UNIVERSITY OF NAIROBI

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MODELING NATURAL FREQUENCIES OF VIBRATION OF THREE DIMENSIONAL FRAMES UNDER TWO DIMENSIONAL LOADING

B Y

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A thesis submitted in partial fulfilment of requirements for the Degree of Master of Science in Civil Engineering at the Department of Civil and Construction Engineering, University of Nairobi.

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I, Gideon Nzioki Mutala, hereby declare that this thesis is my original work. To the best of my knowledge, the work presented here has not been presented for a degree in any other university.

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This research report has been submitted for examination with my approval as university supervisor.

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Dedication

I dedicate this thesis to the Almighty God the Creator of Heaven and Earth who is the source of all good things including knowledge. I reached this far because He enabled me.

Acknowledgement

I am grateful to my wife, Lucy, who encouraged me to proceed with the course. I am also grateful to my supervisor Dr. S.W. Mumenya who patiently guided me in the course of my research design and beyond.

Abstract

The aim of this study was to determine the relationship between natural frequency of vibration, the height of the structure, the stiffnesses of members and number bays of a structure. The relationship was to be developed based on data obtained using two methods. The methods were theoretical whereby Computer Modeling was undertaken based on structural theory, and experimental, whereby physical prototypes of structures were subjected to free vibrations.

In the theoretical method, a matrix approach to analysis was adopted to develop a computer program which generated structural models. A horizontal force would be applied at the top-most joint of each model and deflection at the centre of mass was calculated, which was the amplitude of vibration. The overall stiffness of the structure was calculated using the structural amplitude obtained. The overall stiffness was then used to calculate the frequency of vibration for each structural model.

In the experimental method, physical models of miniature structures were built with different heights, member stiffnesses and number of bays. Each model was subjected to free vibrations and the deflections against time were measured. To simulate a free vibration in a model, a measured horizontal force was applied at a joint located at the top of the structure to produce an initial deflection and then the force was withdrawn to allow free vibrations. The deflections at the centre of gravity for free vibrations were measured against time.

The equipment used to measure the free vibrations was horizontal motion transducer. The instrument has a probe which gets depressed when anobject is pushed against it. The transducer was attached to a TDS 302 data-logger which printed deflection against time.

The data obtained was analysed graphically. It was found that the theoretical values were very close to the experimental values, with very high positive correlation coefficients.A relationship between the natural frequency of vibration and the various parameters was developed. The relationship obtained will enable the engineering design of tall buildings against dangerous resonance with the forces they are subjected to. The relationship will be used to calculate the natural frequency of a proposed structure. If the frequency is found to the same of that of the forces expected to act on the structure, a change will be made in the dimensions of the structure to change its frequency. This way catastrophic resonance will be avoided.

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Abbreviations and Terminologies

Abbreviations

ACSE: American Society of Civil Engineers

FEM: Finite Element Method

Terminologies

MATLAB: A high-level language and interactive software for numerical computation, visualization, and programming

TDS 302: Data logging machine

I/L:Stiffness of structural members where I=area moment of inertia and L=member length

CHAPTER 1: INTRODUCTION

1.1Background

Structures undergo free vibrations if oscillations are induced when there is no further energy input into the structure. The free vibrations occur under natural frequencies of the structure. When energy is input into a structure, for example, due to earthquakes or wind, forced vibrations occur. When an external force causing vibrations has the same frequency as the natural frequency of the structure, resonance occurs and the vibrations can reach extreme levels leading to collapse of the structures. The forces acting on structures in catastrophic resonance situations include wind, earthquake, human movement and machinery.

The parameters associated with vibration characteristics are; natural frequencies, modal shapes and modal damping ratios (Arakawa & Yamamoto, 2004). Variations on vibration characteristics reflect changes in the physical parameters of the structural system and indicate certain cracks or damages caused by failure of members in the system (Arakawa & Yamamoto, 2004).

Winds have different frequencies ranging from 0.6Hz to over 70Hz. Wind tunnel simulations have been used to determine these frequencies. In the United States between 700 and 1100 tornadoes occur each year (Ishizaki & Chiu, 1974). These tornadoes cause major destruction of lives and property. The destruction is more where the frequency of vibration of the building structure is the same as that of the tornado. A structure whose frequency of vibration is different from that of the tornado is sometimes left standing whereas a structure which has the same frequency as the tornado is destroyed. Speed plays an important role in governing the effects of resonance. Fast-moving tornadoes may affect stiffer or smaller buildings while slow-moving tornadoes with lower frequencies may affect taller or flexible buildings (Dutta , et al., 2002). An awareness of both tangential and translation speed may be essential to the understanding of damage caused by a specific tornado event on a building.

The earthquake of September 1985 in Mexico City provides a striking illustration of how resonance can have catastrophic effects on structures. Most of the buildings which collapsed during the earthquake were on average 20 stories high. These had a natural frequency of vibration of about 0.5Hz. These 20 storey buildings were in resonance with the frequency of the earthquake. Other buildings, of different heights and with different natural frequencies, were often found undamaged even though they were located right next to the damaged buildings.There were five parameters that affected the seismic performance. These included the degree of regularity, redundancy of structure, relation between the effective natural period of the structure and the expected predominant period of the seismic motion, the real strength of the structure, and the ability to sustain cycles of inelastic deformation without a loss in strength (Bertero, 1989). In Kenya earthquakes have occurred occasionally even though not of high magnitude.A moderate shallow earthquake meassuring magnitude 4.6 struck at only 33 km from the capital of Kenya (thewatchers.adorraeli.com, 2012), Nairobi on April 17,2012 at 02:01. The epicenter was located 16km (9 miles) NE from Limuru, 22km (13 miles) NNW from Kiambu and 33km (20 miles) NNW from Nairobi. The depth of the epicenter was at 10km (6,2 miles). The intensity of the shaking was not strong enough to inflict serious damage or injuries, caused some cracks in walls of many houses as the houses in Kenya are mostly made of brick and are therefore very vulnerable for damage when serious shaking is taking place. The otherwell built buildings could be affected if they had the same natural vibration frequency as the earthquake leading to resonance. The effect would be high if the vibrations under resonance took a long duration leading to a high gain in energy.

