permitted.

4) Interest rates remain constant and known. That is the continuously compounded risk free interest rate is constant.

5) Assets can be borrowed or lend by investors at the same risk-free rate of interest.

6) All risk-free portfolios earn the same return. That is there are no riskless arbitrage opportunities.

7) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. As a result the distribution of the various stock price at the end of any infinite time interval is log normal and the variance rate of return on the stock is constant.

8) The Black Scholes model assumes the European style option which can be exercised only at expiration.

9) The underlying asset pays no dividend.

One of the major assumption of the original Black Scholes model is that the underlying asset pays no dividend. This paper however uses the modified Black Scholes model for pricing a put option under the crucial assumption that the dividend yields are deterministic. That is, over the remaining time to expiration, the option dividends are at most a known function of time and /or of the underlying asset.

This assumption is realistic given the stable dividend policy that most corporations tend to follow over a short horizon and the short-term life of trade options.

The price of European put option whose underlying asset pay dividend continuously denoted by q is thus given by;

$$P(S_t, t) = Ke^{-r(T-t)}N(-d_2) - S_t e^{-q(T-t)}N(-d_1)$$

Where $\quad d_1 = \dfrac{\ln\left(\frac{S_t}{K}\right) + (r - q + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$q =$ The rate at which the underlying asset pays its dividends.

3.1.2   Drawback of the Black-Scholes Model

It should be apparent that none of these assumptions assumed by the Black Scholes model can be entirely satisfied. This paper analyzes these limitations by putting to challenge the major assumptions underlying the model as follows.

1) **Volatility is constant over time**. Volatility is a measure of how a given stock is expected to move within the near future. While it is likely to be relatively so in the short term, in the long run it can never be constant. In practice, large changes in price tend to be followed by even larger changes and vice-versa, resulting in volatility clustering. This might be the reason why most advanced option valuation models swap the black-Scholes' constant volatility with estimates generated from stochastic processes.

2) **Direction of the market or an individual stock cannot be consistently predicted**. The model assumes that stock prices follow a random walk, in that at any given time , the price can move up or down with equal probability. Unfortunately stock prices are subject to several economic factors that may not be assigned the same probability. In addition, the martingale property of a Brownian motion explains that the stock price at time *t+1* is independent from that at time *t*.

3) **The returns of log-normally distributed stock prices follow a normal distribution**. While this is a reasonable assumption in the real world, it struggles to fit observed financial data. Unlike stable distributions such as the log-normal distribution, asset returns have infinite variance as well as heavy tails (Clark, 1973).

4) **Interest rates are known and constant**. The Black-Scholes model is built around a risk-free rate of interest, which remains constant during the term of the contract. Often, government treasury bills 30-day rates are used. The pitfall here lurks in the fact that these rates can change significantly in periods of increased volatility.

5) **The underlying stock doesn't pay dividends during the life of the option**: The original, Black-Scholes model does not make allowance for dividends, despite the fact that most companies pay out dividends. One way to work around this problem is to subtract the discounted value of future dividend from the stock price. This paper uses the improved version of black-Scholes model that allows for dividend payment.

6) **No commissions or transaction costs**: The model assumes that no fees is charged for the buying and selling of options and that there are no barriers to trading. However, in practice, stock brokers charge varying rates.

7) **Option can only be exercised at expiration date**: American-style options, which can be Exercised on any day before or at maturity, may not be valued accurately by the model.

8) **Markets are perfectly liquid and any amount of options can be purchased at any time**: This is mostly impractical as investors are always constrained by the much they can invest, company policies and wish of the sellers to sell. In addition, the purchase or sell of fractions of options may not be possible.

In the light of these limitations, this paper will seek to address some of the limitations that come with the original Black-Scholes model that are fundamental to the market.

3.1.3   Adaptations of the Black-Scholes Model to Mortgage Insurance

Similarity between option contracts and mortgage insurance contracts

A major part of this study revolves around how the Black-Scholes model that has been widely used in pricing derivatives can be applied in the pricing of mortgage insurance. Before getting into the translation of parameters for the model, it is important to appreciate the similarities between options and mortgage insurance.

1) **Hedging operations**.

Option contracts and Mortgage insurance contracts are both hedging operations. While options cover agents against unexpected price movements, insurance covers the lenders of mortgages against unexpected default by borrowers.

2) **Premium payment**.

In both cases, the purchaser of the contract has to part with a premium. The buyer of an option pays a premium at the time of purchase to obtain the desired hedge while borrowers pay premiums at the start of the mortgage term that will be used to cover the lenders in case of default.

3) **Compensation**.

In the event that the unexpected situation arises, a compensation must be paid in either case. When unexpected price changes occur, the holder of a put option will exercise it,

receiving a compensation equivalent to the difference between the strike price and the market price. In case the outstanding loan at the time of default exceeds the value of the collateral the lender whom the Mortgage Insurance covers will be compensated. In case the covered event does not materialize the premium is forfeited in both case.

3.2   Replicating the Mortgage Insurance Contract as an Option Contract

The essential elements of an insurance contract are:

1) Writer

2) Buyer

3) Premium

4) Underlying asset

5) Strike Price

6) Expiration date

In the case of a mortgage insurance contract, the buyer of the insurance contract is the borrower of the mortgage loan and the beneficiary of the contract is the lender. The writer of the contract is the insurance company that is providing the cover. The strike price is the amount of the outstanding loan in case of default. The underlying asset being the most important part of an option, in this model, the underlying asset is defined as the value of the collateral(house) at the time of default. The expiration date is defined as the end of term of the mortgage loan and the premium paid to the insurance company at the inception of the policy is the "option" premium.

In case the outstanding loan at the time of default is lower than the value of the collateral, upon repossession of the collateral and its subsequent selling the outstanding loan will be settled. And the lender will not exercise their right to "sell " the collateral at the outstanding loan amount (strike price).

In the event that the outstanding loan balance upon default is more than the value of the collateral (market price of the collateral) the insured lender will exercise the option, he/she will exercise the right to "sell " the collateral at the outstanding loan balance. This is achieved by the lender receiving from the insurer the difference of the outstanding loan and the market value of the house at the time of default. The contract is therefore settled by difference.

In both cases the total cost to the borrower (the one who buys the insurance cover) will be the premium paid and the value of the collateral upon default. Clearly, an insurance contract can be replicated as an option contract.

The table below summarizes the variables of mortgage insurance policies that directly translate to our model.

	Options contract	Mortgage insurance contract
Buyer of the contract	Buyer of an option	The insured (in this case both the borrower and the lender.)
Writer of the contract	Writer of an option	Insurance company
Premium	Premium payable at inception	Premium paid by the insured
Underlying asset	Price of the security	Value of the collateral
Strike price	The exercise price	Outstanding loan balance
Time	Time to maturity	Time to end of term of the mortgage.

Table 1: Translation of the Black-Scholes Parameters to a Mortgage Insurance Contract

When purchasing a European put option the buyer guarantees a maximum price for selling the underlying asset when the contract matures. In case the market price is higher than the strike price , the holder of the option is under no obligation to exercise the put option .If he/she wishes to sell the asset , the market price has to be paid. On the contrary, if the market price is lower than the strike price, the holder of the put option will exercise the right and sell the underlying asset at the strike price. Therefore for a put option the payoff is $(K - S(t))^+$ The positive outside the bracket indicates that the payoff is non-negative. Since the payoffs are non-negative, a premium must be paid to buy an option. If no premium was paid, an investor purchasing an option could under no circumstances lose money and would in fact make a profit whenever the payoff turned out to be positive. This would be contrary to the no arbitrage principle. The premium is the market price of the option.

The gain of a put option buyer (writer) is the payoff modified by the premium P, say paid (received) for the put option. At time t, the gain to the buyer of a European Put is $((K - S(t))^+ - Pe^{rt})$. This gain is illustrated in the figure below.

