A TEST OF THE EFFECTIVENESS OF DOWNSIDE RISK FRAMEWORK OVER MEAN-VARIANCE FRAMEWORK IN OPTIMAL PORTFOLIO SELECTION: EVIDENCE FROM THE NAIROBI SECURITES EXCHANGE (NSE).

BY

BONYO KENNEDY OMONDI

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DECLARATION

I hereby declare that this project is my original work and to the best of my knowledge, it contains no material previously published by another person, except where due acknowledgement has been made.

Signature ……………………………….. Date……………………………

STUDENT: BONYO KENNEDY OMONDI

D63/72692/2012

This project has been submitted for examination with my approval as University Supervisor.

Signature ……………………………….. Date……………………………

SUPERVISOR: DR. DUNCAN ELLY OCHIENG'

PhD, CIFA

DEDICATION

I dedicated this project work to The Almighty God for His infinite mercy towards my academics. I also, to my late Dad Mr. Tom Opiyo Bonyo, my Mum Mrs. Jane Achieng Bonyo and brothers Michael Odhiambo Bonyo and Fredrick Otieno Bonyo, for their love, care, understanding and support, may the good Lord continue to bless all of them.

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LIST OF ABBREVIATIONS

CMA	Capital Markets Authority
CVAR	Conditional Value at Risk
ES	Expected Shortfall
HS	Historical Simulation
LPM	Lower Partial Moments
MLPM	Mean Lower Partial Moments
MV	Mean-Variance
NSE	Nairobi Securities Exchange
PT	Portfolio Theory
VAR	Value at Risk

ABSTRACT

Variance is commonly used as risk measure in portfolio optimization to find the trade-off between the risk and return. Investors wish to minimize the risk at the given level of return. However, the mean-variance model has been criticized because of its limitations. The mean-variance model strictly relies on the assumptions that the assets returns are normally distributed and investor has quadratic utility function. This model will become inadequate when these assumptions are violated. Besides, variance not only penalizes the downside deviation but also the upside deviation. Variance does not match investor's perception towards risk because upside deviation is desirable for investors. Therefore, downside risk measures have been proposed to overcome the deficiencies of variance as risk measure. The downside risk measures have better theoretical properties than variance because they are not restricted to normal distribution and quadratic utility function. The downside risk measures focus on return below a specified target return which better match investor's perception towards risk. This study seeks to test the effectiveness of downside risk framework over mean-variance framework in optimal portfolio selection at the Nairobi Securities Exchange. Overall, the study found that the choice of risk measure has a significant effect on portfolio allocation. From the analysis, CVaR as a downside risk measure, outperformed the variance.

CHAPTER ONE: INTRODUCTION

1.1 Background of the Study

There is an ongoing debate in the financial literature on which risk measure to use in risk management and portfolio choice. As some risk measures are more theoretically appealing, others are easier to implement practically. For a long time, the standard deviation has been the predominant measure of risk in asset management. Mean-variance portfolio selection via quadratic optimization, introduced by Markowitz (1952), is used as the industry standard. Two justifications for using the standard deviation in portfolio choice can be given. First, an institution can view the standard deviation as a measure of risk, which needs to be minimized to limit the risk exposure. Second, a mean-variance portfolio maximizes expected utility of an investor if the utility function is quadratic or asset returns normal distribution Ingersoll (1987).

Despite the computational advantages, the variance is not a satisfactory risk measure from the risk measurement perspective. As a symmetric risk measure, the variance penalizes gains and losses in the same way. While the variance term penalizes uncertainties on both sides of the mean, numerous downside risk measures have been proposed to quantify the risk that the investment return is below certain target. Among these downside risk measures, value-at-risk and conditional value-at-risk.

The mean-variance framework do, however, employ a number of assumptions that dissociate their results from real world outcomes. Markowitz (1952, 1959) assumes that asset returns are fully explained under a normal (symmetrical) distribution by the distribution's first two moments i.e. mean and variance. Higher order moments (skewness and kurtosis) were thus not required. Leland (1999) states that the distribution of asset returns are generally not normal and investors have significantly different views pertaining to upside and downside risk. The degree to which asset returns are non-normal was investigated by Cont (2001), who suggests that asset returns display excess kurtosis; that is, they exhibit higher peaks and 'fatter' tails than a normal distribution. This has an effect on the risk metrics of variance and standard deviation. The steeper peaks suggest that asset returns are less deviant about their means, while the 'fatter' tails are indicative of the fact that the majority of returns lie further away from the mean. This implies higher occurrences of extreme positive and negative asset returns. Cont (2001) also observed that there is empirical evidence that asset returns have a tendency to return below-mean returns rather than above-mean returns, a statistical phenomenon referred to as skewness. Empirical testing has observed that asset returns tend to demonstrate negative skewness in general. As a result, mean-variance optimization, while theoretically sound, may not account fully for the empirically-observed behavior of return distributions. A more accurate model would have to include higher order moments such as skewness and kurtosis to compensate for observed asset returns and investor preferences.

It would therefore be more desirable to focus on a measure for risk that is able to incorporate any non-normality in the return distributions of financial assets. Alternative distributional assumptions are now widely used in the risk management literature, where there is greater consensus on the probability of extreme returns being non-normally distributed. Research by Harvey and Siddique (2000), Bekaert et al. (1998) and Das and Uppal (1999) advocate the need to incorporate non-normality into the portfolio allocation decision. The ability to focus on additional moments in the financial return distribution with the possibility of allowing for skewed distributions enables additional risk factors (along with the use of standard deviation) to be included into the optimal portfolio selection problem.

The Value-at-Risk (VaR), defined as the threshold point with a specified exceeding probability of great loss, becomes popular in the financial industry since the mid-90s. However, the VaR has been widely criticized for some of its undesired properties. More specifically, VaR fails to satisfy the axiomatic system of coherent risk measures proposed by Artzner et al. (1999). Most critically, the non-convexity of VaR leads to some difficulty in solving the corresponding portfolio optimization problem. On the other hand, the conditional Value-at-Risk (CVaR), also known as the expected shortfall, is defined as the expected value of the loss exceeding the VaR Rockafellar and Uryasev (2000). CVaR possesses several good properties, such as convexity, monotonicity and homogeneity. Rockafellar and Uryasev (1996, 2000) prove that CVaR can be computed by solving an auxiliary linear programming problem in which the VaR needs not to be known in advance. After the fundamental work of Rockafellar and Uryasev (1996, 2000), CVaR has been widely applied in various applications of portfolio selection and risk management, e.g., derivative portfolio Alexander et al. (2006), credit risk optimization Andersson et al. (2001), and robust portfolio management Zhu and Fukushima (2009)

A downside-risk approach to investment decisions use intuitive measures of risk that focus on return dispersions below a specified target or benchmark return. Downside-risk measures are attractive not only because they are consistent with investors' perception of risk, but also because the theoretical assumptions require justifying their use are very simple. This study tests the effectiveness of the downside risk measure with focus on value-at-risk and conditional value-at-risk over the mean-variance framework on optimal portfolio selection.

1.1.1 Downside risk framework

The concept of downside risk is not new, its existence dates back to 1952. The earlier concern of the downside deviation was addressed by Roy (1952) in the form of a "safety first" rule that measures the probability of outcomes falling below a target return. Kataoka (1963) and Tesler (1956) extended the rule in a single period setting, which was further developed by Tse, Uppal and White (1993) in a dynamic framework.

The safety first rule, together with other measures of risk, namely the expected value of loss, the expected absolute deviation, the maximum expected loss, the semi-variance and the variance, were evaluated when Markowitz (1959) formalized his seminal portfolio theory. The semi-variance, defined as the squared deviation of return below a target return, was found to be a theoretically more robust measure of risk, though the variance was subsequently chosen for technical reasons. Markowitz argued for the importance of the tailed-end return distribution over the upside potential of the investment because he believed that investors' risk perception should be asymmetric. Mao (1970) also found support for the semi-variance measure among business executives who were more sensitive to losses below some benchmark returns compared with the project return going above the benchmarks.

Bawa (1975) presented a formal analysis of downside risk measures for various utility functions in terms of the lower partial moment (LPM) of return distributions. The generalized concept of downside risk is called lower partial variance or lower partial moment (LPM). The appeal of these risk measures partly stems from their consistency with the way individuals actually perceive risk. The LPM measure liberates the investor from a constraint of having only one utility function, which is fine if investor utility is best represented by a quadratic function.

Bawa (1975) generalized the second order LPM function into n-order LPMs to cover a range of risk measures as:

$$LPM_n(\tau, R_i) = \int_{-\infty}^{\tau} (\tau - R_i)^n dF(R_i),$$

Where τ is the "target return," R_i is the return of asset i, $dF(R_i)$ is the probability density function of return on asset i, and n is the order of moment that characterizes an investor's preference of return dispersion below the target rate. The common classes of LPM are the probability of loss $(n = 0)$. In this case; the investor does not care about the size of a gain or a loss. His preferences are characterized by the fact that a shortfall, i.e. a return that falls below the threshold return, does not create any utility for him, while a return that exceeds the threshold level adds positively to his total utility, regardless of the size of the gain. For $(n = 1)$, LPM_1 becomes the expected deviation of returns below the target, or the target shortfall. The target semi-variance $(n = 2)$, and the target skewness $(n = 3)$.The variable n can also be viewed as a measure of risk aversion where risk aversion increases with n. Risk as measured by the $n - LPM$ reflects explicitly the asymmetry and skewness of the probability distribution of asset returns. For computational reasons, if we assume that there are T number of return observation for asset $i,$ then the $n-LPM$ can be described as a discrete distribution:

$$LPM_n(\tau, R_i) = \frac{1}{T-1}\sum_{t=1}^{T}\left[Max\left(0,(\tau - R_{it})\right)\right]^n$$

1.1.2 The Mean-Variance Framework

In 1952 Markowitz made a pioneer work in finance that has resulted in further studies in the field of portfolio theory. He proposed the theory of the mean-variance framework of a portfolio selection since investors want to maximize expected return and minimize the variance (risk). Markowitz stressed that the variance cannot be eliminated by diversification as returns on securities are too intercorrelated. Investors' preferences are different when it comes to selecting an investment portfolio. There is a tradeoff between risk and reward, investors might have to give up expected return to reduce the variance or gain expected return by taking on some risk. Markowitz also discussed that it is not enough to invest in many securities, investment should as well be diversified across industries to obtain a lower covariance. The criticism of the mean-variance framework is that its assumptions are unrealistic. First, it assumes that returns are normally distributed which is not always the case. Second, for a quadratic utility function it assumes that investors prefer a portfolio with the minimum standard deviation for a given expected return. The use of variance has as well been criticized as it is a symmetric risk measure that put equal weights on positive and negative returns when the general assumption is that investors are more concerned about losses than dispersion of high returns.

1.1.2.1 The Efficient Frontier

The efficient frontier is a frontier of efficient portfolios. A frontier portfolio is a portfolio that has the lowest variance of all portfolios with the same expected return. The efficient portfolio that has the lowest risk is the minimum variance portfolio. The investor selects a set of portfolio weights to minimize the variance of any number of assets.

A rational investor makes a decision built on the mean and the variance of a distribution of portfolio returns and he will only select portfolios on the efficient frontier. Only portfolios that are on the frontier are efficient portfolios, portfolios inside the frontier are inefficient and should not be held by utility maximizing investors. Markowitz found out that only portfolios above the minimum variance portfolio were efficient. The most efficient portfolio is the one that achieves highest return for each amount of risk.

1.1.3 Optimal portfolio selection

For typical risk-averse investors, an optimal portfolio that gives a lower risk and a higher return is always preferred Markowitz (1959). Regardless of the optimization method to allocate assets in portfolios, the concept of diversification has been established as very effective to reduce risk. However, the effect diminishes with increasing number of assets in the portfolio. Elton et al. (2009) found the diversification effect is most efficient in portfolios composed of about 20 assets. Nyaraji (2001) evaluated the risk reduction benefits of portfolio diversification at the NSE. He used the mean-variance framework and the period of study was 1996 to 2000. The study found a significant risk reduction at the NSE as the portfolio grew in size up to 13 securities after which risk reduction becomes insignificant. This means that diversification reduces risk only to a certain degree.

Mean-variance is the traditional optimization approach introduced by Markowitz in his seminal paper titled “Portfolio Selection”, published in 1952. Markowitz focused on risk. He established volatility as the major risk measure in portfolio theory and showed how risk can be reduced by diversification. He demonstrated how to generate financial portfolios, which have a maximum expected return for a given level of risk, measured in standard deviation. The portfolio with the lowest risk is called the minimum variance portfolio.

Drake and Fabozzi (2010) stated that portfolios are efficient when they provide the maximum possible expected return for a certain risk. To build efficient portfolios one needs to define some assumptions about investors and their behavior. The first assumption is that investors are risk-averse, which means that they will choose the portfolio with the lowest risk, when faced with several portfolios with the same expected return, but with different risk. On the other hand, a risk-averse investor will choose the portfolio with the highest return, when they have to choose from a set of portfolios with the same risk, but different expected return. That means that efficient portfolios are located on the efficient frontier. From a set of efficient portfolios, the optimal portfolio is the one that is most preferred by an investor.

1.1.4 Downside risk framework and optimal portfolio selection

Portfolio selection under shortfall constraints has its origin in the work by Roy (1952) on the safety-first. Roy defines the shortfall constraint such that the probability of the portfolio value falling below a specified disaster level is limited to a specified disaster probability. Introducing Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) as a shortfall constraint into the portfolio selection decision, so that the portfolio manager or investor is highly concerned about the value of the portfolio falling below the VaR and CVaR constraint, is much more fitting with individual perception to risk and more in line with the constraints which management face. The advantage being that the shortfall constraint is then clearly defined in terms of a widely accepted market risk measure.