A classic modern example of bridge collapse brought about by resonance is the failure of the Tacoma bridge 1940. On the morning of 7th November 1940, the amplitude of vibration increased until the bridge collapsed (Engineering.com, 2016). It was noted that higher wind speed favoured higher frequencies of vibration for the bridge. In a different case in Bangladesh, the collapse of a factory was partly attributed to resonance between the factory machines and the building (Schilling, 2013).

The foregoing illustrates catastrophic resonance where the natural frequency of the structure is the same as that of the force acting on it. It is therefore necessary to gain a better understanding of the relationship between the natural frequency and the dimensions of a structure. Such a relationship will inform engineering design of structures for various forces expected to act on them. This way, destruction of property and lives will be reduced. Moreover, engineers will be able to detect structural deterioration based on changes of frequency of vibration. In addition engineers will also be able to minimize discomfort to users by avoiding resonance during vibrations in buildings.

1.2 Problem statement

In a past study where computer modeling was utilized (Verma & Ashish, 2011) it was found that the natural frequencies of vibration of structures decreased with increase in the height of the structure. The study was undertaken for structures where members were either vertical or horizontal. In the study, two-dimensional structures were considered instead of three-dimensional as proposed in this research. Moreover, no formulation was done for the relationship between the natural frequency and horizontal length of the structure or the stiffnesses of structural members. This research aimed at derivation of the mathematical model for the relationships in three dimensional structures.

1.3 Objective of the study

The main objective of this study was to model the relationship between natural frequency of vibration of a structure , its height, the stiffnesses of its members and the number of bays.

The specific objectives were:

(i) To use stiffness matrix method to develop a software capable of analysing deflections for given initial horizontal forces acting on a structure. The software was to be used to simulate free vibrations and calculate the deflections against time. This was to be done for structures with different heights, number of bays and structural member stiffnesses.

(ii) To make steel model structures and subject them to free vibrations by applying initial horizontal forces. The horizontal deflections against time for the free vibrations were to be measured. This was to be repeated for structures of different heights, number of bays and structural member stiffnesses.

(iii) To model the relationship between natural frequency, the height of structure, the number of bays and the stiffnesses of the structural members.

1.4 Justification of the study

It is important to study natural frequencies of vibration in structures because excessive vibrations due to resonance have been known to cause collapse of the structures. If designers had information regarding the natural frequencies of vibration of the structures under design, there would be a reduced possibility of resonance of the structures under the forces they are likely to be subjected to in their life spans.

1.5 Scope of the study

The scope of the study is two dimensional vibration for the first mode of vibration. It is assumed that the maximum horizontal deflection is at the top of the structure.

CHAPTER 2:LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Vibrations

Vibrations are phenomena that are often unwanted in engineering (James, et al., 1989). When systems start vibrating at the wrong frequencies, there is very high likelihood of failure. In reality all systems are continuous systems, meaning that the displacements of their components depend on many different factors. To simplify the analysis, the system is often modeled as a discrete system whereby the system is split up in parts, which are then evaluated separately.

Two types of vibrations can be distinguished as: free vibrations and forced vibrations. In free vibrations, there is no energy input from the environment, while in forced vibrations there is a continuous energy input from the environment.

Detailed consideration of the vibration of a linear single degree of freedom (SDOF) is essential to the complete understanding of vibration of more complex vibrating systems whether they are composed of lumped elements of mass and stiffness, or whether their mass and stiffness are distributed (Snowdon, 1968).

2.2Free vibrations

2.2.1 Stiffness of an Axially Loaded Rod

To illustrate stiffness concept, the stiffness of an axially loaded rod of negligible mass, having a mass attached to its end is considered. The displacement of the mass is given by Equation 2.1 (Gerre and Goodno, 2008) as follows:

$\delta = FL/EA$ ---*Equation 2.1*

Where F is the tensional force in the bar, L is the length of the bar, E is the Young's modulus of elasticity and A is the cross-sectional area. The stiffness (*k*) is defined as the force needed to cause a unit displacement according to Equation 2.2.

$k = F/\delta$---*Equation 2.2*

Therefore, for an axially loaded rod, the stiffness is as given by Equation 2.3.

$k = EA/L$---*Equation 2.3*

The condition is modeled by replacing the bar by a spring with the stiffness of value *k*.

2.2.2Motion of an Axially Loaded Rod

Information about the movement of the system is desired when it is a known initial displacement and velocity. To find this out, Newton's second law is used; given by; F = Ma (Jacobsen, 1958), where M is mass and a is acceleration. When gravity is not considered, the only force acting on the mass is the spring force, Fs. The spring set up in such a case is shown below.