For the writer of the option the gain is: $Pe^{rt} - (K - S(t))^+$. The potential loss for a buyer of a put option is always limited to the premium paid, but for the writer the loss can be much higher.
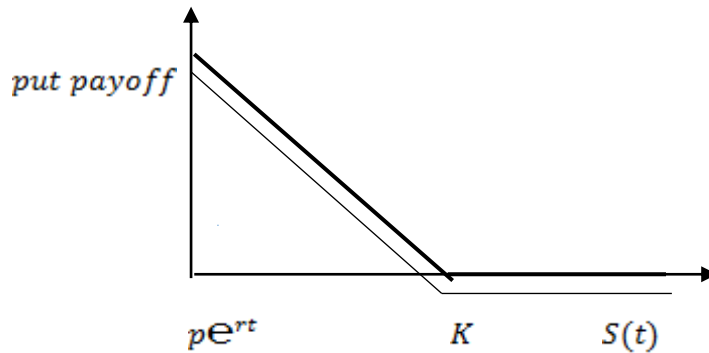

put payoff

pe^{rt} K $S(t)$

Figure 1: Payoffs (Darker Line) and Gains (Lighter Line) for a Buyer of a European Put Option

3.3 The Model

Let $V(t)$ be the price at time t of a risky asset (this asset is referred to as the collateral). Let there exist a risk-free asset in the economy with a constant continuously compounded annual return r. There are three types of agents in the economy: the lender, the borrower, and the insurer.

At origination, which is at time $t = 0$, the lender issues a T-year mortgage, that is secured by the collateral, for the amount of $B(0) = L_v V_0$. Where, L_v is the initial loan-to-value ratio and $V(0) = V_0$ is the initial value of the collateral. The assumption is that the mortgage loan has a fixed interest rate $c > r$ and that installments y are paid annually. With no default or prepayment prior to time t, the balance on the loan at time $0 < t < T$ is given by this expression (equal to the value of an ordinary annuity with an annual payment equal to y and the discount rate equal to the contract rate c)

$$B(t) = \frac{y}{c}\left(1 - \frac{1}{(1+c)^{T-t}}\right)$$

At time $t = 0$, also, the insurer writes a mortgage insurance contract that promises to compensate the lender in the case of the borrower's default. According to a typical MI contract (Kau et al., 1995), if a default occurs at time t, the insurer has to pay the lender the amount:

$$Loss(t) = \max\left(0, \min\left(B(t-1) - V(t), L_R B(t-1)\right)\right) \qquad \text{expression (1)}$$

The quantity L_R in the above equation is called the loss ratio. It is implied by the expression that if the collateral value is greater than the outstanding loan, after the selling of the collateral and the lender is compensated from the proceeds, the lender gets no loss and, therefore, the loss to the insurer is zero. Whereas, if the collateral value is not enough for a full settlement of the loan balance, the maximum loss to the insurer is equal to $L_R B(t-1).$ in the Figure 2 below, it is assumed that the collateral is repossessed instantaneously by the lender at the time of the default. This assumption is relaxed later when time to repossession of the collateral is considered.
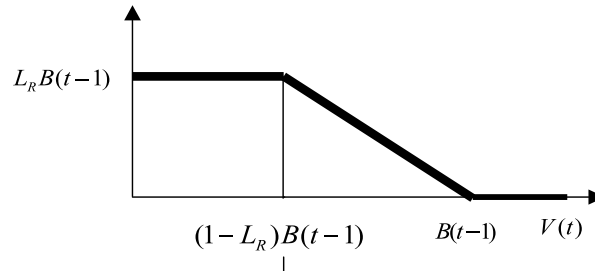
Figure 2: Mortgage Insurance Payoff as a Function of the Collateral Value at the Time of Default, $V(t)$.

According to the literature, the value of the collateral can be modelled as follows:

$$\frac{dV(t)}{V(t)} = (\mu - s)\, dt + \sigma dw(t)$$

Where; μ = the expected return on the collateral

s = maintenance yield.

$\mu - s$ = expected appreciation of the collateral value.

The assumption is that the parameters μ, s, and σ are known scalar quantities and the stochastic process $w(t)$ is a Standard Brownian Motion. As a result, $V(t)$ is a Geometric Brownian Motion

and, thus, its conditional distributions are log-normal .The severity of loss is calculated by taking the expectation of expression(1) ,it generally depends on the expected return μ. It is assumed that the agents in the economy are risk neutral so as to eliminate possible ambiguity related to market incompleteness. The consequence of the risk neutral assumption implies that $\mu = r$. Hull (1999)

It is finally assumed that the unconditional probability of borrower default at time $t \in [1,2,...,T]$ is determined exogenously and set to equal $P_d(t)$ where $P_d(t)$ is a probability distribution function such that $0 < P_d(t) < 1$, for all $t \in [1,2,...,T]$ and $\Sigma P_t(t) = 1$.

3.3.1   Assumptions of the Model

For the above adaptation of the Black-Scholes model to be valid in pricing of mortgage insurance premiums, the following assumptions are made.

1) There are no costs associated with purchasing insurance cover and no taxes. Also, the borrower (the party paying the premium) is free to choose the loan amount he desires. This implies that the strike price is chosen freely.

2) The collateral to the loan is the house and its value is the price of the underlying asset in the option contract i.e. there exist no other loan collateral apart from the house.

3) All policies with the same extent of benefit have the same premium amount. That is, all mortgage loans for houses with the same value have the same premium amount. Implying that the market is free from arbitrage opportunities.

4) There exists a risk free rate of interest that is assumed to hold constant during the term of the loan

5) Policies are purchased at the beginning of the term of the loan

6) Once a particular loan has been offered and the loan schedule determined i.e. the outstanding loan balance at the end of each year, there will be no changes to the amortized loan schedule until the end of the term of the loan.

7) Individual in the market are rational, they prefer more cover to less cover.

3.3.2   The Main Results

The expected loss to the insurer conditional on default happening at time $t \in [1,2,...,T]$ and discounted back to the present time, is given by the following expression:

$$CL(t) = e^{-rt}E\big(Loss(t)\big) = e^{-rt}E(\max(K_1 - V(t),0)) - e^{-rt}E(\max(K_2 - V(t),0)) \quad (4)$$

where $K_1 = B(t-1)$ and $K_2 = (1-L_R)B(t-1)$

The last quantity in (4) comes from the following identity valid for all realizations of V(t).

$$\max(0, \min(B(t-1) - V(t), L_R B(t-1))) = \max(K_1\text{-}V(t), 0) - \max(K_2\text{-}V(t), 0)$$

Equation (4) implies that; The present value of the expected loss to the insurer given default at time $t \in [1,2,...,T]$ can be represented as the value of a portfolio of two put options a long position in a European put option with a strike price K_1 and a short position in a European put option with strike price K_2 both with time to maturity equal to time to default. Bardhan et.al(2003) The value of the collateral is the underlying asset for both of these options. The expected loss can now be represented using the standard pricing results for European put options with constant dividend yields. The assumption of constant dividend yield stems from the expected appreciation of the collateral value. Hull (1999).

$$CL(t) = Put(K_1,t) - Put(K_2,t)$$

Where; $Put(K_i,t) = K_i e^{-rt} N(-d_2(K_i)) - V_0 e^{-st} N(-d_1(K_i)), i = 1,2$

$$d_1 = \frac{\ln\left(\frac{V_0}{K_i}\right) + \left(r - s + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, \; d_2(K_i) = d_1(K_i) - \sigma\sqrt{t} \; i = 1,2.$$

3.4 Parameter Estimation

a) Volatility

Volatility is a measure of the dispersion of returns for a given financial asset relative to a central trend or drift. In option pricing models ,option value depends on the returns of the underlying asset therefore an assumption on the estimate of the expected volatility is required. The higher

the volatility implies that the return of the underlying asset changes dramatically over a short period either up or down.

From the black Scholes model,

$$dS_t = rS_t dt + \sigma S_t dW_t$$

$$S_T = S_t \exp\left(\left(r - \frac{\sigma^2}{2}\right)(T - t) + \sigma\sqrt{T - t} * Z\right)$$

$$\log_e\left(\frac{S_T}{S_t}\right) = \exp\left(\left(r - \frac{\sigma^2}{2}\right)(T - t) + \sigma\sqrt{T - t} * Z\right)$$

$$\log_e\left(\frac{S_t}{S_{t-1}}\right) = \left(\left(r - \frac{\sigma^2}{2}\right)(t - t + 1) + \sigma\sqrt{t - t + 1} * Z\right)$$

$$\log_e\left(\frac{S_t}{S_{t-1}}\right) = \left(r - \frac{\sigma^2}{2}\right) + \sigma * Z$$

$$Var\left(\log_e\left(\frac{S_t}{S_{t-1}}\right)\right) = var\left(\left(r - \frac{\sigma^2}{2}\right) + \sigma * Z\right)$$

$$= \sigma^2 Var(Z)$$

$$= \sigma^2$$

$$Therefore, \quad \sigma = \sqrt{\left(Var\left(\log_e\left(\frac{S_t}{S_{t-1}}\right)\right)\right)}$$

Where the process $\{S_t, t \geq 0\}$ is the price process of the underlying asset. Z is a random variable with a standard normal distribution with mean 0, and variance 1. r is the risk free rate of interest.