In the downside risk frame work, the measure of risk is defined either in terms of VaR and CVaR over and above the risk free rate of return on the initial wealth. The portfolio is then selected to maximize the expected return subject to the level of risk. The final choice of portfolio, including the borrowing and lending decision will therefore meet the specified VaR and CVaR limit. VaR and CVaR are therefore used as ex-ante market risk control measure, extending the richness of VaR and CVaR as risk management tools. The degree of risk aversion is set according to the VaR and CVaR limit; hence avoiding the limitations of expected utility theory as to the degree of risk aversion which an investor is thought to exhibit.

1.1.5 Nairobi Securities Exchange

To carry out portfolio optimization, there must be portfolios in existence, such that one seeks only to find the optimal set of weights for this portfolio. These portfolios are investment portfolios held and traded in the financial markets. Financial markets play a fundamental role in the economic development of a country. They are the intermediary link in facilitating the flow of funds from savers to investors (Aduda, Masila and Onsongo, 2012). The Nairobi Securities Exchange (NSE) is one such market.

In Kenya dealing in shares started in the 1920's when the country was still a British colony. However the market was not formal as there did not exist any rules and regulations to govern stock broking activities. Trading took place on a gentleman's agreement. Standard commissions were charged with clients being obligated to honor their commitments of making good delivery and settling relevant costs. In 1954 the Nairobi Stock Exchange was then constituted as a voluntary association of stockbrokers registered under the Societies Act. Since Africans and Asians were not permitted to trade in securities, until after the attainment of independence in 1963, the business of dealing in shares was confined to the resident European community (NSE market fact sheet file 2012).

Today NSE has 62 companies listed and trading in the different market segments available. Securities that are currently traded at the exchange include government bonds, corporate bonds and ordinary shares. The equity securities investment sector of the Nairobi Securities Exchange (NSE) is divided into four sectors namely the Agricultural sector; the Industrial and Allied sector; the Commercial and Services sector and the Finance and Investment sector. The group of equity securities investment sector in NSE is based on type of products and services provided by the companies whose equity securities are listed in those sectors. There are a number of individual investors, financial institutions and companies that currently hold investment portfolios among these listed companies at NSE. These investors, financial institutions and companies use brokers and investment managers to trade and manage their portfolios. These investment managers or brokers use the qualitative analysis approach of market surveillance intelligence and speculation. However, the Nairobi Securities Exchange (NSE) has developed over time and is still growing as more companies become listed at the NSE; this has made market analysis more complex. Therefore, there is need for a robust approach of using optimization models to analyze and manage the investment portfolios so as to complement the conservative methods currently used.

1.2 Research Problem

In a mean-variance framework Markowitz (1952) risk is defined in terms of the possible variation of expected portfolio return. The focus on standard deviation as the appropriate measure for risk implies that investors weigh the probability of negative returns equally against positive returns. However it is a stylized fact that the distribution of many financial return series are non-normal, with skewness and kurtosis Fama and Roll (1968), Rogalski and Vinso (1978), Boothe and Glassman (1987), Taylor (1986), Jansen and de Vries (1991), and Huisman, Koedijk, Kool and Palm (1998). The choice therefore of mean-variance efficient portfolios is likely to give rise to an inefficient strategy for optimizing expected returns for financial assets whilst minimizing risk. It would therefore be more desirable to focus on a measure for risk that is able to address the drawbacks of standard deviation.

Therefore, downside risk measures such as value-at-risk (VaR) and conditional value at risk (CVaR) or Expected Shortfall have been proposed to overcome the deficiencies of variance as risk measure. These downside risk measures have better theoretical properties than variance because they are not restricted to normal distribution and quadratic utility function. The downside risk measures focus on return below a specified target return which better match investor's perception towards risk.

With increased investment opportunities, investors at the NSE have to make decision on the optimal portfolio to hold in order to maximize return and minimize risk. Many portfolio management applications are based on the traditional mean-variance framework Markowitz (1952). Mwangangi (2006) found that over 60% of fund managers considered mean-variance model in their allocation criteria. Mean-variance theory can either be justified by assuming quadratic utility or by assuming normal returns. Quadratic utility exhibits increasing relative and absolute risk aversion, properties that are considered unrealistic, as they imply that investors invest more in risky assets as their wealth decreases. Failing to account for the distributional characteristics of the return series can therefore cause serious implications in risk management and faulty allocate the assets in the portfolio for investors at the NSE.

However, exactly we measure and estimate risk is still open debate. Rather than focusing on the deviation of returns as the only appropriate measure for risk, the focus is on the more relevant negative domain when defining risk, and thus the notion of downside risk as the correct measure for risk. In fact, Markowitz (1959) recognized the "asymmetrical" inefficiencies inherited in the traditional mean-variance framework, and suggested a semi-variance measure of asset risk that focuses only on the risks below a target rate of return, an intuitively more appealing alternative. However, computational difficulty led to the use of variance. This research aim to address the many of the issues raised concerning the appropriate definition and measurement of risk developed upon the pioneering ideas of Markowitz (1954), Roy (1954) and Kahneman and Tversky (1979), and hope to give some greater insight into the risk-return trade-offs faced investors at the NSE. While empirical testing has been investigated in foreign markets, studies related to Kenya in particular are limited.

1.3 Research Objectives

The objective is to test the effectiveness of downside risk framework over the mean-variance framework of optimal portfolio selection at the Nairobi Securities.

1.4 Justification of the study

Risk is an essential factor to consider when investing in the capital markets. The question of how one should define and manage risk is one that has gained a lot of attention and remains a popular topic in both the academic and professional world. The Nairobi Securities Exchange (NSE) being an emerging capital market, Mwangangi (2006) found that over 60% of fund managers considered mean-variance model in their allocation criteria.

However, little attention was given as the correct definition of risk beyond the mean-variance framework. Indeed the measure for risk is highly debatable. The concern in particular regards the downside risk which in recent years has caused significant losses to the vast majority of investors active in the financial markets. Considerable evidence has since shown that investor preferences do not go beyond mean and variance to higher moments such as skewness and kurtosis. Most investors want to maximize returns without risk consideration. Skewness and kurtosis enables the investor to more correctly quantify the downside risk exposure and has therefore become important considerations in portfolio allocation. This study considers different downside risk measures and tests their effectiveness with the cross-section of returns as well as their performance in portfolio optimization compared to variance. Overall, this study aims to show that the choice of risk measure has a significant effect on the portfolio optimization process.

CHAPTER TWO: LITERATUE REVIEW

2.1 Introduction

This chapter examines risk theory and empirical studies that have been conducted both internationally and locally in the area of downside risk measure in optimal portfolio selection. The theories in this chapter include both qualitative and quantitative elements that lay the foundation for the empirical analysis. Since many downside risk measures exist, in this chapter we will introduce the Value at Risk and Conditional Value at Risk and discuss the advantages and drawbacks.

2.2 Theoretical Literature

In order to attain a comprehensive theoretical basis for the empirical analysis, the study review and apply a broad range of financial literature i.e. articles from academic journals and several finance and risk textbooks. The theory employed in this study is generally based on modern portfolio theory introduced by Harry Markowitz (1952) with the focus mainly on the risk aspect.

2.2.1 Risk Theory

The analyses of risk measures plays a fundamental role in the theory of portfolio selection where the objective is to find a portfolio that maximizes expected return while minimize risk. The following section contains a description of which properties a good risk measure should have and an overview of volatility, the basis of the mean-variance portfolio optimization. In addition, VaR and CVaR are introduced as downside risk measures, which are further used throughout the research for the portfolio optimization.

2.2.1.1 Coherent Risk Measures

Artzner et al. (1999) stated four axioms for risk measure and argued that these axioms should hold for any risk measure used to manage risk. He called the risk measures that satisfy the four axioms coherent. Below is an explanation of the four axioms:

- **Subadditivity.** Merging assets from portfolios P_1 and portfolios P_2 cannot increase risk. The concept of diversification must have a reduction in risk ρ.

$$\rho(P_1 + P_2) \leq \rho(P_1) + \rho(P_2)$$

- **Monotonicity.** If portfolio P_1 has systematically lower returns than portfolio P_2, its risk must be greater.

$$\text{If } P_1 \leq P_2 \text{, then } \rho(P_1) \geq \rho(P_2)$$

- **Translation invariance.** The addition of cash by the amount k to a portfolio should reduce its risk by k.

$$\rho(P + k) = \rho(P) - k$$

- **Homogeneity.** Increasing the size of a portfolio by a factor b should simply scale its risk by the same factor.

$$\rho(bP) = b\rho(P)$$

2.2.1.2 Volatility

Volatility, or the square root of variance, measures the average dispersion over the entire distribution of portfolios gains and losses. Volatility is the central concept in many standard statistics such as risk contribution, beta, and correlation Goldberg, et al. (2010). It is the basis for the traditional portfolio theory and, therefore, used in Markowitz (1952) mean-variance framework and related to portfolio optimization approaches. The advantages of volatility as a risk measure are that it is very easy to calculate. However, even though the risk measure fulfills the axioms of the Artzner's (1999) framework, there are some disadvantages to it.

Although useful, volatility does not describe every aspect of risk. Even as he proposed variance as a risk measure, Markowitz (1952) pointed out that a better risk measure would penalize only losses, and proposed semi-variance as a desirable alternative. The most important disadvantage is that the variance measures treats returns and losses symmetrically. Volatility is, therefore, solely ideal for normal distributed returns. In fact, extreme losses occur more frequently than presumed by a normal distribution, which makes volatility less favorable to measure risk, because investors attach greater importance to losses than to profits of the same magnitude. By using downside risk measures, such as Value-at-Risk and Conditional Value-at-Risk, the risk of so called fat-tail distributions can be expressed and controlled more precisely.

2.2.1.3 Value at Risk

Value at Risk (VaR) is an example of downside risk measure. The VaR was introduced by JP Morgan in 1994. According to Jorion (2007), Value-at-Risk is the worst loss (which is the maximum negative return) over a certain period of time which will not be exceeded with a given level of confidence. It is a statistical measure of the amount of money a portfolio, strategy, or firm might expect to lose over a specified time horizon with a given probability (usually 90%, 95%, or 99%). For example, a portfolio that is expected to lose no more than Ksh1million 95% of the time (or 19 of every 20 days) has a VaR of Ksh1million. On the downside, 5% of the time, or 1 day out of every 20, the portfolio is expected to lose at least Ksh1million. One of VaR's major criticisms is that it provides no information about how much the portfolio could lose (beyond the Ksh1million) during this 5% of the time. The VaR was introduced by JP Morgan in 1994. The parameters' time horizon and confidence interval must both be stated before estimating the VaR. Confidence interval is generally high or between 95% to 99% depending on the probability of loss occurring. The time horizon depends on how often a comparison between actual risk and tolerances are made and how quickly a firm can liquidate or hedge large losses. The time horizon can be from one day up to one year Culp (2001).

VaR has become attractive as a market risk measure for various reasons. First, it combines several sources of market risk into a single measure of potential change in value for a portfolio Johansson (1999). Second, VaR is consistent measure of risk across different positions and risk factors and takes into account how different risk factors interact. Third, it is a holistic measure as it takes care of all driving risk factors Dowd (2005).

On the other hand, VaR also has drawbacks; it only measures the percentile of profit or loss distribution. It gives no information about the size of the loss if a tail event occurs (the loss can be much greater than the VaR indicates). Artzner et al. (1999) showed that VaR is not a coherent risk measure since it does not fulfill the axiom of Subadditivity and thus does not always encourage diversification. Subadditivity is based on the principle of diversification. Jorion (2007), author of Value at Risk: The New Benchmark for Managing Financial Risk, describes Subadditivity this way: “Merging two portfolios cannot increase risk” (p.114).

2.2.1.4 Conditional Value at Risk or Expected Shortfall

The drawbacks and limitations of VaR have motivated many scholars to explore coherent risk measures. One alternative for VaR is Conditional Value at Risk (CVaR) or Expected Shortfall, which has some advantages over VaR. Designed to measure the risk of extreme losses, CVaR is an extension of VaR that gives the total amount of loss given a loss event. CVaR notifies what to expect when a VaR violation occurs and it is a more reliable risk measure during market turmoil. It is a coherent risk measure as it does not fail Subadditivity by discouraging diversification and its estimates can be more accurate than the VaR’s Angelidis and Degiannakis (2007). CVaR is calculated as a portfolio’s VaR plus the probability-weighted average loss expected in excess of VaR. A CVaR estimate cannot be lower than a VaR estimate.

CVaR is superior to VaR because CVaR quantifies tail risk and has been shown to be subadditive. CVaR can capture the minimal probability of a substantial loss for a strategy with an asymmetrical risk profile. In contrast, the VaR for such a strategy would be artificially low and fail to reflect the potential magnitude of losses Jorion (2007). Rockafellar and Uryasev (2000) show that when portfolio losses are estimated using a nonparametric method, portfolio risk is more easily optimized by using CVaR than VaR.

Like any risk measure, CVaR has its shortcomings. VaR estimates tend to be more stable than CVaR estimates for the same confidence level; CVaR often requires a larger number of observations to generate a reliable estimate, and it is more sensitive to estimation errors than VaR Yamai and Yoshiba (2002).

2.3 Portfolio Theory and Expected Shortfall

There are some differences between portfolio theory and the expected shortfall even though the ES is a progression from earlier portfolio theory Dowd (2005):

- Portfolio theory interprets risk using the standard deviation of the return while ES interprets risk in terms of the expected value of the loss (given that a VaR violation has occurred).
- ES is a more flexible measure of risk while PT assumes that returns are normally distributed. ES can however accommodate a wide range of possible distributions.
- PT is often limited to market risk while ES can be applied to broader fields of risk such as credit risk and liquidity risk.