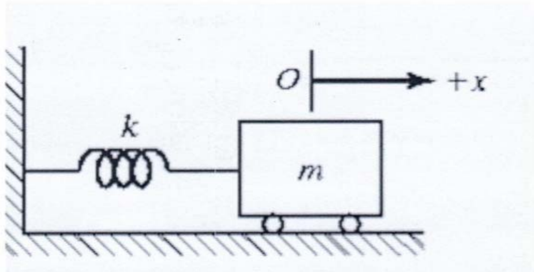


Figure 2.2.2-Spring set up with no gravity consideration

The spring force varies linearly with the displacement "x" times the stiffness "k". However, if the mass moves upward, the spring forces points downward. So there is a negative relationship between the two. The equation of this system becomes; Fs = −kx. If this relationship is combined with Newton's second law, the relationship is described by equations 2.4 and 2.5 (Maurice,1990).

$\frac{m.d^2x}{dt^2} = Fs = -kx$ ---*Equation 2.4*

$\frac{m.d^2x}{dt^2} + kx = 0$ --*Equation 2.5*

The solution is obtained by solving the differential equation to give

$x(t) = C_1cos(\surd(k/m)t) + C_2sin(\surd(k/m)t)$--------------------*Equation 2.6*

Therefore, the system will start vibrating with a fixed angular frequency referred to as angular eigenfrequency, which is denoted by equation 2.7.

$W_n = \surd(k/m)$---*Equation 2.7*

From equation 2.7, the eigenfrequency (f) and vibration period (T) can be derived, according to equations 2.8 and 2.9 (Thomson, 1993).

$f = W_n/2\pi = (1/2\pi)(\surd(k/m))$-------------------------------------*Equation 2.8*

and $T = 1/f = 2\pi/W_n = 2\pi\surd(m/k)$--------------------------------*Equation 2.9*

However a better expression of the solution of the displacement at a given time is described by equation 2.10.

$x(t) = Asin\,(W_n t + \Phi)$ ---*Equation 2.10*

A is the amplitude (usually taken to be positive) and Φ is the phase, both of which follow from the boundary conditions. If the mass is given an initial displacement Xo and an initial velocityV_0, then the solutions for A and Φ are given by equation 2.11.

$A=\sqrt{((X_0)^2+(V_0/W_n)^2)}$ *and* $\Phi=tan\text{-}1(W_n X_0/V_0)$-----------------------*Equation 2.11*

2.2.3 Effects of Gravity

When gravity is considered on a vibrating system, the total force acting on the mass will be Fs + mg. This causes a modification on the differential equation into the relationship given by equation 2.12

$\frac{m.d^2x}{dt^2} + kx = mg$---*Equation 2.12*

When solving differential equations, the first step is to find the homogeneous solution which is obtained after modification of equation 2.12 into equation 2.13, whose solution is given by equation 2.14.

$m.d^2x/dt^2+kx=0$--*Equation 2.13*

$xh(t) = Asin\,(Wnt + \Phi)$---*Equation 2.14*

Equation 2.14 is the homogeneous solution of the differential equation 2.12. After the homogeneous solution is obtained, the particular solution xp(t) is required. Considering that the non-homogeneous term mg is a constant, the particular solution is expected to be a constant too. Therefore, the non-homogeneous term xp(t) is equal to (mg/k); that is:

$xp(t) = mg/k$

Therefore the solution for the differential equation is the sum of the homogeneous and non-homogenous term as given by equation 2.15.

$x(t) = x_h(t) + x_p(t) = Asin\,(W_n t + \Phi) + mg/k$-----------------*Equation 2.15*

From statics if the amplitude A is zero, then the mass will just have a constant displacement of mg/k. In vibration engineering, the homogeneous solution $x_h(t)$ is sometimes referred to as the transient solution x_{tr}(t) and the particular solution $x_p(t)$ is also referred to as the steady state solution x_{ss}(t).

2.2.4Motion of a Laterally Loaded Rod

There are other kinds of vibrations apart from masses on axially loaded rods. For example a laterally loaded rod of a moment of inertia (I) and stiffness, k, when given displacement, δ, will vibrate with natural frequencyW_n, as described by equations 2.16 to equation 2.18.

$\delta = FL^3/3EI$---*Equation 2.16*

$K=F/\delta = 3EI/L^3$-- *Equation 2.17*

$W_n = \sqrt{(k/m)}$--*Equation 2.18*

2.2.5Rotation of a Torsionally Loaded Rod

A disk of mass (m), moment of inertia, *J*, and connected to a rod with polar moment of inertia I_p, undergoes angular displacement, θ which depends on the moment, *M*, that is acting between the rod and the disk. If this moment is known, then the angular displacement is described by equation 2.19 and the torsional stiffness, *k*, by equation 2.20.

$\theta = ML/GI_p$--*Equation 2.19*

Where G is the Modulus of Rigidity (Shear Modulus)

$k = M/\theta = GI_p/L$---*Equation 2.20*

The difference between torsional stiffness and normal stiffness is in the definition. Torsional stiffness is a product of force and displacement (units are Nm per radian) where as normal stiffness is force per unit displacement (N/m).

2.2.6 Modeling of a torsionally loaded rod

Newton's second law for rotations is described by equation 2.21 as follows;

$-k.\theta = j.d^2\theta/dt^2$--*Equation 2.21*

This gives the differential equation;

$j.d^2\theta/dt^2 + k\theta = 0$---*Equation 2.22*

The solution to the differential equation is;

$\theta(t) = \theta a.Sin(W_n.t + \varphi)$--*Equation 2.23*

Where W_n is the angular natural frequency=$\sqrt{(k/j)}$ and θa is the amplitude of vibration, andφ is the phase.

2.2.7Damped Vibrations

If the energy of a free vibrating system is dissipated with time, the amplitude also decreases with time. The generalized equation for a damped vibration is according to equation 2.24.

$m.d^2x/dt^2+C.dx/dt+kx=0$---*Equation 2.24*

where C is the damping coefficient.