 The volatility (standard deviation obtained) must however be annualized to make it suitable for use in valuation. Let σ^* be the annualized volatility ,therefore

$$\sigma^* = \sigma * \sqrt{Sample\ frequency\ in\ years}$$

b) Strike Price

The strike for each particular year t, is set as B(t-1) . Which is the loan balance at the time of default t, calculated as the outstanding loan balance at the beginning of year t. This will always vary from year to year. It is not a stochastic variable but a deterministic variable since its value is determined at the outset.

c) Time

The time component will be taken as one year. Each year ,during the term of the loan ,will be taken as independent periods. The value of severity of loss (CL(t)) will be determined for each year, conditional on default happening at that year.

3.5   Actuarially Fair Price and Probabilities of Default

Actuarially fair price has an expected net pay-off of zero that is the premiums paid are equal to the expected value of the compensation received. Such an insurance policy makes zero economic profit. The actuarially fair price (AFP) is determined as the sum of the expected loss for each year of the mortgage life. It is given by:

$$AFP = \sum_{t=0}^{T} P_d(t)\, CL(t)$$

Where $P_d(t)$ is the unconditional probability of defaulting at time $t \in T$ and it does not depend on the value of the collateral. $CL(t)$ is the present value of the severity of loss, that is the expected loss to the insurer conditional on default happening at time $t \in T$ and discounted back to the present time. AFP is the present value of the accumulated expected loss for an arbitrary set of exogenous probabilities of default (Bardhan et al, 2003).

Following DKY (Deng et al, 2000), for each $t \in T$, there are probabilities of defaulting and prepaying at time t, given that no prepayment or default occurred prior to that time. These conditional probabilities are denoted by d_t and p_t respectively. The probability of neither defaulting nor prepaying, that is staying current, is denoted as π_t and is given by:

$$\pi_t = 1 - d_t - p_t \quad ; t \in T$$

The unconditional probability of defaulting at time $t \in T$ is given by:

$$P_d(t) = \pi_1 \pi_2 \ldots \pi_{t-1} d_t$$

This formula implies that the borrower stayed current only until time *t-1* and then defaulted at time *t*. This unconditional probability, $P_d(t)$, is equal to the probability of defaulting at time $t \in T$ conditional on not defaulting or prepaying at previous times up to time *t*. It is therefore a product of all the previous conditional probabilities of staying current.

The conditional probability of defaulting, d_t , is derived from the conditional default rate (CDR). CDR represents the monthly default rate expressed as annual percentage. It is an annualized value of the outstanding balance of the loans that defaulted in the current month as a percentage

of the outstanding balance of the mortgage portfolio, (Fabozzi et al, 2007). This is given mathematically by:

$$MDR_t = \frac{Default\ loan\ balance_{month\ t}}{Beginning\ balance_{month\ t} - Scheduled\ principal\ payment_{month\ t}}$$

To obtain the CDR, the MDR is annualized, that is:

$$CDR = 1 - (1 - MDR_t)^{12}$$

3.6 The Cost of Legal Inefficiency

An inefficient legal system adversely affects the profitability of the insurer in the event of a default. The time taken by the lender to repossess the collateral when the borrower default is a measure of the inefficiency of the legal structure. A significantly longer period to repossession following a borrower's default depicts an inefficient legal system. Different countries experience different average lag time to repossession with most transition and developing economies experiencing the longest lags.

The pricing model so far assumes that the collateral is immediately transferred to the lender in case of borrower's default. In reality, this is never the case. If a default happens, the lender starts up the court procedure of repossessing the collateral. There is a lag between the stoppage of payments and the repossession of the collateral by the lender. Such delays are a major source of additional expenses to the insurer. During this lag time, the lender faces the opportunity cost of lending the amount $B(t-1)$, the balance remaining at the time of default, to another borrower (at an interest rate equal to c).

The insurer is normally required by law to compensate the lender for the losses accumulated including such opportunity cost losses. The insurance company would have to pay the lender the amount equal to:

$$Loss(t+\tau) = \max\left(K_1^l - V(t+\tau), 0\right) - \max\left(K_2^l - V(t+\tau), 0\right)$$

$$K_1^l = (1+c)^\tau B(t-1), \qquad K_2^l = (1-L_R)(1+c)^\tau B(t-1)$$

Note that this equation coincides with the pay-off from the insurance company to the lender when the collateral value does not cover the loan balance that is when $B(t-1) > V(t)$ and $\tau = 0$, which is:

$$Loss(t) = \max(0, \min(B(t-1) - V(t), L_R B(t-1)))$$

On the other hand, this loss is partially offset by the gains that the insurer has by not paying off the claim right away. Such gains would be equal to:

$$Gain(t+\tau) = e^{r\tau} \cdot Loss(t)$$

Both $Loss(t+\tau)$ and $Gain(t+\tau)$ are expressed in terms of money at time $t+\tau$, that is the moment in time when the court procedure ends. Net loss from legal inefficiency is obtained by subtracting $Gain(t+\tau)$ from $Loss(t+\tau)$. If a default occurred at time $t \in T$, the present value of the expected net (additional) loss resulting from legal inefficiency is given by:

$$LCL(t) = e^{-r(t+\tau)} E\left(Loss(t+\tau)\right) - e^{-r(t+\tau)} E\left(Gain(t+\tau)\right)$$

$$= \left[Put\left(K_1^l, t+\tau\right) - Put\left(K_2^l, t+\tau\right)\right] - \left[Put(K_1, t) - Put(K_2, t)\right]$$

Clearly, the present value of the net expected loss is zero if $\tau = 0$, that is there is no delay in repossession of the collateral. The present value of the expected loss when there is legal inefficiency is given by the summation of the present value of the expected loss when there is no inefficiency and the net present value of the legal inefficiency loss, that is:

$$Updated \; CL(t) = CL(t) + LCL(t)$$

$$Updated \; CL(t) = \left[Put\left(K_1^l, t+\tau\right) - Put\left(K_2^l, t+\tau\right)\right]$$

Substituting the above equation into APF formula gives the expression for the actuarially fair value of a MI contract when the cost of legal inefficiency is taken into consideration. Finally, assuming that the insurer requires a gross profit margin equal to g and that the premium is collected upfront, the MI premium is given by:

$$Premium \; of \; MI = (1+g)AFP$$

3.7   Sensitivity Analysis – The Greeks

Sensitivity analysis studies how any uncertainty in the output of a mathematical model can be apportioned to different sources of uncertainty in its inputs. The Greek letters: Delta, Gamma, Rho, Theta and Vega are used in option pricing to show the sensitiveness of an option price relative to changes in value of either a state variable or a parameter. Each of them shows the rate of change of the option price with respect to different parameters or state variables allowing financial institutions that sell option products to manage their risk they are likely to encounter as option contracts do not usually correspond to standardized products traded in the exchanges. In our study we aim to find out the sensitiveness of the premium obtained using the Black-Scholes Model due to variations in different parameters that we will estimate.

3.7.1   Delta (Δ)

Delta is defined as the rate of change of the option price with respect to the price of the underlying asset. It is the slope of the curve that relates the option price to the underlying asset price.

$$\Delta = \frac{\partial}{\partial S}$$

$$\frac{\partial}{\partial S} = \frac{\partial}{\partial S}\left\{\left[Ke^{-r(T-t)}N(-d_2)\right] - Se^{-q(T-t)}N(-d_1)\right\}$$

$$= Ke^{-r(T-t)}n(-d_2)\frac{\partial}{\partial S}(-d_2) - Sn(-d_1)\frac{\partial}{\partial S}(-d_1)e^{-q(T-t)} - e^{-q(T-t)}N(-d_1)$$

$$= \frac{1}{S\sigma\sqrt{T-t}}\left[-Ke^{-r(T-t)}n(-d_2) + Sn(-d_1)e^{-q(t-t)}\right] - e^{-q(T-t)}N(-d_1)$$

$$\Delta = -N(-d_1)e^{-q(T-t)} \text{ , since } Sn(-d_1)e^{-q(T-t)} = Ke^{-r(T-t)}n(-d_2)$$

Where $n(z)$ and $N(z)$ are the standard normal probability density function and cumulative distribution function respectively.