2.4 Determinants of Optimal Portfolios

2.4.1 Risk

The analyses of risk measures plays a fundamental role in the theory of portfolio selection where the objective is to find a portfolio that maximizes expected return while minimize risk. The capital asset pricing model (CAPM) assumes that the total risk of a security consist of systematic (non-diversifiable) risk and unsystematic (diversifiable) risk. The systematic risk of an asset is typically conceived as a measure of the contribution of the asset to the total risk of a diversified portfolio. Further Elfakhani and Zaher (1998) found smaller stocks may be riskier than the larger stocks.

2.4.2 Number of Stocks in a Portfolio

Based on the traditional portfolio theory, increasing the number of stocks in a portfolio will reduce the risk. The study by Worthington, Andrew, Higgs and Helen (2004) state that diversifying across markets could reduce portfolio risk while holding expected returns constantly, when there is an existence of low correlations of return between various markets. Karasin (1986) and Sang and Lerro (1973) find that, it is necessary to calculate the correlation between all assets and all possible combinations of assets, together with the expected returns and risk of each asset included in the portfolio.

2.5 Empirical Review

In the literature for portfolio risk measurement, scholars seems to agree that in the presence of non-normal returns and non-quadratic utility functions, a downside risk measure should be preferred instead of portfolio variance. Markowitz (1959) proposed a portfolio optimization procedure based on the semi-variance measure. Markowitz argued for the importance of the tailed-end return distribution over the upside potential of the investment because he believed that investors' risk perception should be heuristically asymmetric. Mao (1970) also found support for the semi-variance measure among business executives who were more sensitive to losses below some benchmark returns compared to the likelihood of the project return going above the benchmarks.

Lower partial moment (LPM) was introduced by Bawa (1975), measuring different utility functions in terms of LPM of a return distribution. Furthermore, it assumes that investors have more than one utility function. With numbers of utility functions investors can be risk taking, risk neutral, and risk averse. Fishburn (1977) employed a utility function model that translates downside risk depending on a risk aversion parameter and a target return. His findings indicated that investors' risk perception significantly changes below this individual threshold. However, downside risk is not only meaningful from an individual investor's perspective but also from an asset pricing perspective.

Blazer (1994) compared a set of risk measures for portfolio selection: standard deviation, probability-based measures, minimum shortfall, expected shortfall, moment-based measures and relative semivariance. He suggested some things to consider when dealing with risk quantification, first acknowledge that risk depends entirely on the investor's needs and second measure risk relative to one or more benchmarks. Among the measures he considered, whenever a single measure is needed, he suggests relative semivariance. Grootveld and Hallerback (1999) used LPM as their risk measure and found that the downside risk approach did produce on average slightly higher bond allocations than the mean-variance approach. Rajan and Gnanendran (1998) compare variance and semivariance for portfolio selection. For the data set consisting of 15 to 27 countries they found that the returns are not normally distributed and therefore they suggested using semivariance instead of variance.

Vorst (2000) views the optimal portfolio selection problems when a VaR constraint is imposed. He showed that this provides a way to control risk in the optimal portfolio. Krokhmal et al. (2002) proposed a model with CVaR constraints. They showed that multiple CVaR constraints with various confidence levels can be used to shape the profit/loss distribution. Byrne and Lee (2004) compared different portfolio compositions got by using different risk measures. In particular, they compared portfolio composition got by using variance, semivariance, lower partial moment and absolute deviation. They show that none of these risk measures behave better in the domain of the other measures and therefore they are incomparable. Instead they compare the holdings in terms of composition and found variance and absolute deviation have similar composition. Alexander et al. (2006) compared the portfolio selection problem under VaR and CVaR and showed that solving Mean-CVaR is better alternative to Mean-VaR. Zakamouline (2010) focuses on VaR as the risk measures as it can evaluate both downside risk, upside return potential and can be used for non-normal distribution.

The Nairobi Securities Exchange (NSE) being an emerging capital market, studies carried out have focused only on the application of mean-variance framework and benefits of diversification. Mwangangi (2006) surveyed the applicability of Markowitz's portfolio optimization model in overall asset allocation decisions by pension fund managers in Kenya. He used a questionnaire and secondary data from RBA on fund allocation for three years from 2003 to 2005. The result of the study showed that 60% of the fund managers applied the Markowitz's optimization model in their allocation criteria despite its shortcomings. From the survey he concluded that most fund managers considered the model in their allocation criteria and the key challenge faced was client investment constraints.

CHAPTER THREE: RESEARCH METHODOGY

3.1 Introduction

This chapter looks at the procedures and methods that were employed in conducting the study in order to answer the research question and achieve the research objectives. It entails the research design, target population, sampling, data collection and data analysis.

3.2 Research design

This study adopted descriptive research design. It is a type of non-experimental design that collects and analyzes data to describe the problem in its current status for the purpose of clarification. This method was appropriate due to its capacity to establish whether the classical mean-variance optimization theory outperform downside risk framework in optimal portfolio selection.

3.3 Population

The population of this study was all the 62 companies listed at the NSE as at December; 2014.This was used because of the availability of the relevant information on the quoted companies. A census survey of all companies was used.

3.4 Data collection procedure

The study utilized secondary data that was obtained from the Nairobi Securities Exchange (NSE) official website. Daily closing prices, for 7 years; 2007, 2008, 2009, 2010, 2011, 2012, 2013, and 2014 was used. It is important that all the data came from the same source and this increases the strength of comparability of each time series.

3.5 Data Analysis

This study used both descriptive and inferential statistics to analyze the secondary data. Descriptive is usually used at the beginning of the analysis phase in order to provide preliminary analysis of the data and guide the rest of data analysis process Cooper and Schindler (2008). All the calculations analysis was carried out in Microsoft Excel (2007) and R statistical programming language.

3.5.1 Return Calculation

It was worth noting that there are two ways to calculate returns. Equity returns can be calculated as arithmetic returns and logarithmic returns. An advantage of the logarithmic returns is that the log normal distribution of returns has infinite upside potential while losses cannot exceed 100%. The arithmetic returns are particularly useful when mutual funds and other financial players must determine the performance of their investments. Another advantage of the logarithmic return is that the log return is more convenient to calculate total return for a certain period. In this study, return was obtained by calculating logarithmic returns of closing prices.

3.5.2 Return Frequency and Period

The secondary data used span over a 7-year period; from January 2007 to December 2014. This period was chosen because of the need to include recent developments and at the same time analyze over a longer period.

The study included one return frequency within the selected 7-year period when examining the relationship between risk and return. The frequency was daily returns. The reason for not including monthly, quarterly and yearly returns is because this would result in fewer observations that would be critically low for some of the statistical tests used in our analysis. Furthermore, using monthly, quarterly or yearly returns would also contain less information since stock prices may exhibit high fluctuations within short time intervals, which would then not be captured.

3.5.3 Outliers

Any abnormal return was included in the analysis since the study aimed to include all fluctuations. This is because we were analyzing the downside risk of returns where extreme values may account for an important part.

3.5.4 Normality Test

The study examined whether the returns were normally distributed by applying different statistical techniques. Many financial models rely heavily on the normality assumption, including the mean-variance criterion. It is an important assumption, since the assumption makes calculations involving risk more straightforward than they are with non-normal distributions. The normality assumption in asset returns has been tested many times, reaching the same conclusion that the historical returns are usually *not* normally distributed. In order to make the analysis complete, the normality assumption was tested with our specific data set.

In order to prove the assumption of the returns following a non-normal distribution, the same model validation process as Alexander (2008) and Kourouma et al., (2011) was followed, where Jarque-Bera test for normality was conducted on daily returns of securities at the NSE. Alexander (2008) states that *"the Jarque-Bera test applies to any random variable whenever we need to justify an assumption of normality"*. The Jarque-Bera test is an asymptotic test in which the reliability of test results increases with number of observations. The Jarque-Bera test is defined as follows:

$JB = T\left(\frac{skewness^2}{6} + \frac{kurtosis^2}{24}\right) \sim X^2(2)$ which follows a chi-squared distribution with two degrees of freedom. The test was conducted under the null hypothesis that the daily returns were normally distributed.

3.5.5 Estimation of VaR and CVaR

There are three ways to compute VaR: the analytical method (also called "delta-normal method" or "variance-covariance method"), historical simulation and Monte Carlo simulation. This approach, called the analytical method, calculates a parametric VaR based on estimating returns, variances and correlations for a portfolio's assets. Analytical methods are not recommended when estimating the VaR of portfolio with asymmetric returns Beder (1995).

The historical, or nonparametric, approach can account for fat tails and skewness in distributions if they have been represented in past return patterns; prices can be weighted to emphasize more-recent movements. This approach, naturally, assumes future prices will behave as past prices have.

The Monte Carlo simulation is done by generating random scenarios for the future returns within a given distribution and computing VaR for these varied scenarios. The distribution used in simulating can be of a variety of forms-from Gaussian bell shaped to the family of extreme value distributions. The flexibility gained in this approach comes at the expense of introducing greater model risk. Not surprisingly, the three different calculation methods can yield very different estimates. This study computed VaR and CVaR using historical simulation of returns.

CHAPTER FOUR: DATA ANALYSIS, RESULTS AND DISCUSSION

4.1 Introduction

The main objective of this study was to test the effectiveness of downside risk framework over the mean –variance frame work in optimal portfolio selection at the Nairobi Securities Exchange. Data was obtained from the Nairobi Securities Exchange. A total of twenty (20) securities out of the sixty two listed companies at NSE was analyzed because they had complete information on daily prices for the study period January 2007 to December 2014. The study used descriptive statistics to analyze the data obtained. Application of the portfolio optimization algorithms was made simple with the use of statistical software R and Microsoft Excel. Name of the twenty securities chosen from different industries are listed in Appendix I.

4.2 Descriptive Statistics

This section presents the descriptive statistics and the distribution of the daily log returns. The descriptive statistics considered were minimum, maximum, mean, standard deviation, skewness and kurtosis. Table 1 shows the summary of the descriptive statistics of the twenty (20) securities. Twelve (12) securities had negative mean daily rate of return while eight (8) had positive returns. The positive returns adhere to the expectation that stocks increase on average while the negative returns means that the stocks decrease on average. The mean daily rate of return range from -0.16334% to 0.06414%.

Table 1: Summary of the descriptive statistics for daily returns.

Security	Mean	Std.Dev	Minimum	Maximum	Kurtosis	Skewness
Sasini Tea	-0.001237678	0.0572407	-2.142954947	0.265703166	997.5331571	-27.05527261
Kenya Airways	-0.001285963	0.025541148	-0.307273082	0.30196803	25.47391836	-0.112014119
Nation Media Group	-0.000151497	0.018691479	-0.253846075	0.159064695	29.33430217	-1.950360074
Standard Group	-0.000339055	0.018691479	-0.167722757	0.182321557	2.510894688	-0.133622519
Scan Group	0.000304736	0.025809817	-0.232331977	0.274222919	12.10548632	0.019848258
Centum(ICDC)	0.000206854	0.025546183	-0.202764389	0.137127225	5.891859834	-0.385367141
Kenya Commercial Bank	-0.000787408	0.052433194	-2.174751721	0.149035579	1494.980284	-36.04225139
Housing Finance Company	4.3193E-05	0.0263422	-0.223143551	0.242264593	8.530158242	0.074076841
Standard Chartered Bank	0.000133866	0.015234335	-0.176781376	0.083381609	14.87800893	-1.133422579
Barclays Bank	-0.000797495	0.033849841	-1.337504197	0.094665227	1230.291584	-31.21013806
Equity Bank	-0.00071015	0.061702554	-2.210917904	0.30330739	897.9258784	-26.63989941
Cfc Stanbic Bank	2.07869E-05	0.036726345	-0.392042088	1.087801373	396.0412878	12.44146202
East Africa Brewaries	0.000370145	0.017291608	-0.191350964	0.09763847	14.35269732	-0.731603859
British American Tobacco	0.000641404	0.019701623	-0.22943948	0.168965766	21.45668047	-0.921447815
Mumias Sugar	-0.001633406	0.037856172	-1.123046314	0.247096792	391.9520125	-13.23230534
Bamburi Cement	-0.000265208	0.017368482	-0.088947486	0.087434299	5.954810606	-0.177417781
Kenya Electricity Generating Co	-0.000486714	0.025079105	-0.223143551	0.23697503	15.04381603	0.577569176
Kenol Kobil	-0.001337809	0.058288364	-2.302585093	0.356674944	1230.593468	-31.16903979
Kenya Power	-0.001576929	0.052336072	-2.083895892	0.175048797	1268.214653	-31.91228927
Jubilee Holdings	0.00013566	0.090553104	-2.738137493	2.717303407	831.8484896	-0.3420204

It is observable that the kurtosis values are all positive, a clear indication of heavy tails. Due to the nature of kurtosis it does not give information as to whether the distribution has one or two heavy tails and if it only has one which end is it at. Using the bar chart of skewness we get a clear view as to the dispersion of the mass in the distribution as shown in Figure 4.1 below.

Figure 4.1: Bar Chart of Daily Skewness.