Let $cc=2m\sqrt{(k/m)}$, Wdbe damped angular frequency, $W_n$be natural angular frequency for undamped oscillation. The damping ratio, $\zeta, W_d,$ *and* $W_n$are described by equations 2.25 to 2.27 below.

ζ, *damping ratio=C/cc,* --*Equation 2.25*

$Wd = \sqrt{(1 - \zeta 2w_n)}$ --*Equation 2.26*

$W_n=\sqrt{(k/m)}$--*Equation 2.27*

If the damping ration is less than 1 (ζ<1), the system is under-damped. In this case, the motion is described by equation 2.28.

$x(t)=A_0\ e^{-\zeta\ Wnt}(cos(Wdt- \Phi_0\ )$---*Equation 2.28*

If the damping ratio is equal to 1 (ζ=1), the system is said to be critically damped. In this case equation 2.29 describes the motion.

$x(t) =e^{-Wnt}(x_0+(v_0+Wnx_0)t)$---*Equation 2.29*

If the damping ration is greater than (ζ >1), the system is said to be over-damped. In this case

$x(t)=x_0+(v_0/2\ \zeta Wn)(1-e^{2\ \zeta Wnt})$---*Equation 2.30*

2.3 Forced Vibrations

Free vibrations die away with time because the energy trapped in the vibrating system is dissipated by the damping. In order for the damping ratio to be less than zero (therefore negative), the opposite of damping occurs. In such a case energy is eternally added into the system instead of being removed. As energy is externally added into the system the amplitude grows. Such vibrations where energy is externally supplied are called forced vibrations.

In engineering, many structures are prone to vibrate when excited at or near the natural frequency. This is a situation called resonance and can lead to catastrophic collapse of structures. It is this understanding that this thesis attempts to address.

2.4 Vibrations of Trees

A study was conducted to measure frequencies of vibration of trees(Moore and Maguire, 2003). They first reviewed and synthesized previous studies that measured the natural frequencies and damping ratios of conifer trees. In their analysis of natural frequency measurements from six hundred and two trees, belonging to eight different species, they showed that natural frequency was strongly and linearly related to the ratio of diameter at breast (DB) to total tree height (H) squared (DB/H^2). After accounting for their size, pines were found to have a significantly lower natural frequency than both spruce and Douglas fir (Moore and Maguire, 2003). Natural sway frequencies of de-branched trees were significantly higher than those of the same trees with the branches intact, and the difference increased with increase in the ratio (DB/H^2). Damping mechanisms were analysed and methods for measuring damping ratio were suggested (Moore and Maguire, 2003). It was found that internal damping ratios were typically less than 0.05 and were not related to tree diameter. It was also found that external damping was mainly due to aerodynamic drag on the foliage and contact between the crowns of adjacent trees. Analysis of data from previous wind-tunnel studies indicated that damping due to aerodynamic drag is a nonlinear function of velocity (Moore and Maguire, 2003). Damping due to crown contact has been suggested by a previous author to be a function of both the distance to and the size of adjacent trees (Moore and Maguire, 2003). It was found that in uniformly spaced stands, it may be possible to model crown contact damping as a function of stand density index (SDI), a common forestry measure which incorporates both of these variables (Moore and Maguire, 2003).

2.5Comparison of Structural Earthquake Response and Computer Modeling

Maison and Neuss (1985), members of ASCE (American Society of Civil Engineers) performed the computer analysis of an existing forty four story steel frame high-rise building to study the influence of various modeling aspects on the predicted dynamic properties and computed seismic response behaviours. The predicted dynamic properties were compared to the building's true properties as previously determined from experimental testing. The seismic response behaviours were computed using the response spectrum and equivalent static load methods. It was found that the theoretical values were very close to the experimental values.

Other researchers (Maison and Ventura, 1991)extensively computed dynamic properties and response behaviours of a thirteen-story building and the results were compared to the true values as determined from the recorded motions in the building during two actual earthquakes and showed that state-of-practice design type analytical models can predict the actual dynamic properties.

Previous studies have been carried out (Awkar and Lui, 1997) on responses of multi-story flexibly connected frames subjected to earthquake excitations using a computer model. The model incorporated connection flexibility as well as geometrical and material nonlinearities in the analysis. The conclusion was that the connection flexibility tends to increase upper stories' inter-storey drifts but reduce base shears and base overturning moments for multi-storey frames.

Vasilopoulos and Beskos (Vasilopoulos and Beskos, 2009) undertook study on rational and efficient seismic design methodology for plane steel frames using advanced methods of analysis in the framework of Eurocodes 8 and 3 (Vasilopoulos and Beskos, 2009). This design methodology employed an advanced finite element method of analysis that took into account geometrical and material nonlinearities and member and frame imperfections. It could sufficiently capture the limit states of displacements, strength, stability and damage of the structure.

Ozyigit (Ozyigit, 2002) undertook study on free and forced in-plane and out-of-plane vibrations of frames. The beams had straight and curved parts and were of circular cross section. A concentrated mass was located at different points of the frame with different mass ratios. FEM (Finite Element Method) was used to analyse the problem. It was found that frequencies of vibration decreased as mass increased.