3.7.2   Gamma (Γ)

The gamma (Γ) of a portfolio of options on an underlying asset is the rate of change of the portfolio's delta with respect to the price of the underlying asset. It is the second partial derivative of the portfolio with respect to asset price.

Therefore the gamma for the Black Scholes model is:

$$\Gamma = \frac{\partial^2}{\partial S^2} = \frac{\partial \Delta}{\partial S} = \frac{\partial}{\partial S}\left[N(-d_1)e^{-q(T-t)}\right]$$

$$= -e^{-q(T-t)}n(-d_1)\frac{\partial}{\partial S}[-d_1]$$

$$= \frac{e^{-q(T-t)}n(-d_1)}{S\sigma\sqrt{T-t}}$$

3.7.3 Vega (υ)

The Vega of a portfolio of derivatives is the rate of change of the value of the portfolio with respect to the volatility of the underlying asset. If the absolute value of Vega is high, the portfolio's value is very sensitive to small changes in volatility. If the absolute value of Vega is low, volatility changes have relatively little impact on the value of the portfolio.

$$\upsilon = \frac{\partial}{\partial\sigma}$$

$$= \frac{\partial}{\partial S}\left[Ke^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1)\right]$$

$$= Ke^{-r(T-t)}n(-d_2)\left[\frac{\partial d_1}{\partial\sigma} - \sqrt{T-t}\right] - Se^{-q(T-t)}n(-d_1)\frac{\partial}{\partial\sigma}(-d_1)$$

$$= \frac{\partial d_1}{\partial\sigma}\left[Ke^{-r(T-t)}n(-d_2) - Se^{-q(T-t)}n(-d_1)\right] - Ke^{-r(T-t)}n(-d_2)\sqrt{T-t}$$

Where; $Se^{-q(T-t)}n(-d_1) = Ke^{-r(T-t)}n(-d_2)$

And $\frac{\partial d_2}{\partial\sigma} = \frac{\partial d_1}{\partial\sigma} - \sqrt{T-t}$

$$\upsilon = Sn(-d_1)e^{-q(T-t)}(T-t)$$

3.7.4 Rho (ρ)

The rho of a portfolio of options is the rate of change of the value of the portfolio with respect to the interest rate. It measures the sensitivity of the value of a portfolio to a change in the interest rate when all else remains the same.

$$\rho = \frac{\partial}{\partial r}$$

$$= \frac{\partial}{\partial r}\left[Ke^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1)\right]$$

$$= \left[K e^{-r(T-t)} n(-d_2) - S e^{-q(T-t)} n(-d_1) \right] \frac{\partial(-d_1)}{\partial r} - K(T-t) e^{-r(T-t)} N(-d_2)$$

$$\rho = -K(T-t) e^{-r(T-t)} N(-d_2)$$

Name	Symbol	Definition	Value in Black-Scholes Model
Delta	Δ	$\dfrac{\partial}{\partial S}$	$-N(-d_1) e^{-q(T-t)}$
Gamma	Γ	$\dfrac{\partial}{\partial \Delta}$	$\dfrac{e^{-q(T-t)} n(-d_1)}{S \sigma \sqrt{T-t}}$
Vega	υ	$\dfrac{\partial}{\partial \sigma}$	$S \sqrt{T-t}\, n(-d_1) e^{-q(T-t)}$
Rho	ρ	$\dfrac{\partial}{\partial r}$	$-K(T-t) e^{-r(T-t)} N(-d_2)$

Table 2: Summary of the Greeks of the Black-Scholes Model

4   Data Analysis

4.1 Description of the Data

For the empirical application of our proposed valuation model for valuing mortgage insurance contracts, we used data obtained from the Hass Consult Limited. Specifically, we used the *HassConsult index* developed by the company since the year 1998 to track the movement of the house prices up to the year 2014. The decision to settle on this company was informed by the fact that the company has been the market leader in the Kenyan Real Estate industry for the last 15 years and has a vast experience in the Kenyan real estate industry, having been in the market since its establishment in 1992. In addition, HassConsult Limited is the only real estate company that has an active House price index that dates back to the year 1998. The other house price index by the Kenya Association of Bankers is under development since it was established in January 2015.

The company has won numerous awards for the accuracy of its index in tracking of property prices and its engagement of the public in sharing of real estate information. It has won two top real estate prizes by the British based International Property Awards forum. It has also won the Best Real Estate agency marketing in Kenya as well as the Best Real Estate Agency in Kenya.

To ensure the accuracy of its information, the company employs statistical skills to ensure that the mixture of different types of properties sold on quarterly basis does not yield a false impression of the actual changes in house prices. The quarterly prices released by the company measures the mix adjusted average price of the house for upper and middle sections of the real estate market only. These are the main relevant sections in our model since they represent the actual prices of property based on the location. These prices included in the index are the average prices for the three types of homes, which are Apartments, House and Villas. The main purpose of the adjustment is mainly isolation of the pure price changes.

4.2 Reliability of the Data

As a leading property, company in Kenya, HassConsult Limited transacts many transactions involving property development, rental, selling and management. Majority of the information used in the construction of its index is obtained from the *HassConsult* sales data that is recorded at the date of transaction to show the true prices of the property as at that date. The data is usually recorded on a monthly basis after the price agreement has been reached. To supplement

its data, the company also obtains other information from the public domain that includes more than 20 estate agencies that are located in Nairobi and Mombasa as well as data from the *propertyleo* database.

4.3 Data Filtration

We obtained data for the overall property prices, prices for detached, apartments and semi-detached properties and rental prices for detached, apartments and semi-detached houses for the quarterly periods between January 2010 and December 2014 showing the individual prices for each category and average quarterly and annual changes. We also obtained data on Hass Consult sales index and rental sales index for detached, semi-detached, apartments and the adjusted mix for all properties from for the period ranging between January 2004 and December 2014.

The index covers residential property for the following areas:

Muthaiga, Kitisuru, Nyari, Lower Kabete, Runda, Karen , Rosslyn, Wispers, Gigiri, Kyuna Estate, Spring Valley, Brookside, Muthaiga North, Riverside, Lavington, Westlands, Mountain View, Upperhill, Kilimani, Langata, Kileleshwa, Nyali, Parklands, Valley Arcade, State House, Hurlingham, Kiambu, South C, South B, Fedha Estate, Nairobi West, Thika, Industrial Area, Kitengela, Ngong, Madaraka, Embakasi, Ongata Rangai, Imara Daima, Athi River, Nyayo Estate, Tena, Koma Rock.

For the empirical application of our model, we settled on the Hass Consult annual property index from the year 2000 that we used to track the movement of house prices for 15 years. Our decision to settle on this index was because it was more relevant to our study since it included the adjusted mix for all the three property types sold in the upper and middle suburbs in Nairobi. This enabled us to track the price changes for 15 years in order to determine the volatility of house prices over different economic periods within which the housing industry has experienced growth, depression and stability. These periods were seven years that preceded the year 2007, during which the housing industry experienced a boom. This boom busted in 2008 during the global economic crisis. However, the industry gained stability from the year 2010 as global economy started to recover from the effects of 2008 financial crisis.

This index was justifiably accurate for tracking the house prices because of its adjustment in the year 2010 to reflect the international standards recommended by the International property

association. It was also suitable for our study because it was developed by a Property Company that has direct contact with the Kenyan market.