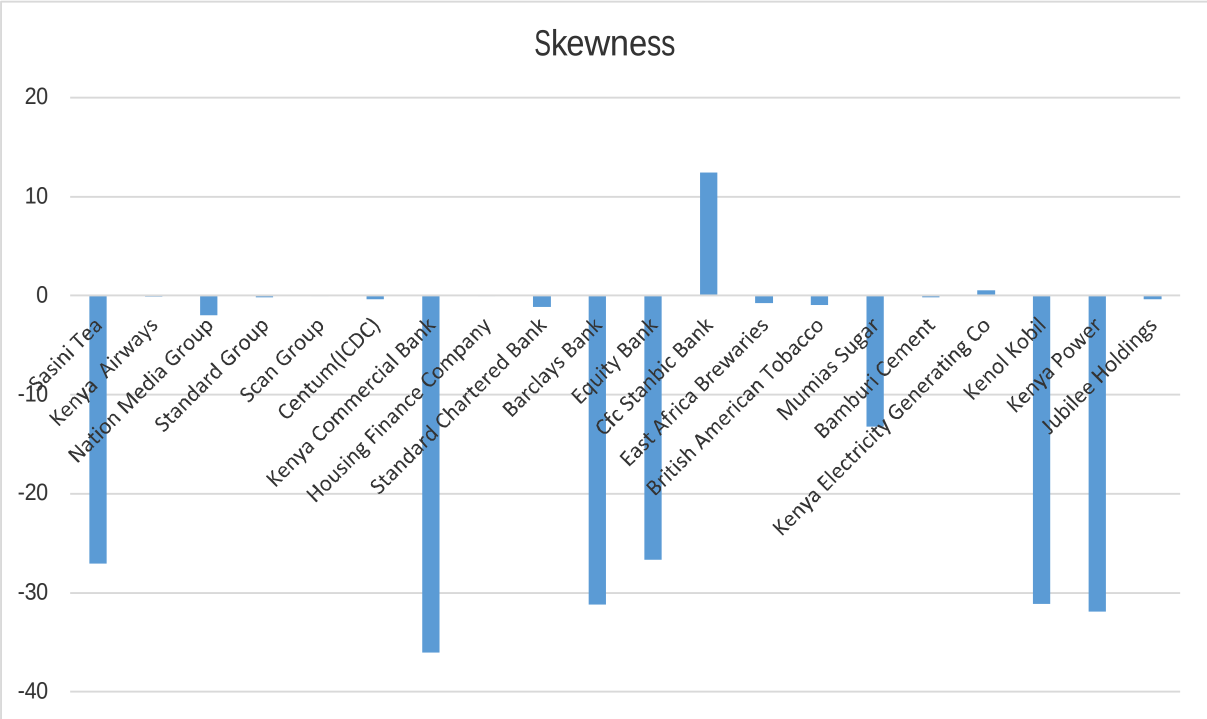
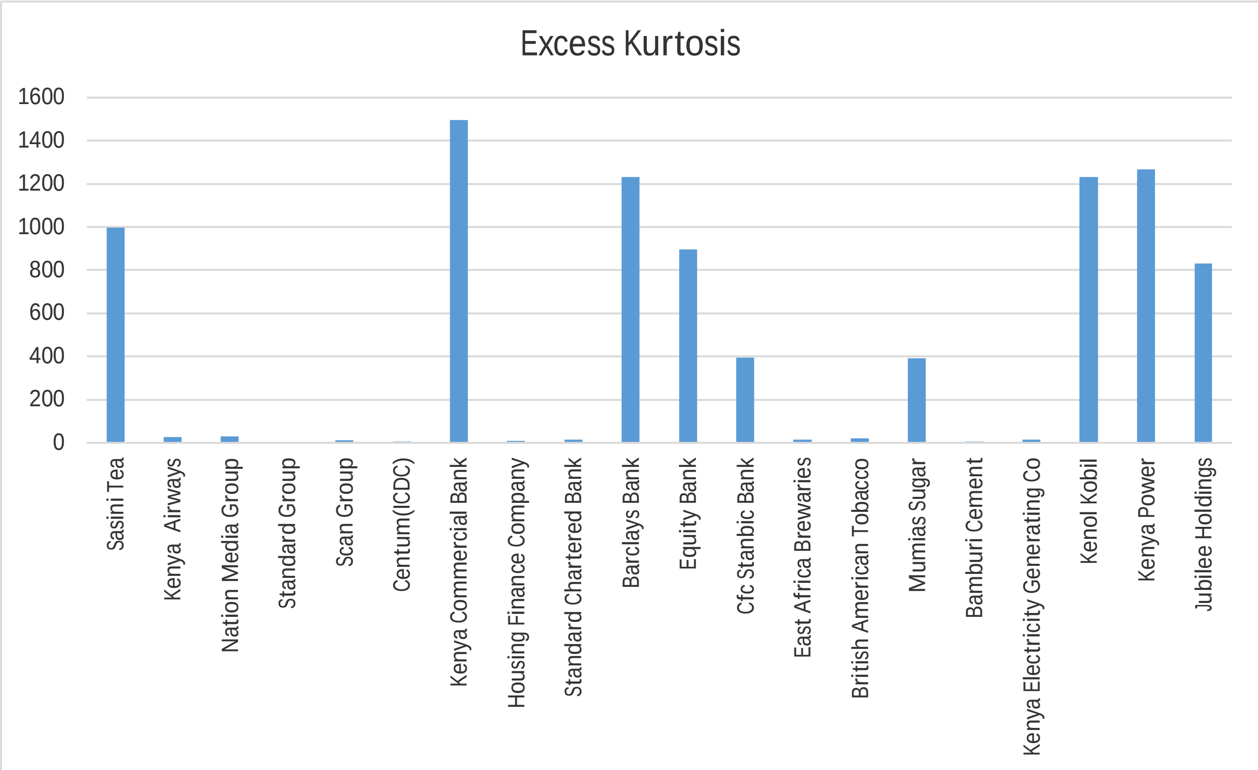


Figure 4.1 exhibit daily return data from Sasini Tea, KCB, Barclays Bank, Equity Bank, Mumias Sugar, Kenol Kobil and Kenya Power have a sizeable negative skewness value, thus supports the concept of heavy lower tails. Daily return data from Kenya Airways, Nation Media Group, Standard Group, Centum, Housing Finance, Standard Chartered Bank, East Africa Brewaries, British American Tobacco, Bamburi Cement, and Kenya Electricity Generating Co are reasonably symmetrical at a daily frequency. Daily return data from CFC Stanbic Bank have a sizeable positive skewness value, thus it has a heavy upper tails.

Figure 4.2: Bar chart on Daily Kurtosis



From Figure 4.2 above, Sasini Tea, KCB, Barclays Bank, Equity Bank, CFC Stanbic Bank, Mumias Sugar, Kenol Kobil and Kenya Power log returns have shown considerable skewness and kurtosis which means it is heavily skewed and heavy-tailed. The expectation follows that at this frequency (daily) its returns will completely be rejected as being normal by normality tests.

4.2.1 Properties of Stock Prices

Figure 4.3: Daily Price Plot for Stocks and NSE 20-Share Index from Jan 2007-Dec 2014.

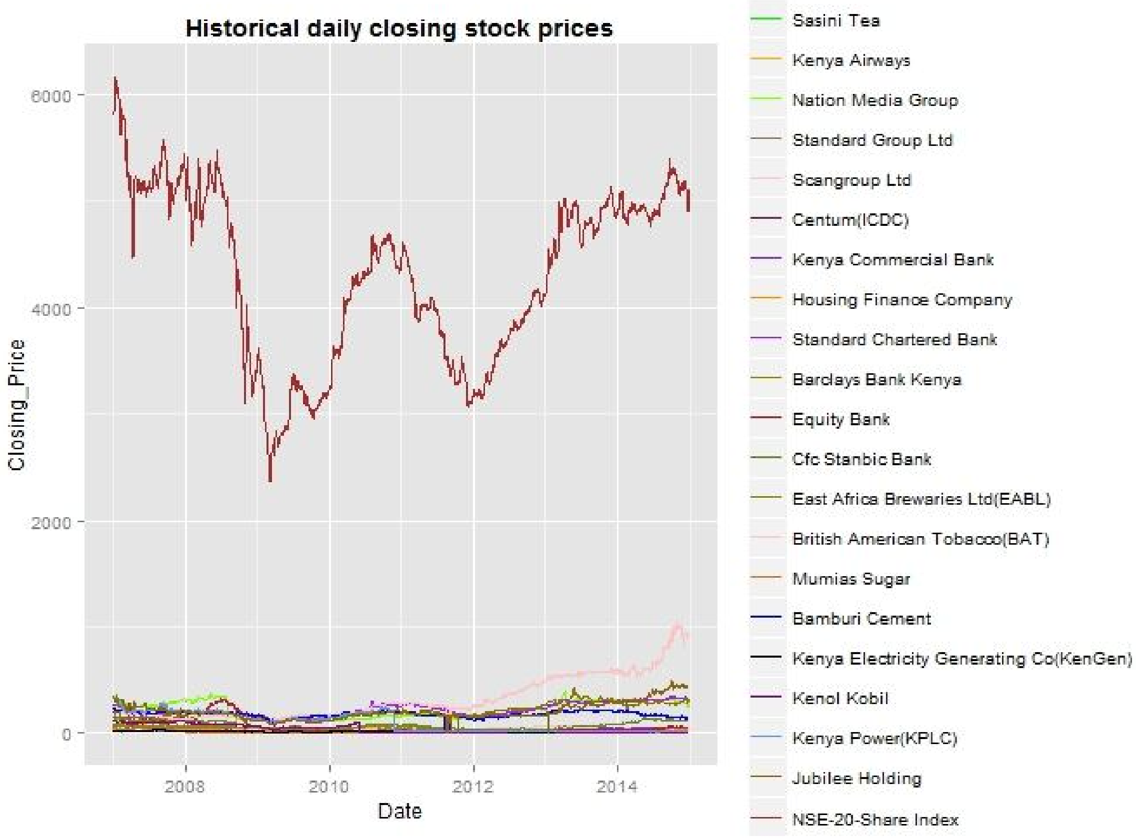


Figure 4.3 above presents the daily closing prices for each trading day from January 2007 – December 2014 for the twenty (20) selected stocks and the NSE 20-Share Index. We observe that the series has a trend, i.e. then mean is obviously non-constant over time. The volatility (i.e. fluctuation of returns about the mean) appears to change over time. The series show higher volatility over the periods 2008 -2009 and 2011 – 2012 than over 2007 – 2008 and 2012 – 2014. This is an indication of possible non-stationarity in volatility. Also, the

coincidence of high and low volatility periods across assets suggests a common driver to the time varying behavior of volatility. Thus it is sufficient to say that price series are clearly non-stationary.

4.2.2 Volatility of log-returns cluster.

Most financial studies involve returns of assets instead of prices. According to Campbell et al. (1997), there are two main reasons for using returns. First, for average investors, returns represent a complete and scale-free summary of the investment opportunity. Second, return series are easier to handle than price series because the former have more attractive statistical properties.

Figure 4.4: Plots daily returns for different stocks under the study

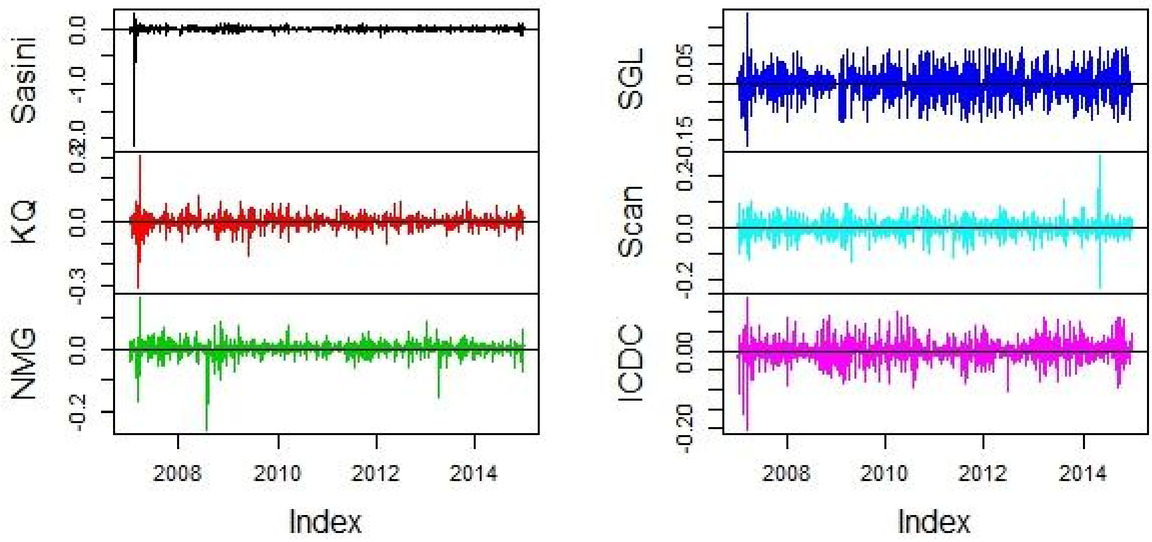
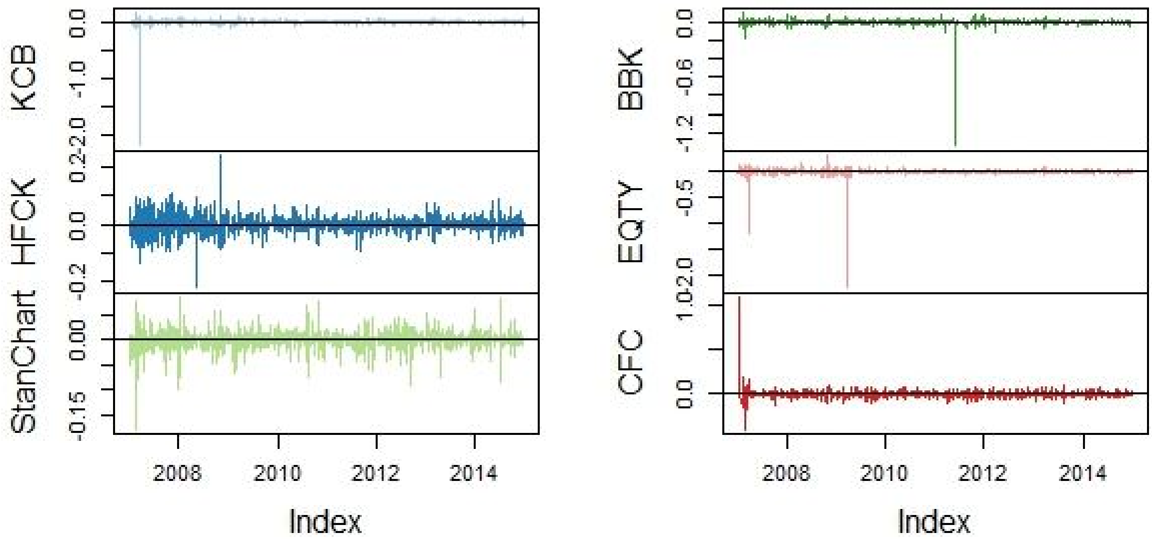