2.6 Computer Modeling of Two Dimensional Multi-storey Frame Vibrations

Generally the stress and deformation analysis of any structure is done by constructing and analysing a mathematical model of the structure. One such technique is Finite element method (FEM). A frame is subjected to both static and dynamic loading with dead load comprising the static load and all other time varying loads making up the dynamic load. In the past research was undertaken (Verma and Ashish, 2011) on vibrational analysis of frames, aimed at analysing two dimensional structural frame both statically and dynamically using the matrix approach of FEM. In that research, generalized codes in MATLAB software were developed and analysis was done for static loads and also for the variation of various parameters such as displacement and moment with increasing number of storeys. Dynamic analysis was also undertaken whereby a code was developed to determine the natural frequency of the structure along with the various other parameters. The structures analysed had only vertical and horizontal members (beams and columns). The three findings of that research by Verma and Ashish were:

(i) After static analysis of single bay multi-storey frame, it was found that the deflection at any node increases with the increase in number of storeys. Also the sway or deflection of the topmost node increases steeply with increase in number of storeys.
(ii) That the natural modal frequencies decrease as the numbers of storeys increased.
(iii) Both the static and dynamic formulations can be extended to any number of bays of a frames

2.7 Relationship between Amplitude and Natural Frequency

Some studies were undertaken in the past (Tamura, et al., 1993) using the random decrement technique which have shown that for a given external load the amplitude was depended upon natural frequency and modal damping ratio. The random decrement technique has been applied to structures such as buildings to evaluate structural damping under random excitation (Jeary, 1986). In a study on low frequency noise induced by lorries on bridges (Tsubomoto et al, 2015), it was found that concrete bridges produced lower volume noise than steel bridges but the resonance frequency was the same for both materials. The lower volume of noise was attributed to lower amplitudes of vibration.

2.8 Damage Assessment in Structures using VibrationCharacteristics

A research was conducted (Shih, 2009) which showed that damage in structures can be established by detecting changes in vibration characteristics. The study used variation in vibration parameters to provide a multi-criteria method for damage assessment. It incorporated the changes in natural frequencies, modal flexibility and modal strain energy to detect damage in main load bearing structures in bridges. Other related researches showed that the procedure to identify changes in parameters of structure must factor the following seven aspects (Aktan, et al., 1997):

(i) Structural conceptualization
(ii) Structural modeling (may include Finite Element method)
(iii) Designing and executing various experiments
(iv) Data processing and identifying modal and other characteristics
(v) Model calibration and validation
(vi) Simulation and interpretation
(vii) Decisions and heuristics

Classification of bridges (Baker and Puckett, 1997) can be done according to the structural layout, namely; single span, multi-span and cantilever. Modal analysis is an important tool in the analysis, diagnosis, design and control of vibration. Modal testing includes instrumentation, signal processing, parameter estimation and vibration analysis (De Silva, 2007). An experimental vibration system consists of three measurement mechanisms as follows:

(i) The excitation mechanism
(ii) The sensing mechanism
(iii) The Data acquisition and Processing Mechanism

It has been found that the damaged positions in structures experienced increased amplitudes (Baker and Puckett, 1997)). According to Beck and Jennings(Beck and Jennings, 1980), identification of system parameters of buildings is important for the structural monitoring or the damage detection of buildings due to some excitation or the passage of time.

2.9Resonance between Structure and Ground

Vibration resonance between a structure and the ground is possible (Navarro, et al., 2004). According to some past research the ground natural frequency is affected by water content and temperature in the soil (Clinton, et al., 2006). Data on probable resonance phenomena in Granada city, comparing predominant period of soil and natural period of Reinforced Concrete buildings, shows that a significant number of buildings have dominant periods close to the ground motion ones and consequently resonant phenomena would be able to appear if an earthquake occurred in the zone (Navarro, et al., 2004).

The literature reviewed showed that the true behaviour of structures can be predicted using computer modeling. It was also found that there is need to come up with a formulation for prediction of frequency of structures. This research utilized the following tools to be able to analyse vibrations experimentally;

(i) The excitation system
(ii) The sensing system
(iii) The data acquisition and processing system.

2.11 Effect of Height and Number of Floors on natural Frequency

In a study carried out in the University of Gujarat (Nilesh and Desai, 2012) it was found that the natural frequency of vibration decreased as the number of storeys increased. They prepared computer models of reinforced concrete buildings ranging in height from 60m to

90m with number of storeys varying from 20 to 30. They calculated the natural frequency of vibration for each model and found the frequency decreased as number storeys decreased.

2.12 Effect of Horizontal Member stiffness and Number of Bays on Natural Frequency

In a study carried out in 1996 (Anwar and Hossain,1996), the effect of horizontal member stiffness and number of bays evaluated. In the study a software named "ANYSYS" was used to analyse the effect of bay width and number of bays on the natural frequency of vibration. It was found that as the bay width increased in the direction of motion, and therefore horizontal member stiffness decreased, the natural frequency of vibration decreased. It was also found that as the bay width increased in the direction transverse to motion the frequency of vibration increased. In the same study, as the number of bays in the direction of motion increased, it was found that the frequency of vibration increased. Moreover, it was found that as the number of bays increased in the direction transverse to motion, the frequency of vibration decreased.

2.13 Theoretical Framework

The theoretical framework for this research is outlined in the remaining section of this Chapter.

2.13.1 Theory and Code for Computer Modeling

Based on the theory given in this Chapter, a software, named "Structuresoft", was developed to simulate free vibrations and calculate deflections against time. The computer code is given in the appendix. A three dimensional structure has six degrees of freedom at each joint which can fully move. Similarly, a structure with twelve fully movable joints has seventy two degrees of freedom at the fully movable joints. Such a structure has 72 natural frequencies at the joints. However in this research a single degree of freedom (SDOF) at the centre of gravity is of interest. Therefore after analysing the structure for the initial deflection at all the degrees of freedom at the joints, interpolation was used to determine the idealized initial horizontal deflection at the centre of gravity of the structure.