4.4 Parameter and Variable Estimation

After showing that a mortgage insurance contract can be replicated as an option contract, the parameters invariant to the model had to be estimated to make it possible to apply the proposed Black-Scholes model to pricing the mortgage insurance contract. The parameters were estimated from the relevant data obtained. Below are the main parameters that needed to be estimated in order to apply the proposed model:

1. Price of the underlying asset (S)
2. The Strike price (K)
3. Volatility of the underlying asset (σ)
4. The continuous time interest rate for the time of option valuation (r)
5. The time to maturity of the option contract (T-t)

a) **Price of the Underlying Asset (S)**

The underlying asset is the most important part of an option. In the proposed model, the price of the underlying asset is the value of the collateral (house). The value of the house/collateral varies from time to time just like the price of an option in an option contract. Thus, valuation of the option at time t is based on the price of the underlying asset at that time, S_t. In the proposed model therefore, the price of the mortgage insurance contract at time t, say, depends on the value of the collateral at that time. Since our valuation was annual, the price of the underlying asset was the value of the house at the beginning of each year.

b) **The Strike price (K)**

The strike price was taken to be the outstanding loan balance at the time of default. This assertion is justified because the buyer of the option (the insured lender) may decide to exercise or not to exercise the right to "sell" the collateral at the outstanding loan amount at the time of default if the default occurs.

c) **Volatility (price variation) of the underlying asset**

The volatility of the underlying asset is critical in the valuation. To estimate the volatility, we used the approach proposed by the Black-Scholes Merton model (1973).

The quarterly house prices from the year 2004 to 2014 were used to calculate the standard deviation of the house prices. The following formula was used for this purpose.

$$\hat{\sigma} = \sqrt{Var\left(\ln\left(\frac{S_t}{S_{t-1}}\right)\right)}$$

Where S_t and S_{t-1} are the price of the underlying asset at time t and the price of the underlying asset in the previous year respectively.

However, since we needed the annual volatility to apply it in our model, we expressed this quarterly volatility in annual terms. To achieve this, we multiplied the quarterly volatility with the square root of the number of years that our mortgage loan goes for. The number of years is 15 in our case.

The results gave an annualized volatility of 18%. We then used this value as a forecast of the volatility of house prices of the 15-year period that we considered as the average time to maturity of a typical mortgage in Kenya.

d) **Continuous Time Interest Rate (r)**

In the Black-Scholes model, an annualized risk-free interest rate is used to calculate the price of an option. In our proposed model, we used the interest rate for the Central Bank of Kenya's 364-day Treasury Bills (T-Bills) as the risk-free rate. This assumption was justified because the interest rate for the T-Bills is usually used as a benchmark risk-free rate of interest since government securities are usually risk-free. The risk-free rate was therefore taken to be the 9th March 2015 interest rate for the Treasury Bills, which was 10.636%.

e) **Time to maturity (T-t)**

In our proposed model, we used a time to maturity of 1 year since we are interested in calculating the severity of loss to the insurer for each year. To achieve this, we need to calculate the price of the option contract (mortgage contract) for each year.

4.5 Premium Calculation under the Proposed Model

Determining the price of a mortgage insurance premium in emerging economies is a complex process due to the scantiness of relevant data, unlike in developed countries where it is straightforward because mortgage markets have a long tradition. We assume that the collateral is stochastic and that it follows a Geometric Brownian motion when option pricing is used as a method of pricing mortgage insurance.

In order to calibrate the model, we have to determine parameters that describe the pricing process, for instance volatility of collateral. HassConsult Limited provides the only available source of available data on residential real estate prices in Kenya. This has quarterly data on the aggregate real estate transaction prices from 1998 to 2014. We estimate historic volatility σ to be quite high (around 18%) because of the very uncertain environment of the Kenyan economy in the last several years. Maintenance yield s is set at 1%, half the typical US number since in many emerging markets, properties are less maintained than in the developed world (Kau et al, 1995).

Default and prepayment rates also play a significant role in determination of MI premium. In countries where long-term series data are collected on mortgage prepayment and default, for example US and Sweden, one can use such data to forecast empirically future values of these variables. However, in many developing and underdeveloped countries, there are only rudiments of a mortgage system and consequently no reliable data is available about default or prepayment rates. For instance, in Kenya, defaults rates used by some commercial banks are integrated with mortality probabilities.

In order to price MI we need to find plausible guesstimates. We settle on a comparables method. We start by analyzing historical prepayment and default experience in the US since data is readily available from the Federal Housing Administration (FHA). Since it is well known that default curve has an inverted U profile, we assume that the target experience curve for Kenya would have a similar shape. We use FHA default experience for a 15-year term mortgage loan from US General Accounting Office report on homeownership (May 1997) as our benchmark.

We then assume that the default probabilities in Kenya are similar to the findings on low income-high credit risk customers in US as reported by Deng and Gabriel (2002). According to them, default probabilities for such customers to be 3 times higher than the average FHA experience.
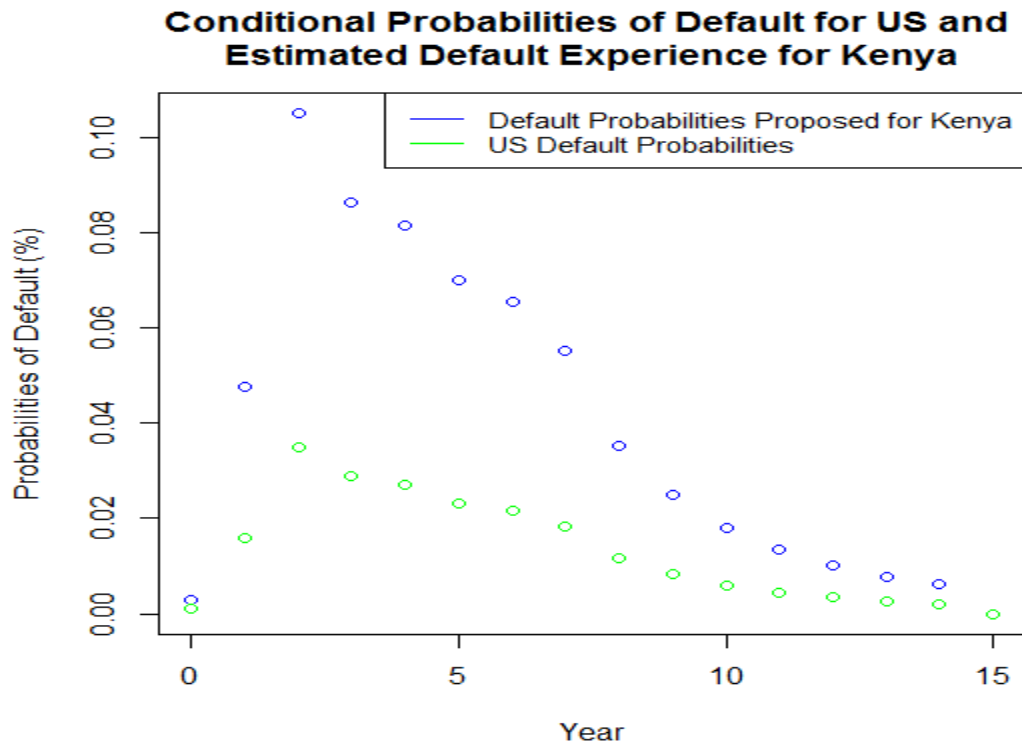

Figure 1: Unconditional Probabilities of Default for Actual US 1996 Experience and Estimated Default Experience for Kenya

Figure 3 depicts both the American default experience for 15-year residential mortgages in 1996 and the best estimate for Kenya on a 15-year scale.

The other parameters required by the model are mortgage amortization schedule, loan-to-value ratio LTV, loss ratio L_R, gross profit margin g, risk free interest rate r, contract rate c and the average time to repossession. Banks in Kenya are unwilling to issue mortgages with amortization periods greater than 20 years because they do not want the repayment period to go past retirement age. Pension-backed mortgage loans and those for special customers go upto 25 years. Most loans are between 10 to 20 years. We use the average (15 years) to value this model. LTV and L_R depend on the value the Insurance Regulatory Authority (IRA) proposes. $LTV = 80\%$ and

L_R = 60% are the most common ratios in Kenya. We therefore use these values as our base case scenario. We then select g = 10%. This is the margin protecting against unexpected losses.

The risk free rate r in Kenya is approximately equal to the 364-day Treasury bills, that is 10.636% according to the Central Bank of Kenya. The average mortgage contract rate is around 16.3% according to World Bank report on the mortgage market in Kenya. This value is high in emerging markets and reflects the shortage of long-term sources of funds, inflationary expectations and political uncertainity.

At present, the Kenyan legal system makes it impossible for a lender to reposses collateral without a lawsuit that could typically last several years, that is 3 years on average with some cases going to 5 years. It is because of this that lenders prefer to try settle such cases outside court. Therefore, there is strong pressure to significantly simplify and speed up such processes. Our model will consider 3 years to repossession of collateral. The average size of the insured property in Kenya is KSh. 4,000,000.