Figure 4.5: Plots daily returns for different stocks under the study



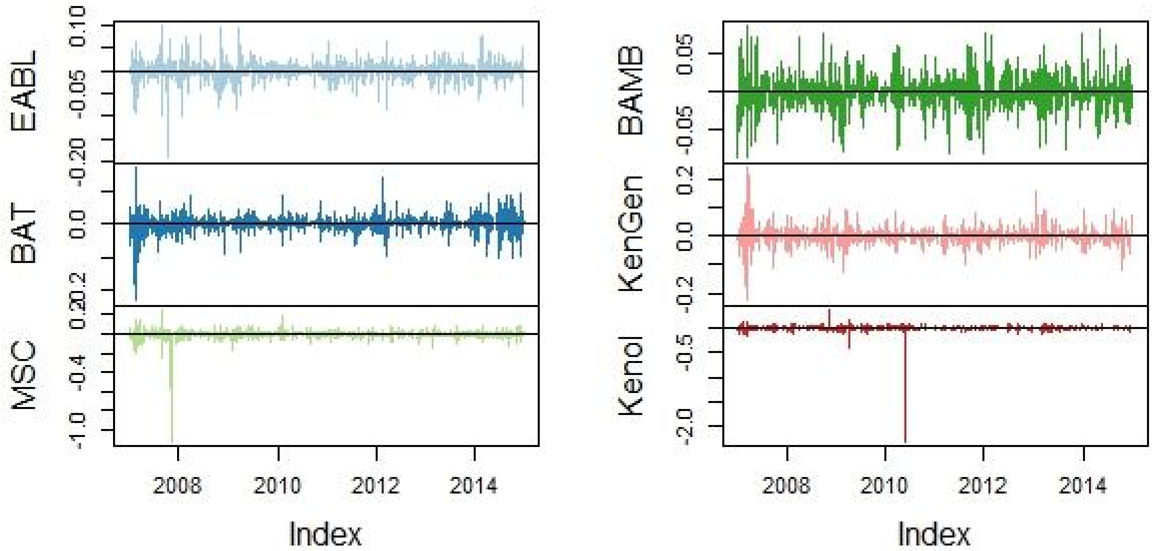


Figure 4.6: Plots daily returns for different stocks under the study

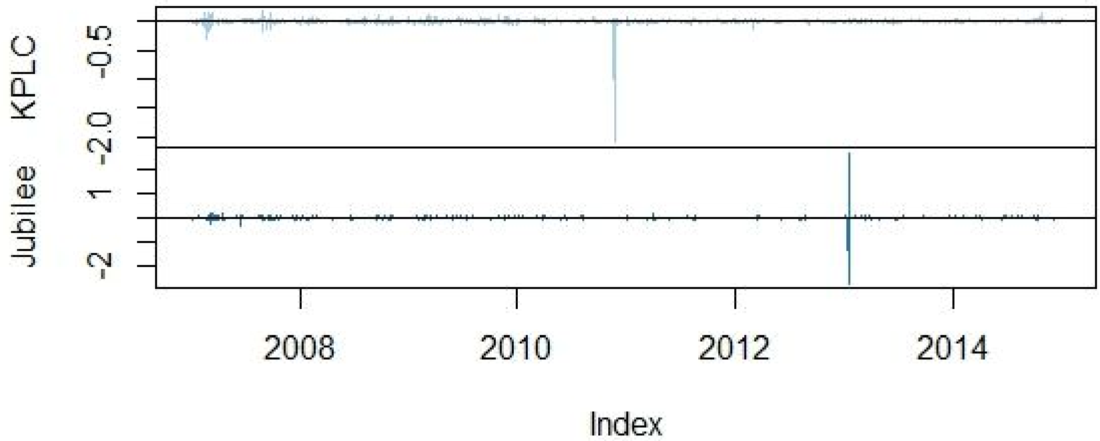


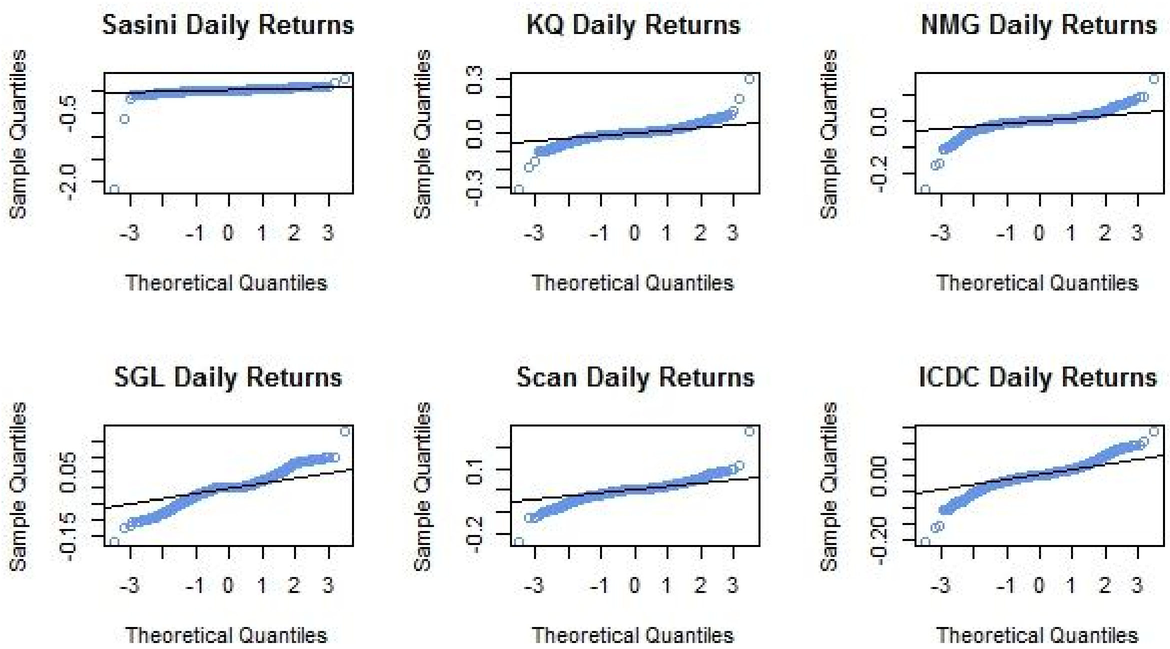
Figure 4.4, 4.5 and 4.6 shows the volatility of daily returns on the stocks under the study. Daily returns for Housing Finance Company (HFCK), Standard Chartered Bank, Standard Group, Scan Group, Centum (ICDC), Bamburi Cement, British American Tobacco and KenGen have larger volatility compared to the other securities in the study. Also, the daily returns show some large and small "spikes" that represent unusually large (in absolute value) daily movements.

4.2.3 Fat Tail

A random variable is said to have *fat tails* if it exhibits more extreme outcomes than a normally distributed random variable with the same mean and variance (Danielsson, 2011). This implies that the market has more relatively large and small outcomes than one would expect under the normal distribution. The *kurtosis* measures the degree of peakedness of a distribution relative to its tails. High kurtosis generally means that most of the variance is due to infrequent extreme deviations than predicted by the normal distribution that has kurtosis equal to 3. Such leptokurtosis is a signal of fat tails. As seen in Table 1 above, all stock returns have excess kurtosis, well above 3, which is evidence against normality.

The most commonly used graphical method for analyzing the tails of a distribution is the quantile-quantile (QQ-plot). The quantile-quantile plot (QQ-plot) gives a graphical comparison of the empirical quantiles of a data sample to those from a specified reference distribution. The QQ-plot is xy-plot with the reference distribution quantiles on the x-axis and the empirical quantiles on the y-axis. If the quantiles exactly match up then the QQ-plot is a straight line. If the quantiles do not match up, then the shape of the QQ-plot indicates which features of the data are not captured by the reference distribution. The QQ-plots for the stock returns against some theoretical distributions are shown in Figure 4.6 and 4.7:

Figure 4.6: Normal quantile-quantile plot (QQ-plot) for log daily returns of securities under study



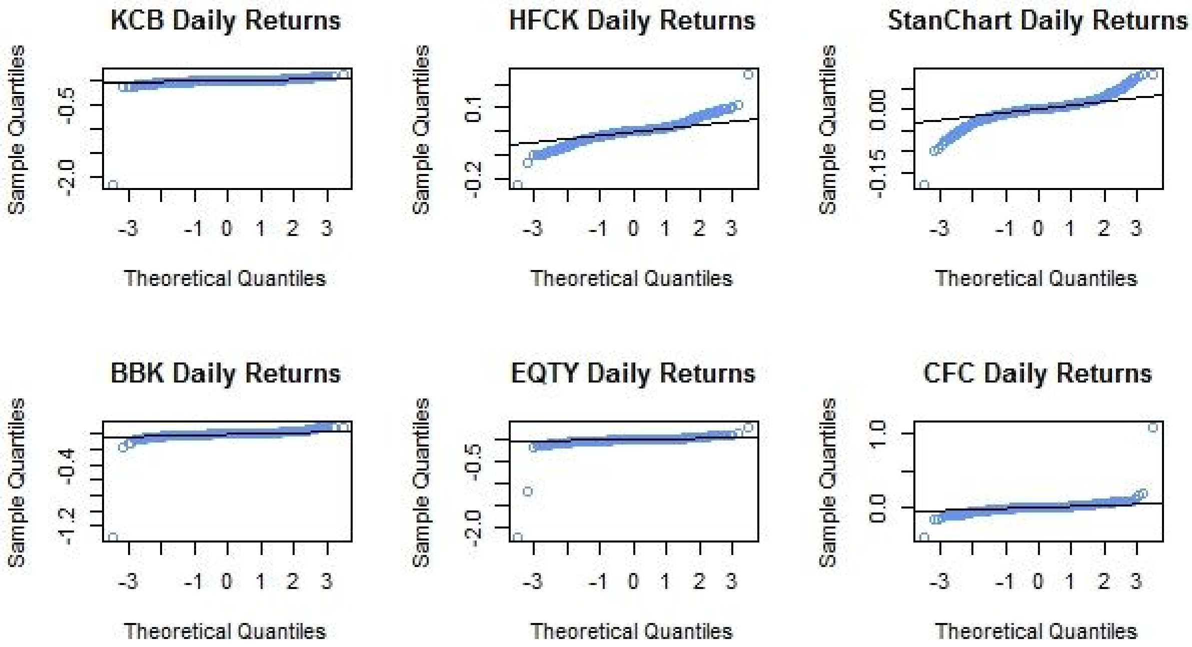
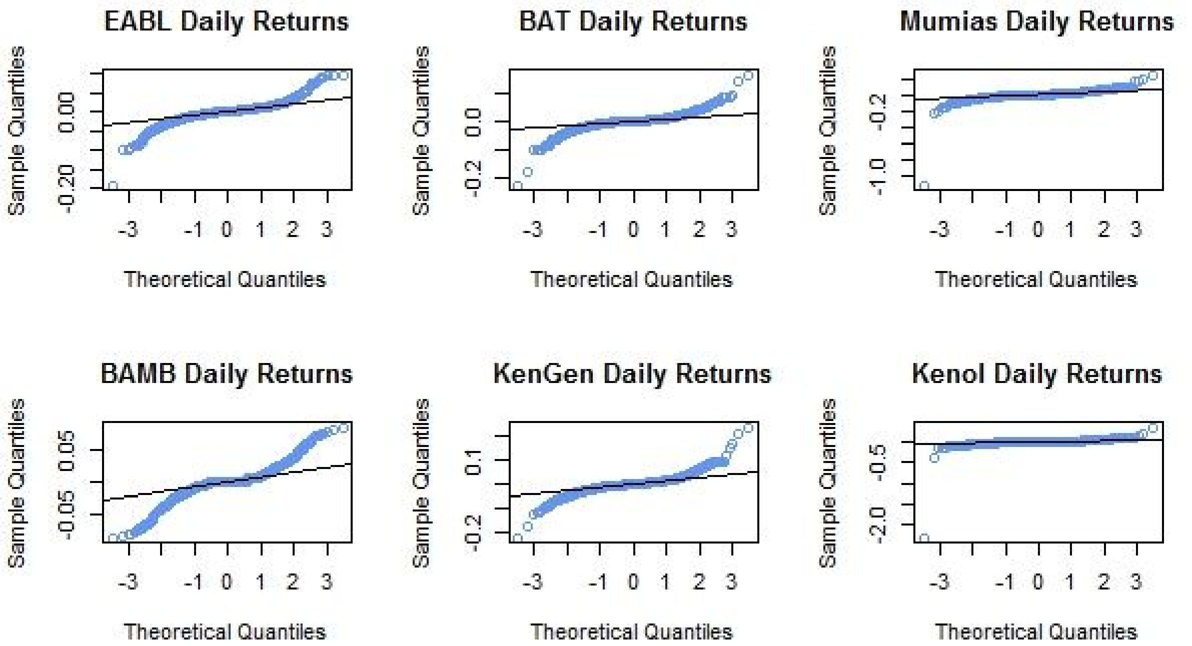


Figure 4.7: Normal quantile-quantile plot (QQ-plot) for log daily returns of securities under study



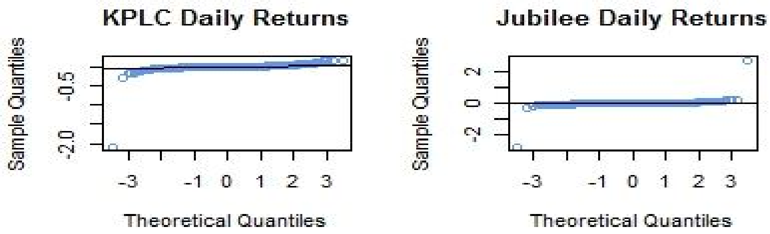


Figure 4.8: Shows the boxplots of daily log returns on stocks under study

From Figure 4.6 and 4.7 the normal QQ-plots for KQ, NMG, SGL, Scan, ICDC, HFCK, StanChart, EABL, BAT, Mumias, BAMB and KenGen daily log returns exhibit a pronounced tilted S-shape with extreme departures from linearity in the left and right tails of the distribution. The normal QQ-plots for the Sasini, KCB, BBK, EQTY, CFC, Kenol, KPLC and Jubilee daily log returns are linear in the middle of the distribution but deviate from linearity in the tails of the distribution. Returns seem to have fatter tails to fit the normal distribution.