The stiffness matrix that was assembled to analyse structures was of the form;

$K_{1,1}$ $K_{1,2}$ $K_{1,3}$ -------------------$K_{1,n}$

$K_{2,1}$ $K_{2,2}$ $K_{2,3}$--------------------$K_{2,n}$

$K_{n,1}$ $K_{n,2}$ -------------------------$K_{n,n}$

Where K_{ij} is the force in coordinate *i* due to a unit deflection in coordinate *j* and n is the total number of joints.

The relationship between the external force, the stiffnesses, the deflections, and the internal forces at coordinates 1 and 2 is given by equations 2.31 and 2.32:

$0=P_{1}+P_{1'}+K_{11}\Delta_1+K_{12}\Delta_2+K_{13}\Delta_3$----------------$K_{1n}\Delta_n$ --------------------Equation 2.31

$0=P_2+P_2'+K_{21}\Delta 1+K_{22}\Delta_2+K_{23}\Delta 3$-----------$K_{2n}\Delta_n$ ---------------------------Equation 2.32

The equations at the other coordinates are similar.

The general expression for equations 2.31 and 2.32 is given by equation 2.33;

$0=P_J+P_{J'}+K_{J1}\Delta_1+K_{J2}\Delta_2+K_{J3}\Delta_3$--------------$K_{Jn}\Delta_n$--------------------------Equation 2.33

Where P_J is the external force at coordinate J, $P_{J'}$ is the internal force at coordinate J and $\Delta_1,\Delta_2,\Delta_3$-----------$\Delta n$ are deflections at coordinates 1,2,3---- up to n. The force can be in kN or be a moment in kNm.

2.13.2 Enumeration of Coordinates

It is necessary to enumerate coordinates by considering the degrees of freedom at all the joints. The maximum number of degrees of freedom in space for any joint are six as described below:

1. Linear displacement in X-direction
2. Linear displacement in Y-direction
3. Rotation about Z-axis
4. Linear displacement in Z-direction
5. Rotation about Y-xis
6. Rotation about X-axis

If a joint is a support, it has zero degrees of freedom unless restraint is removed in any of the above mentioned degrees of freedom. The none support joints were rigidly welded otherwise there would be some pin-joints above the base. If pin-joints arose above the base the number of degrees of freedom would increase per joint to be more than six(6).

2.13.3 Calculation of Stiffnesses for Various Coordinates for a Given Member

Consider a member with two joints J1 and J2 at the left and right respectively. The XYZ axes are shown in Figure 2.2. The displacement coordinates at the two end joints are shown in Figure 2.3.

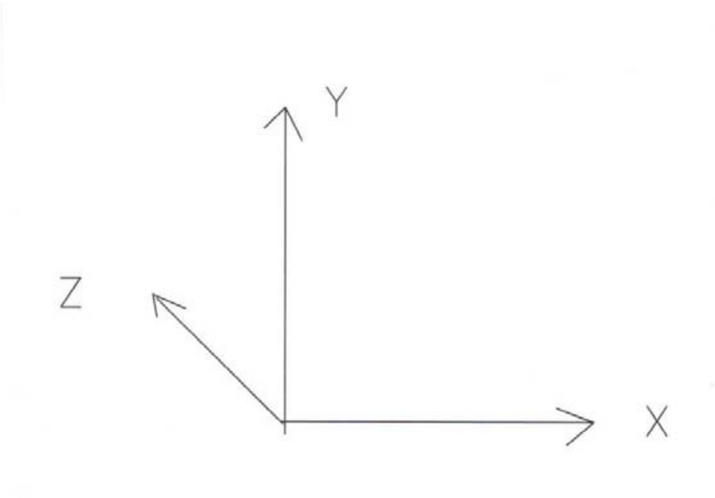


Figure 2.2 XYZ AXES

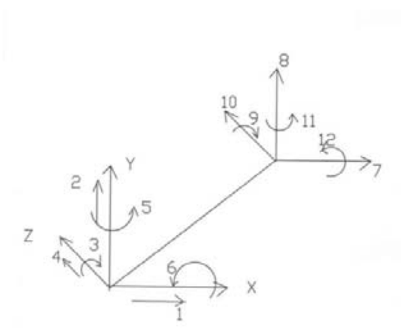


Figure 2.3-The Displacement Coordinates at the 2 ends of a Structural member

Joint 1 has coordinates 1 to 6. Coordinate 1 represents displacement in x –direction, 2 represents deflection in y-direction, 3 represents rotation about z-axis,4 represent displacement in z-direction, 5 represents rotation about the y-axis and 6 represents rotation about the x-axis.

Joint 2 has coordinates 7 to 12. Coordinate 7 is deflection in x-axis, coordinate 8 is deflection in y-axis, coordinate 9 is rotation about z-axis, and coordinate 10 is deflection in z-axis, coordinate 11 is rotation about y-axis and coordinate 12 is rotation about x-axis. The equilibrium condition of the members is described in equation 2.34.

0=External load +Internal load+ sum of (stiffness x deflection) at any coordinate--*Equation 2.34*

2.13.4 Simulation of Vibrations

The model structures were subjected to an initial horizontal force F_{ot}at one of top most joints to cause deflection x_{ot} at the top and deflection x_o at the centre of mass. The deflection at the centre of mass was calculated based on the deflection at the top using interpolation. The structures were thereafter released to undergo free vibrations. The overall stiffness of the structure K_o is defined as the force required at the top to cause a unit horizontal deflection at the centre of mass. Application of the force at the top results in the first mode of vibrations. For theoretical approach, the deflections at the centre of mass were determined by use of the model structure's stiffness matrices. On the other hand, in the experimental approach the deflections were physically measured. The force applied at the top is divided by the deflection at the centre of mass to give the stiffness of the structure K_o.