Table 3 shows a summary of all the base case scenario input values required for pricing and caliberating the model.

Parameter	Value
Expected annual return on the collateral μ	10.636%
Volatility of the value of the collateral σ	18%
Maintenance yield s	1%
Risk free rate r	10.636%
Loan-to-value ratio LTV	80%
Loan ratio L_R	60%
Gross profit margin g	10%
Morgage contract rate c	16.3%
Mortgage amortization period T	15 years

Average time to repossession τ	3 years
Average size of insured property V_0	KSh. 4,000,000

Table 3: Input Parameters in the Model

The MI premium for the base case values is estimated to be 6.0279% of the initial loan value. Taking into account the underwriting gross profit margin g, the adjusted MI premium is 6.63069%. This is an upfront premium. Clearly, the most significant influence on the price of MI is due to average time to repossession, probabilities of default, volatility of the collateral value, loss ratio and loan-to-value ratio.

Figure 4 depicts the sensitivity of MI premium towards the change of time to repossession in the case of borrower default, fixing the other parameters. As seen, as the average time to repossession increases, the MI premium increases exponentially. Decreasing the repossession time to a very small period would decrease the price of MI contract in Kenya drastically.
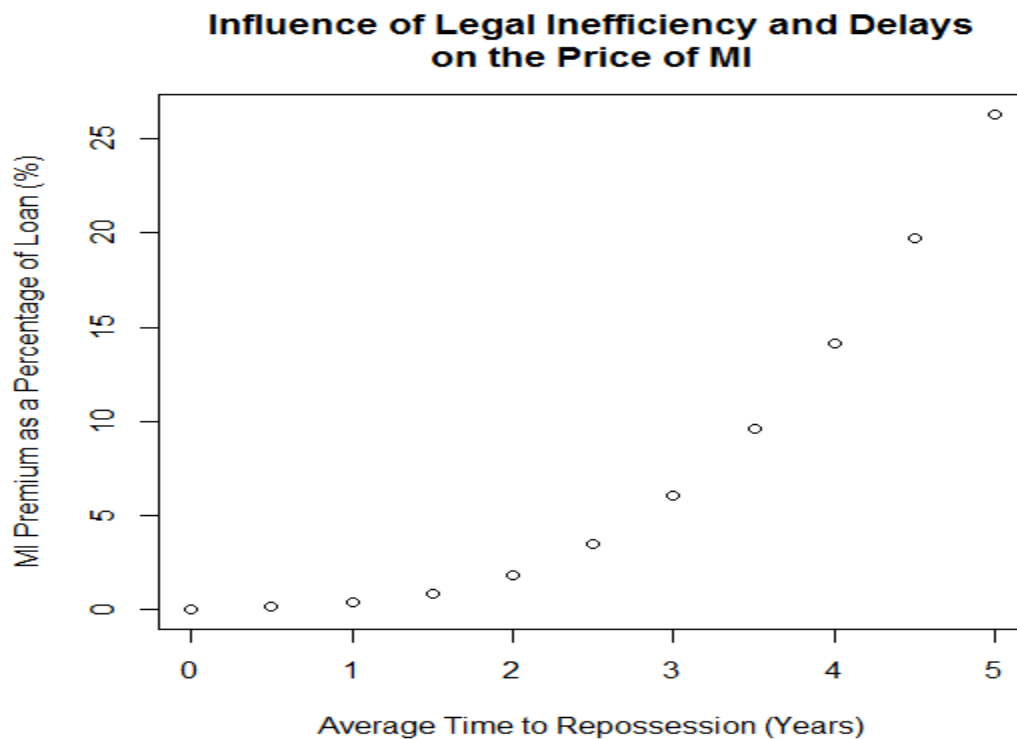

Figure 2: Influence of delay τ in repossession of the collateral on the price of MI contract

We can obtain a variety of plausible values for loss ratio and loan-to-value ratio. Sensitivity of the price of MI premium to changes in the loss ratio and the loan-to-value ratio is presented in figures 5 and 6 respectively.
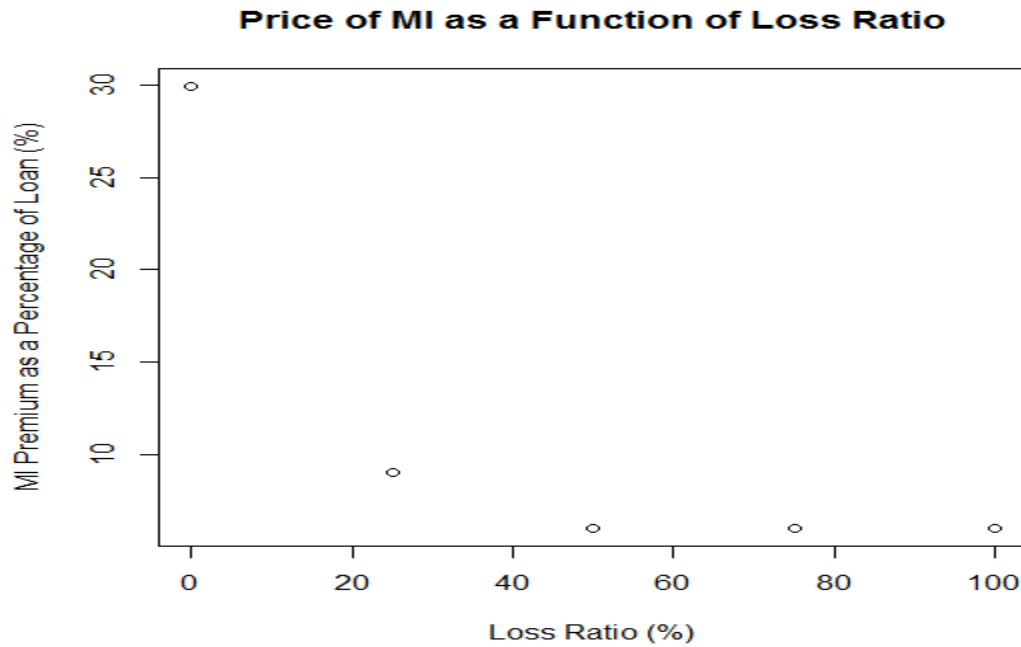

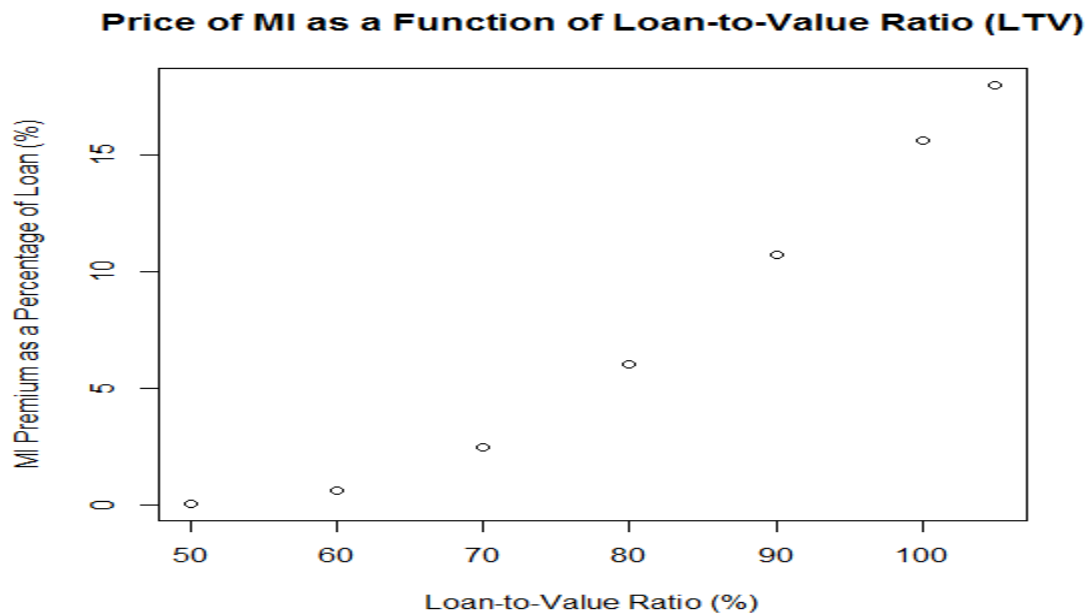**Figure 3: Price of MI as a Function of Loss Ratio**

Figure 4: Price of MI as a Function of Loan-to-Value Ratio

It can be seen from figure 5 that 50% is the optimal loss ratio. A loss ratio of 60% is therefore very acceptable. A very low loss ratio results in a very high MI premium because it implies that the insurance industry is very unstable and unfavorable economically.