4.2.4 Outliers

The decision to not reject any abnormal returns is made since all fluctuations must be included. This is done because when analyzing the downside risk of returns, extreme values may represent an important part. Figure 4.8, 4.9 and 4.10 shows the boxplots of daily log returns on stocks under study

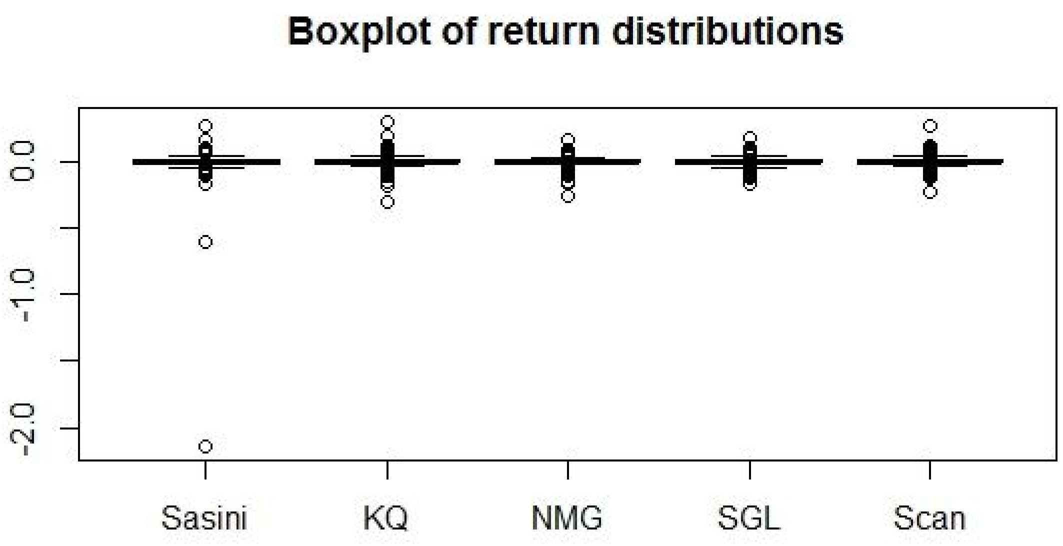
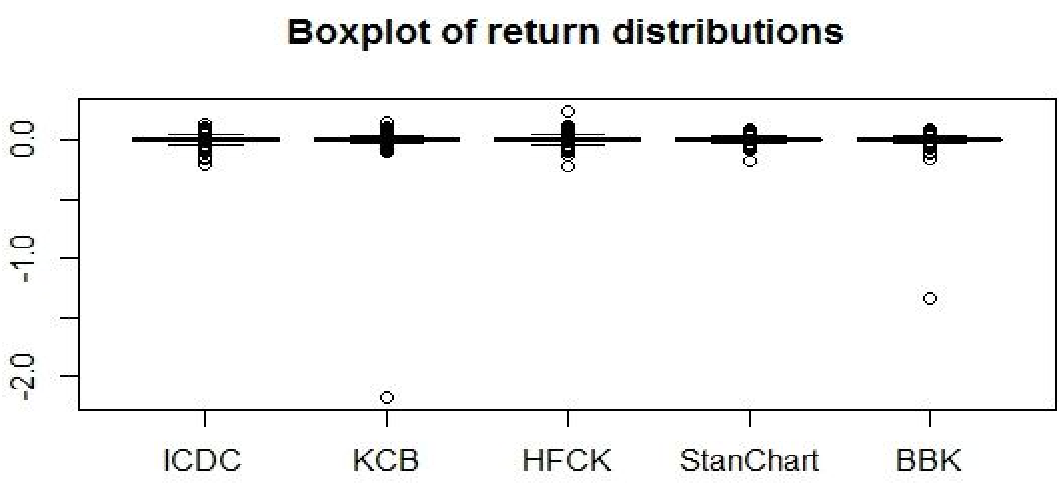


Figure 4.9: Shows the boxplots of daily log returns on stocks under study



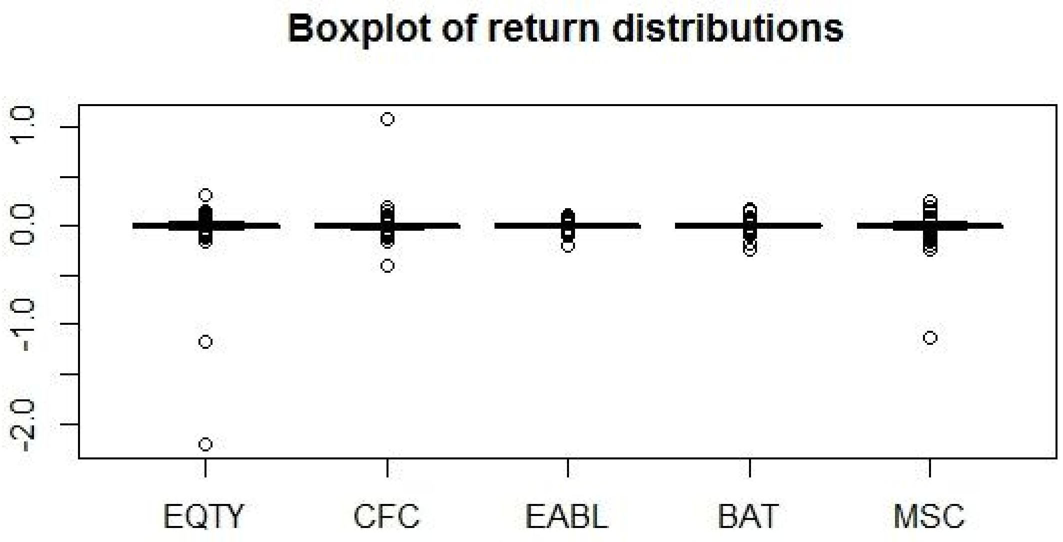
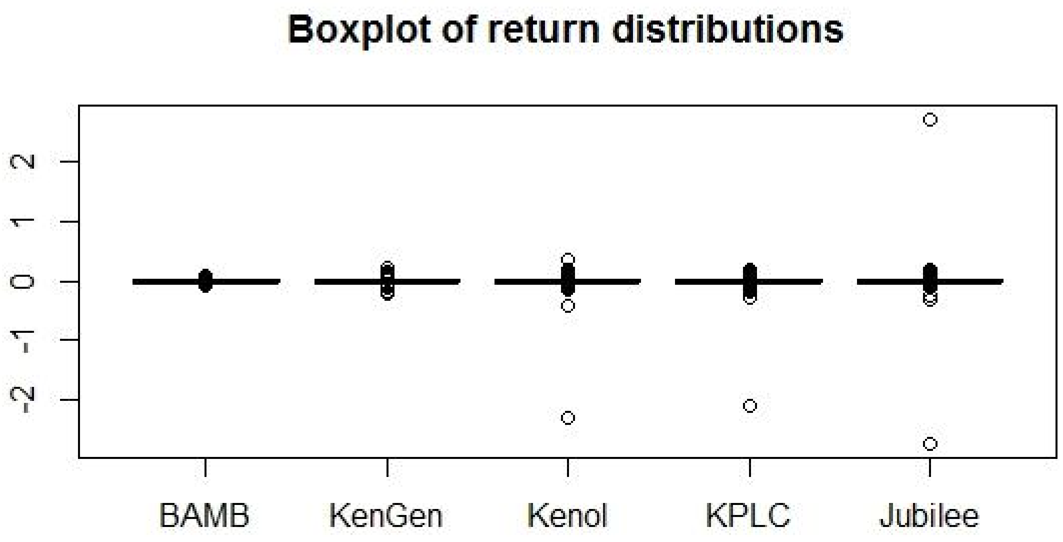


Figure 4.10: Shows the boxplots of daily log returns on stocks under study



From the Figure 4.8, 4.9 and 4.10, Sasini, KCB, BBK, EQTY, MSC, Kenol and KPLC daily returns have negative outliers whereas CFC returns have positive outliers. Jubilee returns have both positive and negative outliers.

4.2.5 Statistical Test of Normality

The Jarque-Bera (1982, 1987) test of normality compares the sample skewness and kurtosis to 0 and 3, their values under normality.

Table 2 summarizes the test results from applying Jarque-Bera test of normality on the daily return data:

Table 2: Outcome of the Jarque-Bera test on the daily returns

All Daily Data Log Returns	Jarque-Bera Test		
	X-squared	df	p-value
Sasini Tea	81920000	2	p-value < 2.2e-16
Kenya Airways	53258	2	p-value < 2.2e-16
Nation Media Group	71872	2	p-value < 2.2e-16
Standard Group	522.14	2	p-value < 2.2e-16
Scan Group	12023	2	p-value < 2.2e-16
Centum(ICDC)	2895.5	2	p-value < 2.2e-16
Kenya Commercial Bank	183880000	2	p-value < 2.2e-16
Housing Finance Company	5970.3	2	p-value < 2.2e-16
Standard Chartered Bank	18586	2	p-value < 2.2e-16
Barclays Bank	124560000	2	p-value < 2.2e-16
Equity Bank	66415000	2	p-value < 2.2e-16
Cfc Stanbic Bank	12926000	2	p-value < 2.2e-16
East Africa Brewaries	17079	2	p-value < 2.2e-16
British American Tobacco	38060	2	p-value < 2.2e-16
Mumias Sugar	12668000	2	p-value < 2.2e-16
Bamburi Cement	2918.1	2	p-value < 2.2e-16
Kenya Electricity Generating Co	18679	2	p-value < 2.2e-16
Kenol Kobil	124620000	2	p-value < 2.2e-16
Kenya Power	132360000	2	p-value < 2.2e-16
Jubilee Holdings	56800000	2	p-value < 2.2e-16

X-squared with 2 degrees of freedom (df) is the test statistics for the Jarque-Bera test. The column next of this contains the corresponding p-value for each test statistics. It is this figure that is measured against the significance level. As the test was carried out on a 32-bit platform due to memory restrictions any values less than 0.00000000000000022 are simply denoted as less than $2.2e^{-16}$ ($< 2.2e^{-16}$).

Table 2 above shows under the Jarque-Bera test of the daily log returns we should reject the null hypothesis that they are normally distributed. Note, because the p-values are extremely small, we have strong evidence to reject the null hypothesis that the continuously compounded daily returns for securities are normally distributed. Not surprisingly, the null hypothesis of a normal distribution is rejected in all cases. Thus, risk management decisions based on the assumption of normality of asset returns will be flawed.

4.3 Efficient Frontier and Minimum Risk Portfolio

The study looked at the portfolio optimization problem and analyze the dynamics between mean-variance framework and the downside risk framework. Throughout the analysis, assumption made was that investor is highly risk-averse and therefore focus on the minimum risk portfolios, which are optimal for such an investor.

4.3.1 The Mean-Variance Framework

Within the mean-variance framework, the study followed algorithm described in Markowitz (1991) and Sharpe (2000). The 20 stocks in Appendix A selected from different industries acted as the market and by using R statistical program, results were obtained on the efficient frontier and minimum variance portfolio. The weights were obtained by minimizing the portfolio variance, thus obtaining the minimum variance portfolio.