The relationship between the overall structure's stiffness, K_o, the horizontal deflection, x, the acceleration, a and total mass of the structure, m is governed by Newton's second law of motion (*which states that force =mass * acceleration*).

Total Mass* acceleration=stiffness *deflection at centre of mass, which is given by equation 2.35 below.

$m*a=K_o*x$ --*Equation 2.35*

Which is also the same as equation 2.36 after inclusion of displacement and time in place of acceleration.

$m.d^2x/dt^2=-K_ox$ ---*Equation 2.36* This is a second order differential equation

To determine K_o, a horizontal force F_{ot} at a top joint of the structure is input in a computer programme and the deflection, x_o, of the structure at the centre of mass is solved.

The stiffness Ko is then calculated using equation 2.37 below;

$K_o=F_{ot}/x_o$--*Equation 2.37*

The angular velocity, W, is then calculated as follows;

$m.d^2x/dt^2=-K_o x$ as shown in equation 2.35

$m.d^2x/dt^2+K_o x=0$ ---*Equation 2.37(i)*

This a second order differential equation. The characteristic equation is:

$m.\lambda^2+K_o=0$---*Equation 2.37(ii)*

The solution for the characteristic equation is:

$\lambda=\lambda_1$ or $\lambda=\lambda_2$ where $\lambda_1= -\sqrt{(K_o /m)}j$ and $\lambda_2=+\sqrt{(K_o /m)}j$-------------------*Equation 2.37(iii)*

Therefore $x=c_1e^{\lambda 1t}+c_2e^{\lambda 2t}$---*Equation 2.37(iv)*

Therefore $x=d_1\cos(\sqrt{(K_o /m)})t+d_2\sin(\sqrt{(K_o /m)})t$ ----------------------------*Equation 2.37(v)*

where d_1,d_2 are constants

However if at time $t=0$, $x=0$ therefore $d_1=0$

Therefore final displacement equation is:

$x= d_2\sin(\sqrt{(K_o /m)})t$---*Equation 2.37(vi)*

where d_2 is the amplitude deflection and $W=(\sqrt{(K_o /m)})$ is the angular velocity in radians per second. The equation is illustrated in Figure 2.4

$W=\sqrt{(K_o/m)}$---*Equation 2.38*

From equation 2.38, the eigen frequency f and vibration period T are derived, according to;

$f=W/2\pi=(1/2\pi)(\sqrt{(Ko/m)}$---*Equation 2.39*

and $T = 1/f = 2\pi/W = 2\pi\sqrt{(\frac{m}{Ko})}$--*Equation 2.40*

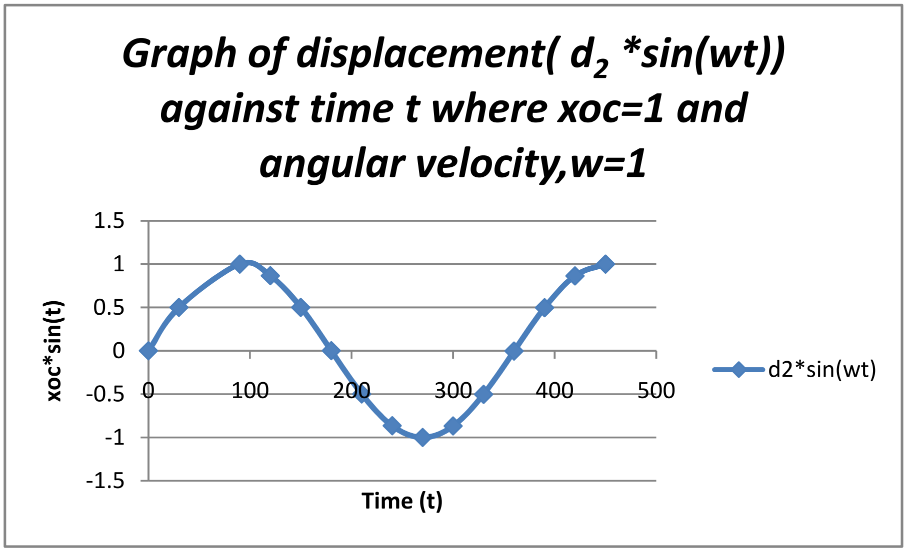


Figure 2.4 –Displacement against time for an oscillating system

At any time, *t*, the horizontal deflection, *x(t)* at centre of mass is given by

$x(t) = xoc * sin(Wt + \Phi)$ --*Equation 2.41*

where xoc is the amplitude and also the initial deflection at centre of mass. Φ is the phase difference which is $\pi/2$ in the case where initial deflection is maximum deflection and equal to the amplitude at time t=0.This case applies for the experiments done for this thesis. The motion is illustrated by the red graph in Figure 2.5.