Figure 6 shows that the price of MI increases with increasing loan-to-value ratio. This increase of MI premium is associated with the moral hazard resulting from a high loan-to-value ratio.

We can also check how the price of MI changes with various probabilities of default. For simplicity, we use a simple stretching transformation. In our case, we propose the Kenyan default experience to be a simple multiple of the US default experience. Figure 7 presents the sensitivity of the price of MI premium to changes in the US default probabilities multiplier.
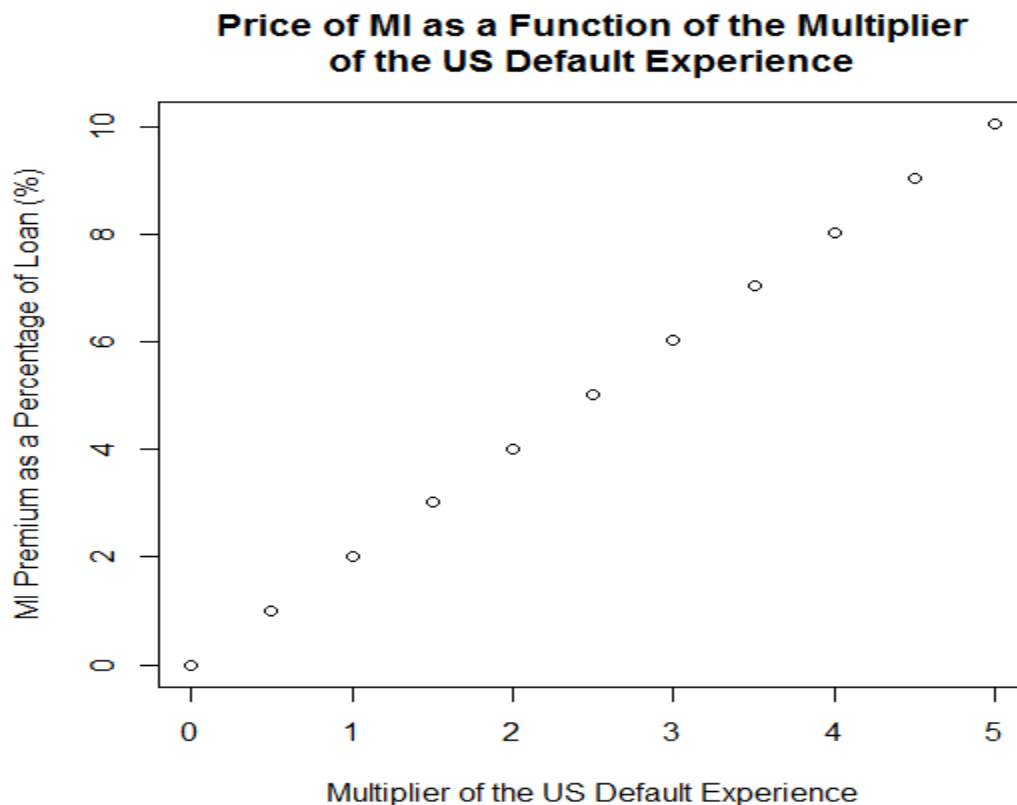

Figure 5: Price of MI as a Function of the Multiplier of the US Default Experience

It can be seen clearly that the price of MI increases with increasing US default experience multiplier, that is with increasing probabilities of default.

4.6 The Greeks
1) Delta

This is the rate of change of the MI premium with respect to the changes in house prices. It is the slope of the curve that relates the premium to the house prices. From the data on the premium obtained the delta of the data fell within the range (-0.03077997, 0.00000000) with mean value of -0.00619715. This can be interpreted to mean that an increase by one unit of house prices leads to the premiums decreasing by 0.6%.

2) Gamma

It is the second partial derivative of the MI premiums with respect to the Value of collateral (House prices). This is the rate of change of the delta with respect to the value of the collateral.
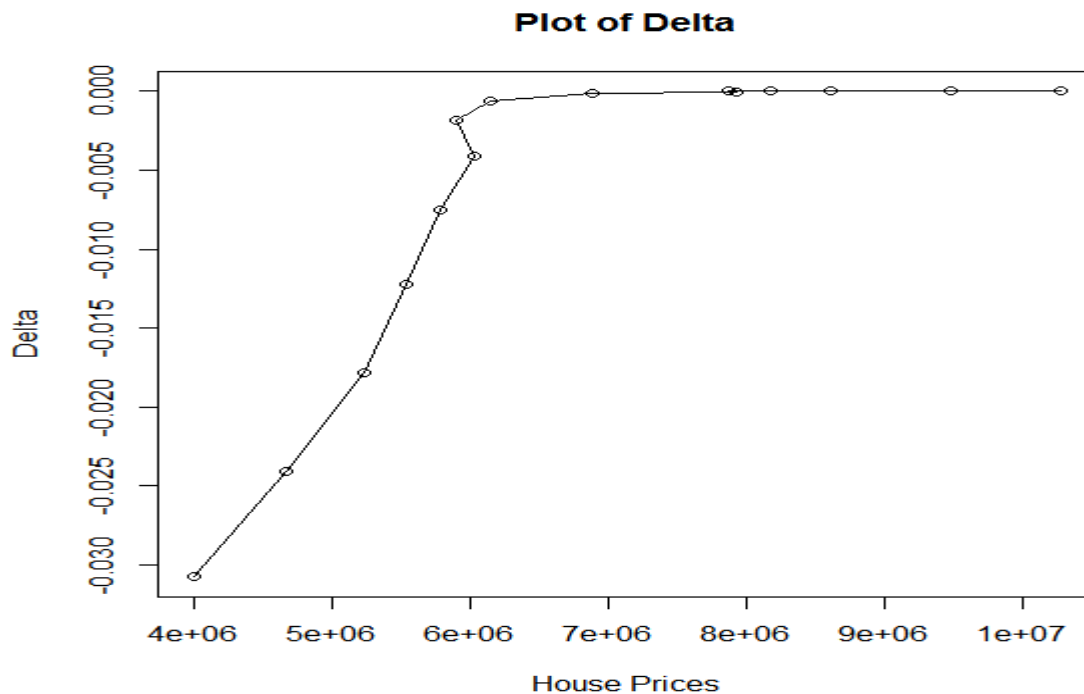

Figure 6: Plot of Delta against House Prices

From the data on premiums obtained, the gamma of the data fell within the range (0.00000, $9.63677*10^{-8}$) with a mean value of $1.755749*10^{-8}$. This shows that the effect of changes of the house prices (value of the collateral) on delta is insignificant. As shown in the graph above.