The result of the optimization of the minimum-variance is shown in Table 3:

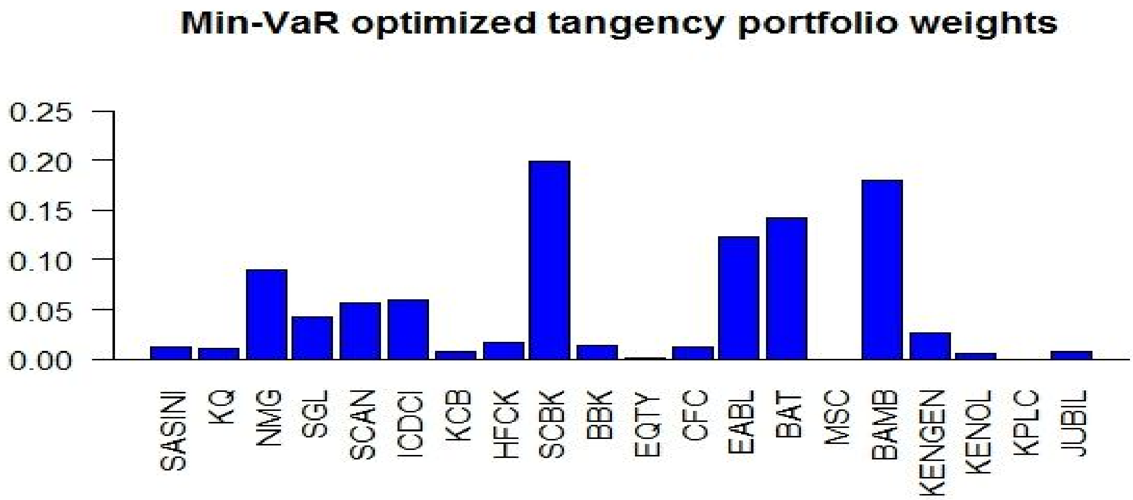
Table 3: Minimum Variance Portfolio Results

The Optimal Portfolio Weights:									
SASINI	KQ	NMG	SGL	SCAN	ICDCI	KCB	HFCK	SCBK	BBK
1.26523%	1.11502%	8.88785%	4.13227%	5.70848%	5.89405%	0.70010%	1.65092%	19.84266%	1.30864%
EQTY	CFC	EABL	BAT	MSC	BAMB	KENGEN	KENOL	KPLC	JUBIL
0.10674%	1.13756%	12.33408%	14.19563%	0.00000%	17.96290%	2.55756%	0.52644%	0.00000%	0.67386%

Target Returns and Risk	
Mean	Standard deviation
0.00534%	0.89136%

As we can see, Standard Chartered Bank (SCBK), Bamburi Cement (BAMB), British American Tobacco (BAT), and East Africa Brewaries (EABL) take the largest portion of where investment should be placed respectively. This makes sense since these securities have the lowest volatilities of all securities under study as exhibited in Table 1 above. The optimal portfolio expected return is 0.00534% and the volatility is 0.89136%.

Figure 4.11: Bar Chart of Minimum-Variance Portfolio Weights



Having determined the minimum variance portfolio, the efficient frontier was computed in the mean-variance framework. This was done by means of linear programming where the minimum variances given different levels of portfolio return is solved, thus identifying several sets of weights that all compose the efficient portfolios.

Figure 4.12: Efficient frontier of Minimum Variance optimal portfolios.

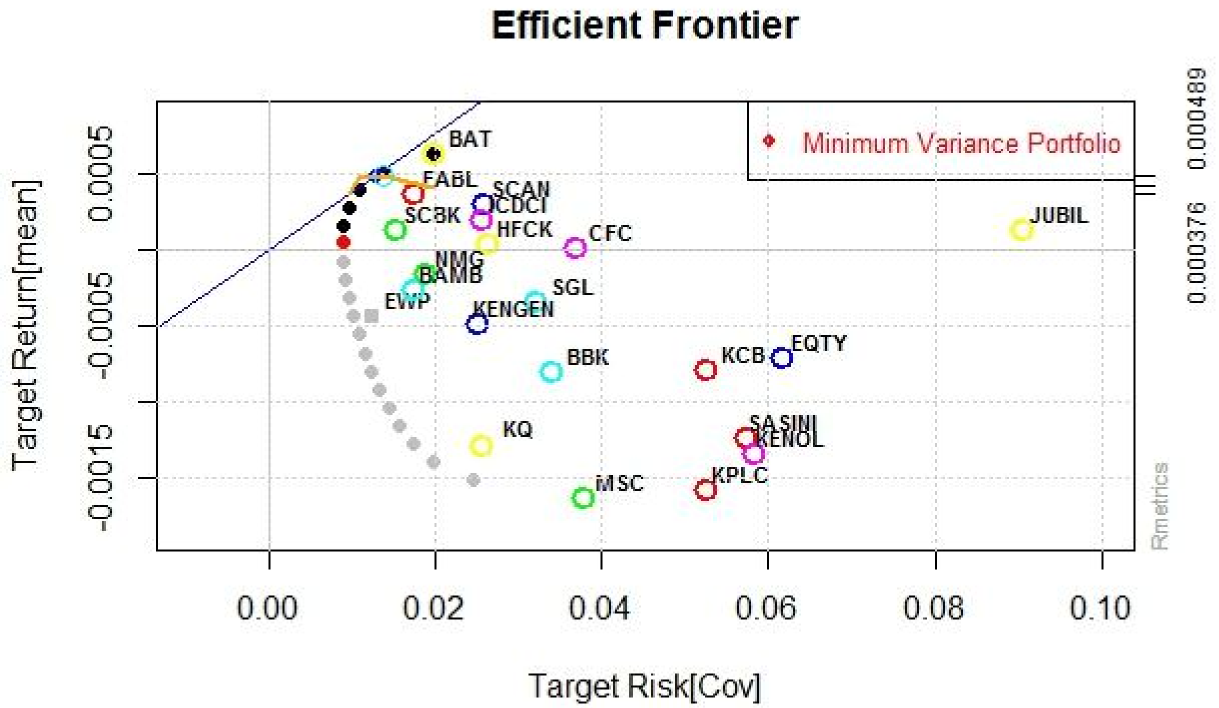


Figure 4.12 above illustrates the mean-variance frontier of investing in our twenty securities. Note that the efficient frontier does not include the portfolios below the minimum variance portfolios, as these are clearly inferior.

4.3.2 The Mean-CVaR Framework

Within the Mean-CVaR framework, the study followed algorithm described by Rockafellar and Uryasev (1999, 2001). The CVaR-optimal portfolio was obtained by minimizing CVaR subject to the constraint on portfolio's CVaR with 95%-confidence level ($\alpha = 0.95$). Minimizing CVaR also provides us with the corresponding VaR.

Table 4: Minimum CVaR Optimal Portfolio Results

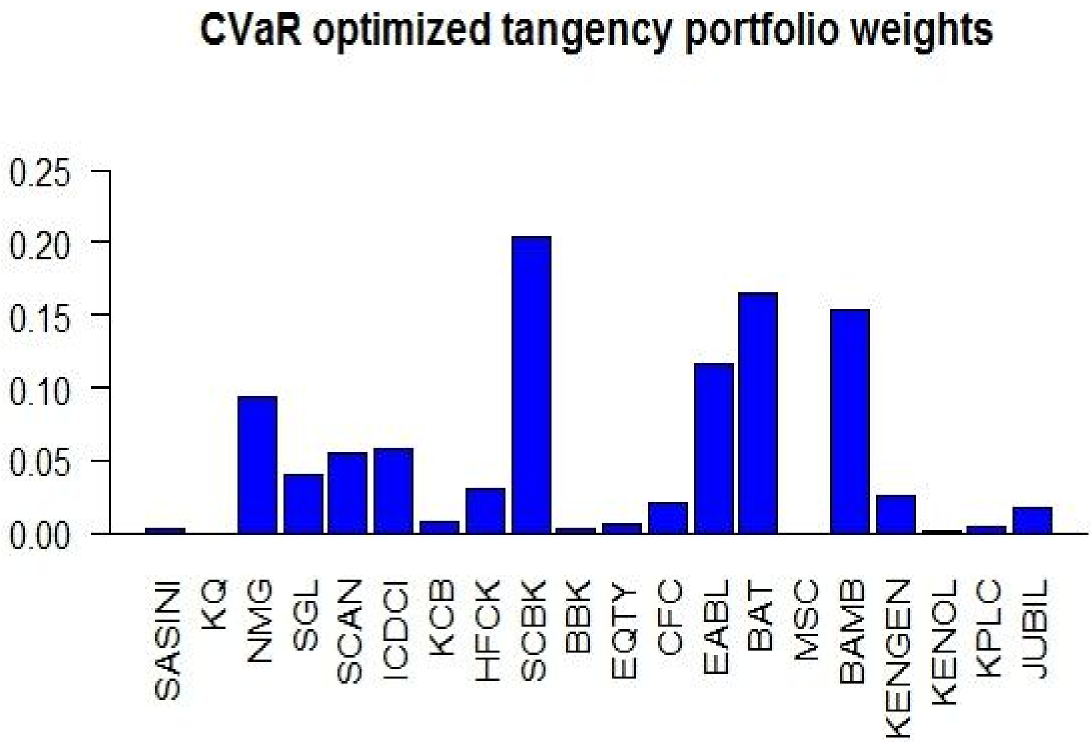
The Optimal Portfolio Weights:

SASINI	KQ	NMG	SGL	SCAN	ICDCI	KCB	HFCK	SCBK	BBK
0.28360%	0.00000%	9.35602%	4.01341%	5.39489%	5.74454%	0.79134%	3.04765%	20.34150%	0.24655%
EQTY	CFC	EABL	BAT	MSC	BAMB	KENGEN	KENOL	KPLC	JUBIL
0.52858%	2.11397%	11.55400%	16.56022%	0.00000%	15.34369%	2.49480%	0.09089%	0.34694%	1.74739%

Target Returns and Risk		
Mean	CVaR	VaR
0.04500%	2.72767%	1.75000%

The minimum CVaR portfolio yields a CVaR of 2.72767% and a corresponding VaR of 1.75%. CVaR is always larger than VaR since it also considers losses exceeding VaR. The gap between the two depends on how far we move into the tail of the distribution and since the study looked at a 95% confidence level, this gap is relatively small. Since the study used daily observations since 2007, there is enough data to set a 95% confidence level.

Figure 4.13: Bar Chart of Minimum-CVaR Portfolio Weights



The result given in Table 5, we see that Standard Chartered Bank (SCBK) again has received the highest weight in the minimum CVaR portfolio.

Figure 4.14: Efficient frontier of Minimum-CVaR optimal portfolios.

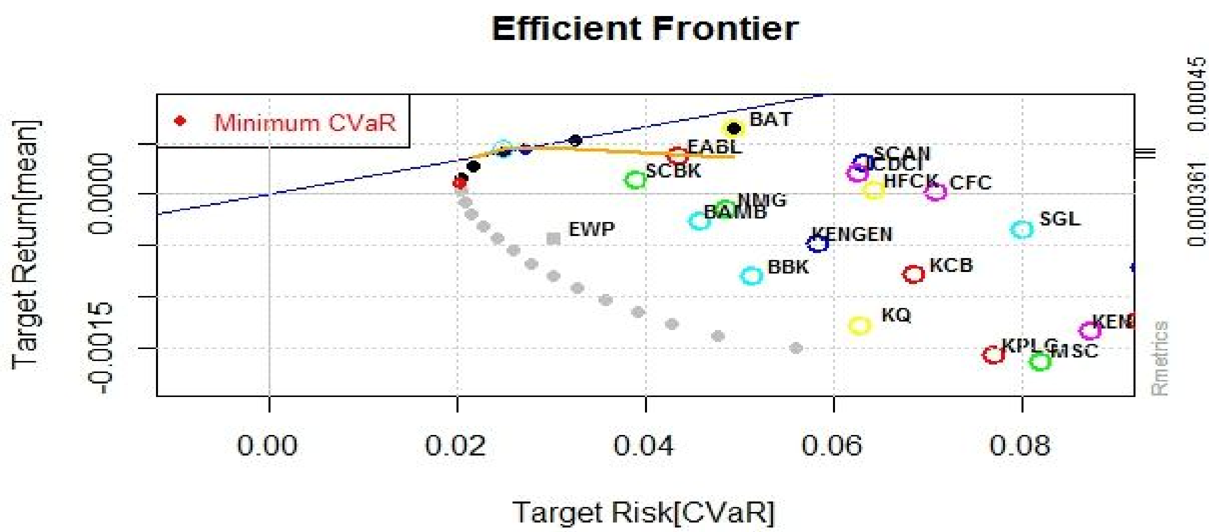


Figure 4.14 illustrate the Minimum-CVaR efficient frontier. Rate of return is the expected rate of return of the optimal portfolio during the period under study (January 2007-December 2014). The Target Risk scale displays the risk tolerance level in the CVaR risk constraint ($\alpha = 0.95$). Again, note that only the portfolios above the minimum CVaR are the efficient portfolios.

4.3.3 Back Testing

Back testing portfolio optimization strategies (minimum-variance and minimum-CVaR) provides valuable feedback about the accuracy of the models. Back testing involved testing how well the minimum-variance and minimum-CVaR portfolio optimization strategies estimates would have performed in the past. In the study, daily back test was implemented for MV and M-CVaR portfolio optimization strategies from January 2009 to December 2014.

Figure 4.15 and 4.16 exhibit the result of the back-test:

Figure 4.15: Wealth Trajectory of Minimum-CVaR versus Minimum-Variance Portfolio Values Back-Test.