3) Vega

The Vega measures the rate of change of the MI premium with respect to changes in the volatility of the value of the collateral (house prices). From the data the Vega lies in the range of (0.08, 0.26) with an average of 0.17. This shows that on average every unit change in volatility causes 0.17 units change in the premium. This shows that the model is slightly affected by minor changes in volatility. As shown in the graph below. In figure 9 below, we show how changes in volatility of collateral value σ influence the price of MI. It can be observed that when $\sigma = 0\%$, the required insurance premium is very close to MI premium values in developing economies.
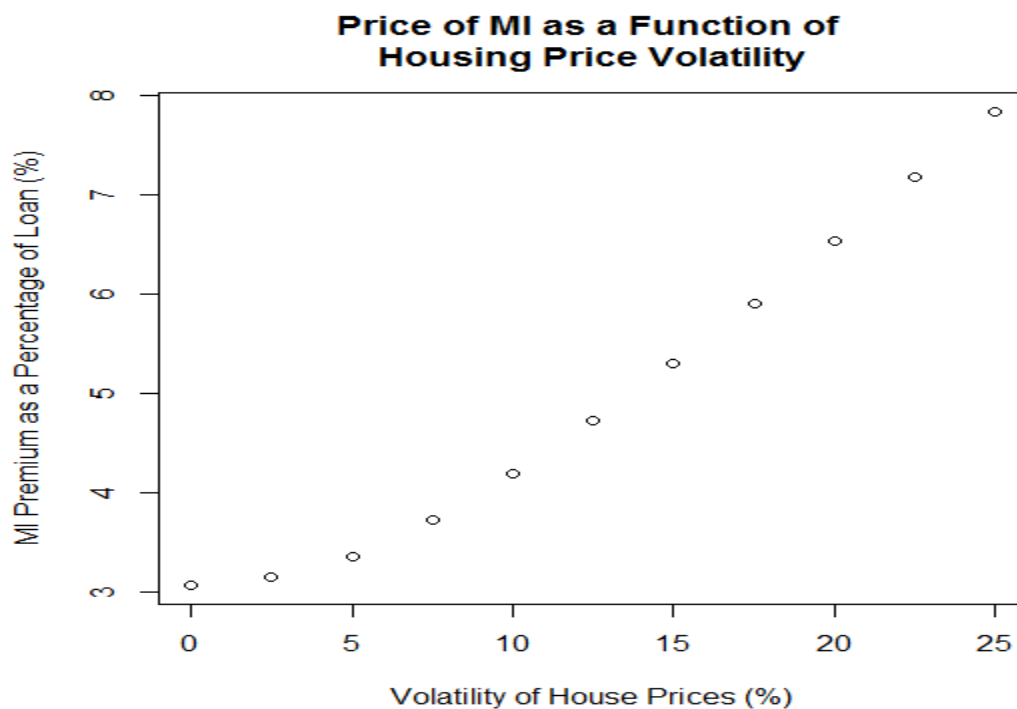

Figure 7: Price of MI as a Function of House Price Volatility

4) Rho

This is the measure of the rate of change of the MI premium with respect to the change in the risk free rate of interest. From the data the Rho lies in the range $(-1.069913*10^{-5}, 0.00)$ with an average of $-1.162485*10^{-6}$. This shows that on average, every unit change in the risk free rate of interest causes a -0.000116% change in the premium. Thus, the model is not affected by minor changes in the risk free rate of interest.

Conclusion

Our first objective was to replicate an MI contract as an option and develop an option-based model for pricing mortgage insurance contracts. We can therefore conclude that an MI contract can be modelled as the value of a portfolio of two put options, both with the time to maturity equal to the time to default.

 Based on existing work published in journals we were able to show that assuming that agents in the economy are risk neutral, the collateral value follows a geometric Brownian motion. Making further assumptions that there exists a risk free asset in the economy with a constant return, it is possible to translate the parameters of the Black-Scholes model to suit an MI contract context. Consequently, it is also possible to price an MI contract using the Black-Scholes model. The proposed valuation method is general and particularly useful in emerging markets where other methods currently in existence may be either inappropriate or are too difficult to implement because of the lack of relevant data.

Another objective was to incorporate the time to repossession of the collateral in the case of the borrower's default. This is a measure of the inefficiency of the legal system and leads to additional costs to the insurer. The average time required for the repossession of the collateral varies from country to country. In Kenya, it is 3-5 years. By incorporating the cost of legal inefficiency, the cost of an MI contract increases significantly. For instance, with our typical mortgage of KES. 4 Million, and assuming immediate repossession of the collateral, we obtained a premium of 0.0442%. When the cost of legal inefficiency was incorporated, the new actuarial fair price was 6.63069%. This shows a reasonable difference of 6.58649%.

Therefore, we can conclude that delays in repossession have a very significant effect on the price of an MI contract. Thus, the shorter the time to repossession, the lower the actuarial fair value of the Mortgage insurance contract. For this reason, one of the key factors that would contribute to the success of a mortgage insurance scheme Kenya and other emerging economies is, therefore, a comprehensive legal reform that would shorten the time delay in repossession of the collateral in case of borrower default.

Finally, the sensitivity analysis reveals that the premium calculated by our option-based model is robust to small changes in parameter values. The most significant parameter is the volatility of the collateral.

Recommendations

Based on the results of our study and the above conclusions, we propose the following recommendations:

i.   Kenyan insurance companies take into account the cost of legal inefficiency when pricing their MI contracts.

ii. That Mortgage insurers already using other valuation methods to adopt our option based model as a reasonable check for their premium levels.

iii. Further analysis should be done to provide a more comprehensive view since our study used data from HassConsult a Kenyan real estate agency that deals with high-end houses. This will reveal how our proposed model behaves in different scenarios

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Appendix

R Code:

```
data<-read.csv("mortdata.csv")

data


#severity of loss assuming immediate repossession of collateral

#k=strike,t=one year,r=risk-free interest rate,v0=initial house price;s=maintenance yield

sigma<-0.18;v0<-4000000;r<-0.10636;t<-1;k1<-data[,9];k2<-data[,10];s<-0.01;g<-0.1

d1<-(log(v0/k1)+(r-s+0.5*(sigma^2))*t)/(sigma*sqrt(t))

d2<-d1-(sigma*sqrt(t))

putk1<-k1*exp(-r*t)*pnorm(-d2)-v0*exp(-s*t)*pnorm(-d1)

putk1

d1<-(log(v0/k2)+(r-s+0.5*(sigma^2))*t)/(sigma*sqrt(t))

d2<-d1-(sigma*sqrt(t))

putk2<-k2*exp(-r*t)*pnorm(-d2)-v0*exp(-s*t)*pnorm(-d1)

putk2

CLt<-putk1-putk2

CLt

#actuarially fair price assuming immediate repossession of collateral

AFP<-CLt*data[,14]

AFP

sum(AFP)

MIpremium<-sum(AFP)/3200000

MIpremium


#severity of loss with cost of legal inefficiency

sigma<-0.18;v0<-4000000;r<-0.10636;t<-1;k1l<-data[,11];k2l<-data[,12];s<-0.01
```

```r
d1<-(log(v0/k1l)+(r-s+0.5*(sigma^2))*t)/(sigma*sqrt(t))

d2<-d1-(sigma*sqrt(t))

putk1l<-k1l*exp(-r*t)*pnorm(-d2)-v0*exp(-s*t)*pnorm(-d1)

putk1l

d1<-(log(v0/k2l)+(r-s+0.5*(sigma^2))*t)/(sigma*sqrt(t))

d2<-d1-(sigma*sqrt(t))

putk2l<-k2*exp(-r*t)*pnorm(-d2)-v0*exp(-s*t)*pnorm(-d1)

putk2l

LCLt<-putk1l-putk2l

LCLt

#actuarially fair price with cost of legal inefficiency

AFP2<-LCLt*data[,14]

AFP2

sum(AFP2)

MIpremium2<-sum(AFP2)/3200000

MIpremium2

UpdatedMI<-(1+g)*MIpremium2

UpdatedMI


#plotting probabilities of default

plot(data[,1],data[,14],

main="Conditional Probabilities of Default for US and

Estimated Default Experience for Kenya",

xlab="Year",ylab="Probabilities of Default (%)",col="blue")

points(data[,1],data[,13],col="green")

legend("topright",legend=c("Default Probabilities Proposed for Kenya","US Default
Probabilities"),

lty=1,col=c("blue","green"),cex=.9)
```

#plotting MI premium against time to repossession

plot(data[,16]~data[,15],main="Influence of Legal Inefficiency and Delays

on the Price of MI",xlab="Average Time to Repossession (Years)",

ylab="MI Premium as a Percentage of Loan (%)")

#plotting MI premium against probabilities of default

plot(data[,24]~data[,23],main="Price of MI as a Function of the Multiplier

of the US Default Experience",xlab="Multiplier of the US Default Experience",

ylab="MI Premium as a Percentage of Loan (%)")

#plotting MI premium against loss ratio

plot(data[,18]~data[,17],main="Price of MI as a Function of Loss Ratio",

xlab="Loss Ratio (%)",ylab="MI Premium as a Percentage of Loan (%)")

#plotting MI premium against property volatility

plot(data[,20]~data[,19],main="Price of MI as a Function of

Housing Price Volatility",

xlab="Volatility of House Prices (%)",ylab="MI Premium as a Percentage of Loan (%)")

#plotting MI premium against loan-to-value ratio

plot(data[,22]~data[,21],main="Price of MI as a Function of Loan-to-Value Ratio (LTV)",

xlab="Loan-to-Value Ratio (%)",ylab="MI Premium as a Percentage of Loan (%)")