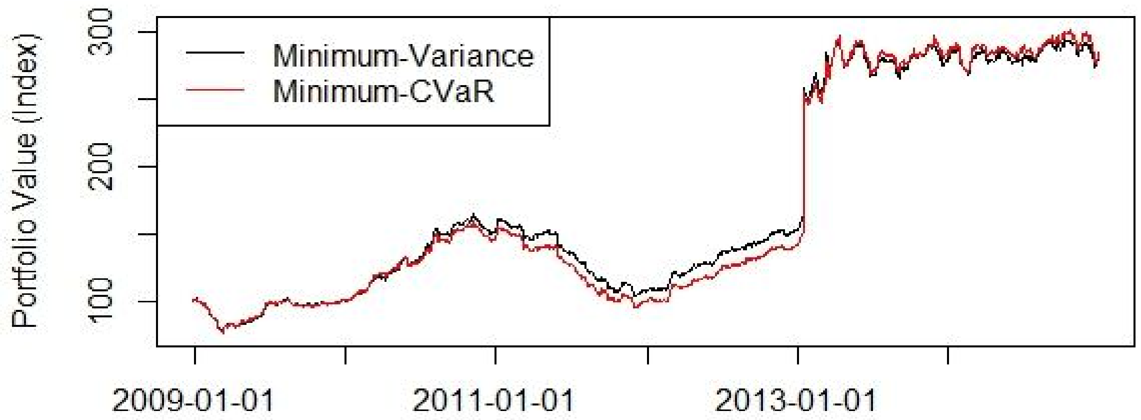
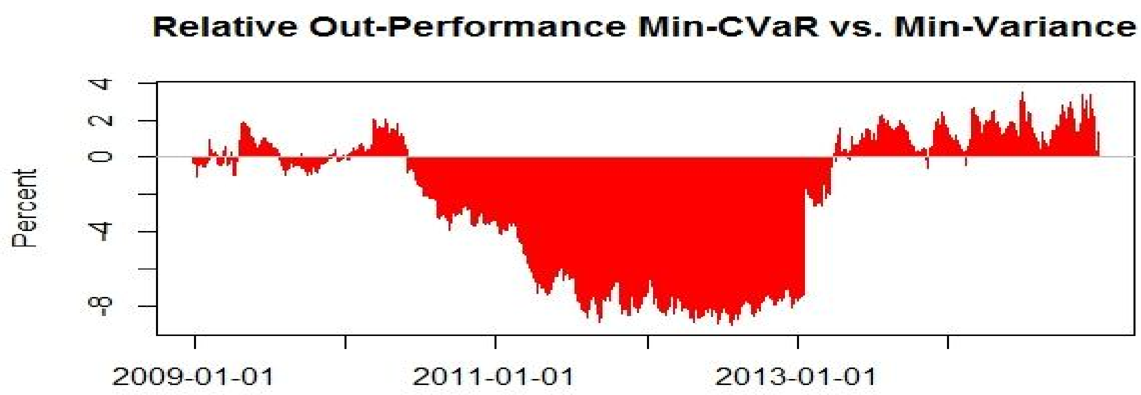


Figure 4.16: Relative Performance of Minimum-CVaR and Minimum-Variance Portfolios



From the result of the back-test, Figure 4.15 and 4.16 exhibit that the Minimum-Variance portfolio approach outperformed the Minimum-CVaR portfolio approach from mid-year 2010 towards the end of year 2012. From early year 2013 to year 2014, the Minimum-CVaR portfolio approach outperformed the Minimum-Variance portfolio approach. Relatively close graphs of CVaR- and MV- optimal portfolios from year 2009 to mid-year 2010.

4.4 Discussion of Findings

The descriptive results of this study were concerned about the dispersion, central tendency and normality assumption of the returns of equity securities of the twenty companies quoted in the various sector of the NSE. This is important because dispersion including range, variance and standard deviations are established measures of riskiness of returns of investment portfolios in finance. Over the study period, most of the securities had an average negative daily returns. This implies that the daily prices and therefore the holding returns had a general declining tendency. However, this was not the case for the CFC Stanbic Bank, Scan Group, Centum, Housing Finance Company, Standard Chartered Bank, East Africa Brewaries Ltd, British American Tobacco and Jubilee which hade average positive daily returns.

The results indicates that the probability density function of returns for the 20 securities under the study was skewed and fat tailed. Many securities under study had negatively skewed daily returns, indicating large market returns are negative. However, this was not the case for the Scan Group, Housing Finance Company, CFC Stanbic Bank and KenGen which had positively skewed returns, implying possible investment opportunities. Furthermore, fat-tails existed for all equity securities with kurtosis far in excess of the corresponding to the normal distribution. Normal distribution of the daily returns was rejected using graphical and Jarque-Bera test.

The Conditional Value-at-Risk (CVaR), an example of downside risk measure was used in the downside risk framework. The study compared the CVaR methodology with the MV approach by running the optimization algorithms on the same set of securities and scenario. The table 3 shows the summary result of the mean-variance optimal portfolios while table 4 shows the summary results of the CVaR optimal portfolios. It indicates that the mean return of the CVaR model (0.045%) is the highest whereas the mean-variance model has a mean return of (0.00534%). The CVaR model (2.72767%) is the most risky portfolio while the mean-variance model (0.89136%) generates the less risky portfolio. However, it is important to note that under the mean-variance model, asset returns are assumed to be normally distributed. This model becomes inadequate when the normality assumption is violated as shown from the result of the analysis that the daily log returns are not normally distributed.

It was shown in Rockafellar and Uryasev (2000) that for normally distributed loss functions these two methodologies are equivalent in the sense that they generate the same efficient frontier. However, in the case of non-normal, and especially non-symmetric distributions, CVaR and MV portfolio optimization approaches may reveal significant differences. Indeed, the CVaR optimization technique aims at reshaping one tail of the loss distribution, which corresponds to high losses, and does not account for the opposite tail representing high profits. On the contrary, the Markowitz approach defines the risk as the variance of the loss distribution, and since the variance incorporates information from both tails, it is affected by high gains as well as by high losses.

Overall, the study found that the choice of risk measure has a significant effect on portfolio allocation. From the analysis, CVaR as a downside risk measure outperform variance. This suggest that downside risk can be a better tool in investment management than variance.

CHAPTER FIVE: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter presents summary of the key findings presented in chapter four, conclusions drawn based on such findings, recommendations, limitations and suggestions for further research.

5.2 Summary of Findings

It is widely known that the distributions of financial asset returns exhibit fatter tails than the normal distribution. The study empirically tested the normality of the daily log returns of the twenty securities under the study. The result indicates that the daily log returns of the twenty securities are not normally distributed. The evidence that distribution of the daily log returns are not normally distributed and have fatter tails than indicated by the normal distribution meant that the mean-variance approach underestimate the true risk. It therefore becomes apparent that in order to capture full risk from fat tailed assets, Conditional Value-at-Risk, a downside risk measure should be used.

After testing for the normality of daily log returns of the twenty securities under study, the focus moved to the implications of using the different risk measures in optimal portfolio selection. The study started out by demonstrating how to compute the efficient frontiers and calculating the different minimum risk portfolios under the mean-variance framework and the downside risk framework. This study empirically tested the effectiveness of CVaR over the mean-variance model. Findings from the study suggests the efficient frontiers comparison seems to indicate that the CVaR model could provide clearer indications in terms of asset allocations that lead to an optimal portfolio that not only matches if not exceeds those expected returns from the traditional mean-variance framework, but also lowers downside risk. In other words, the results indicate that the CVaR model could provide investors with asset allocation not only in those assets with higher variance but also in those assets with higher CVaR in particular. By constructing a portfolio as suggested by the CVaR model, investors can simultaneously minimize the downside risk and achieve similar or better expected returns. In contrast, the mean-variance model tends to lead to a portfolio that has limited upside gain and higher downside risk.

The level of risk, as measured by the empirical CVaR for the portfolio, is higher than for the mean-variance framework captured by the use of standard deviation alone. The greater the deviation from normality the greater the underestimation of risk by standard deviation. The greater probability of extreme negative returns in the empirical distribution from the results implies greater downside risk than is captured by the measure of standard deviation alone. The use therefore of the normal distribution to assess the risk-return trade-off will result in an incorrect allocation of assets for investors with low risk tolerance.

The study results relate to findings by Krokhmal et al. (2002) who proposed a model for optimal portfolio selection when a CVaR constraints is imposed. They showed that multiple CVaR constraints with various confidence levels can be used to shape the profit/loss distribution. The findings also relates to the study by Alexander et al. (2006) who compared the portfolio selection problem under VaR and CVaR and showed that solving Mean-CVaR is better alternative to Mean-VaR.

5.3 Conclusions

Risk is an essential factor to consider when investing in the capital markets. The question of how one should define and manage risk is one that has gained a lot of attention and remains a popular topic in both the academic and professional world. Since the dawn of modern portfolio theory, there has been a consensus that investors should not minimize uncertainty in general but rather minimize "bad" uncertainty. In other words, investors should minimize downside risk. The measure for risk depends on a portfolio's potential loss function, itself a function of portfolio CVaR. Introducing CVaR into the measure for risk has the benefit of allowing the risk-return trade-off to be analyzed for various associated confidence levels. Since the riskiness of an asset increases with the choice of the confidence level associated with the downside risk measure, risk becomes a function of individual's risk aversion level. The portfolio selection problem is still to maximize expected return, however whilst minimizing the downside risk as captured by VaR.

The aim of this paper was to test the effectiveness of downside risk framework over the mean-variance framework in optimal portfolio selection at the Nairobi Securities Exchange. The performance as well as portfolio composition of the mean-variance model and CVaR model, a downside risk measure, were compared with the use of twenty securities listed at the NSE in period from January 2007 to December 2014. The result of the study indicate that the difference between the MV and CVaR approaches is not very significant. However, portfolio allocation for the two different optimal portfolio strategies were different depending on the risk measure. In order to determine the performance of the different risk measures, the study back tested the mean-variance model and the CVaR model from January 2009 to December 2009. The result showed relatively close graphs of CVaR- and MV- optimal portfolios. This indicated that a CVaR optimal portfolio is “near optimal” in MV-sense, and vice versa, a MV-optimal portfolio is “near optimal” in CVaR-sense. This agreement between the two solutions should not, however, be misleading in deciding that the discussed portfolio optimization methodologies are the same. The obtained results are dataset-specific, and the closeness of solutions of CVaR and MV optimization problem is caused by apparently “close-to-normal” distributions of the historical returns used in this case studies. However, the picture somewhat changed from early year 2013 to year 2014, where the CVaR model approach outperformed the mean-variance model. Arising from the normality test of daily log returns, the study suppose the CVaR model to perform better than the mean-variance model.

Overall, the study found that the choice of risk measure has a significant effect on portfolio allocation. The study illustrated just how great the impact is on the portfolio selection decision from non-normality, time horizons and alternative risk specifications. From the analysis, CVaR as a downside risk measure, outperformed the variance. The CVaR meets all quality requirements that are set for a coherent risk measure in accordance with the scientific literature. In particular, the use of the CVaR allows diversification effects in a portfolio context to be adequately taken into account. This is key in large multi asset mandates, as it means a diversified portfolio always has the same or less risk than the sum of the individual risks. This characteristic is also referred to as a Subadditivity condition. This suggest that downside risk can be a better tool in investment management than variance.

5.4 Recommendations

Based on the findings and conclusions drawn thereof, a number of recommendations are made. Investors should take care to ensure that risk measurement is as realistic as possible, because fund management impulses arise directly from it. This applies to risk management, but also for issues related to portfolio optimization and performance measurement. In reality, return distributions are frequently skewed and have many outliers, and symmetrical risk measures such as volatility have a limited ability to provide adequate risk measurement. Downside risk measures such as CVaR address this issue and measure risk in the area of the return distribution that causes concern for investors and that reflects real risk in the sense of loss. Among downside risk measures, CVaR possesses several attractive properties that make it superior to the traditional VaR.

CVaR model have practical implications for both individual investors and institutional investors for asset allocation and portfolio optimization while managing their downside risk exposure. It is especially important for insurance and banking sectors, the industries that have a higher risk aversion for downside risk. Insurance companies and commercial banks in Kenya, for example, are required by the regulators to maintain a certain level of capital, which is determined by the level of the risk in their invested assets. While insurance companies and banks seek to reduce the required capital to a minimum level, they are very concerned about minimizing the downside risk while maintaining a certain level of return on investment. The CVaR model could be very instrumental to these companies for their risk management. Moreover, the CVaR model allows portfolio managers to have a clear definition of risk that combines the objectives and constraints of the entire investment portfolio.

5.5 Limitations of the Study

This study is limited by its lack of analysis of transaction costs and taxes. This is because the study wanted to preserve a focus on how the different risk measures affect the performance of optimized portfolios. This limits the generalization of findings to only returns and volatility.

The study also is limited by its inability to incorporate a model that forecast how market returns will become in the future which would be useful to investors who are risk averse and try to minimize their risk. This owes to lack of forecasting model that can give a precise for expected returns of assets. This is because of the fact that all security prices are always going to vary a lot, since investors have different opinions about how risky and profitable a security. Due to this they have different believes on how the market will be in the future.

In attaining its objective, the study was limited to NSE in Kenya. However, case studies like this cannot be generalized to other securities as they might have different dynamics. The study only concentrated on the stocks and did not look at other securities such as bonds and treasury bills. It also important to acknowledge that the study only analyzed twenty securities out of sixty two securities listed at the NSE because of the availability of complete data for the period of the study.

5.6 Suggestions for Further Studies

However, there is still room for further research. Firstly, the study was based on simple scenario generation procedure which could be substituted for more sophisticated methods as Monte Carlo simulations, etc. Furthermore, the same simulation framework can be done with a use of multi-period optimization routine, instead of single period optimization in several steps.

The study recommends that future studies can be conducted by looking at other downside risk measure such as semi-variance, conditional drawdown at risk.

Further studies can also be done by looking at other security investments like bonds and treasury bills. This would help argument this study as the performance of various security instruments differs.

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APPENDICES

Appendix I: Securities selected from different industries in study

Security	**Sector**
Sasini Tea Ltd	Agricultural
Kenya Airways Ltd	Commercial & Services
Nation Media Group	Commercial & Services
Standard Group	Commercial & Services
Scan Group	Commercial & Services
Centum(ICDC)	Investment
Kenya Commercial Bank	Banking
Housing Finance Company	Banking
Standard Chartered Bank	Banking
Barclays Bank	Banking
Equity Bank	Banking
Cfc Stanbic Bank	Banking
East Africa Brewaries	Manufacturing & Allied
British American Tobacco Kenya Ltd	Manufacturing & Allied
Mumias Sugar	Manufacturing & Allied
Bamburi Cement	Construction & Allied
Kenya Electricity Generating Co	Energy & Petroleum
Kenol Kobil	Energy & Petroleum
Kenya Power	Energy & Petroleum
Jubilee Holdings	Insurance