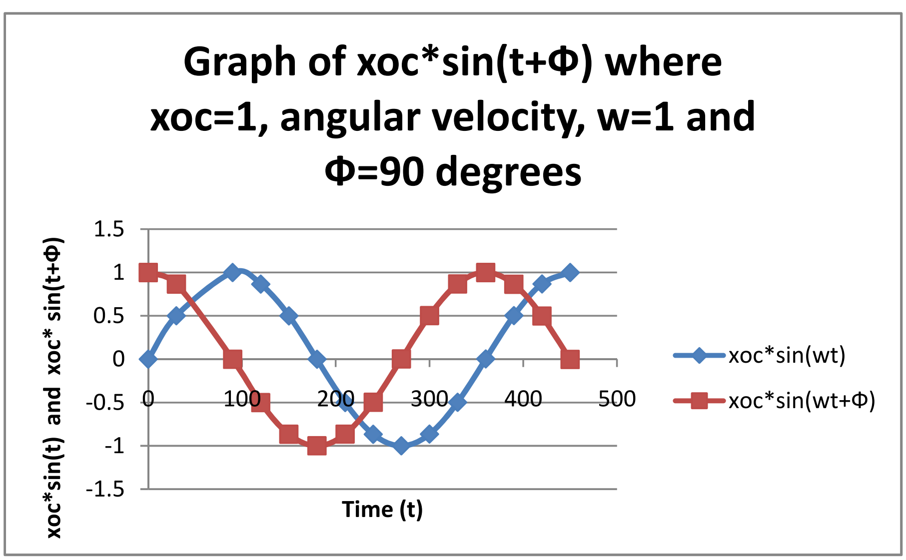


Figure2.5-Displacement against time compared where there is a phase difference of 90 degrees

CHAPTER 3: MATERIALS AND METHODOLOGY

3.0 OVERVIEW OF THE METHODOLOGY

The stiffness matrix method was used to develop a software capable of analysing deflections for given initial horizontal forces acting on a structure. The software was then used to simulate free vibrations and calculate the deflections against time. This simulation was done for structures with different heights, number of bays and structural member stiffnesses.Steel model structures were made and subjected to free vibrations by applying initial horizontal forces and releasing them. The horizontal deflections against time for the free vibrations were measured. This was repeated for structures of different heights, number of bays and structural member stiffnesses.The logarithmic values of the natural frequencies obtained were plotted against the logarithmic values of the heights of the models, number of bays and stiffnesses. The natural frequencies were the dependant variables and the heights of the models, the number of bays and the stiffnesses were the independent variables. The relationship between natural frequency, the height of structure, the number of bays and the stiffnesses of the structural members was determined using a graphical method.

3.1Theoretical Simulation of Free Vibrations

In the theoretical method, vibrations were simulated using a software, named Structuresoft", developed in Visual Basic language using the stiffness matrix theory. The code for the programme is given in Appendices 1 to 6 whereby:

- Appendix 1- is the code for enumerating the coordinates
- Appendix 2- is the code for calculation of external loads at joints
- Appendix 3- is the code for calculation of internal loads at joints
- Appendix 4- is the code for generation of stiffnesses
- Appendix 5- is the code for evaluating the stiffness matrix
- Appendix 6- is the code for equilibrium equations and their solutions.

The theory is presented in earlier section 2.10 in Chapter 2. Each of the computer generated miniature structures was subjected to free vibrations by application of an initial force at a top joint. The deflection at the top joint where the force was applied was computed. The deflection at the top enabled estimation of deflection at the centre of mass. The deflection at centre of mass enabled determination of the stiffness of the structure (*K*), whereby, the structure stiffness, $K = ForceApplied/deflection$. The frequency, F,was calculated using formula $F = (1/2(3.14))\sqrt{(K/\ M)}$ where *M*= mass of structure.

3.2 ExperimentalFree Vibrations

In the experimental method, three dimensional physical models of multi-storey structures were built using mild steel bars. The miniature structures had different heights and height to total horizontal length ratios. The miniature structures were subjected one at a time to an initial horizontal force at one of the highest joints and then released to vibrate freely. To avoid eccentricity due to horizontal loading where there was no top joint centrally placed horizontally, two equal initial forces were applied simultaneously to two top joints which were equidistant from the line of symmetry of the model.

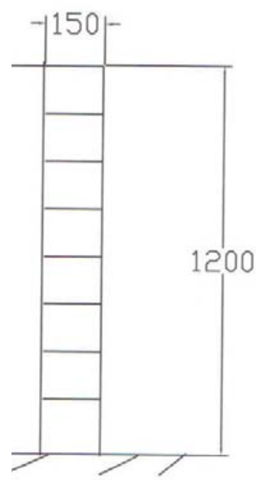


Figure 3.1-Diagram showing miniature structure and initial load (units are in millimeters)

The deflections against time due to the free vibrations were measured at the centre of mass using horizontal motion transducers. The initial horizontal force was varied for each miniature structure. For each new initial horizontal force, the deflections against time were measured for each miniature structure. This enabled determination of the average frequency of vibration for each model. The initial force was applied using a magnet which would lose contact at a force of 0.05KN. The miniature structures were mounted on concrete base using stiffened base plates and bolts to ensure there was no movement at base joints. The Data logger and horizontal motion transducer are shown in Figures 3.2 and 3.2b respectively. The strain gauge used, the transducer, was a static one. However it was connected to the Data Logger which could record strain values as time changed. The output was therefore dynamic.

The accuracy of the TDS Data Logger is 95%. Some of the errors are due to temperature change.

Figure 3.2-Data Logger

Figure 3.2b –Horizontal motion Transducer

The miniature structures subjected to free vibrations are tabulated below:

Table 3.1 Unbraced Miniature Structures Subjected to Free Vibrations

Miniature Structure number	Height(mm)	Bay length (mm)	Number of bays in x-direction	Number of bays in z-direction	Floor to floor height(mm)	Member type and size
1	2100	150	1	1	150	6mm by 6mm square section steel
2	1800	150	1	1	150	6mm by 6mm square section steel
3	1500	150	1	1	150	6mm by 6mm square section steel
4	1200	150	1	1	150	6mm by 6mm square section steel
5	900	150	1	1	150	6mm by 6mm square section steel
6	1500	150	1	1	150	6mm by 6mm square section steel
7	1500	150	1	2	150	6mm by 6mm square section steel
8	1500	150	1	3	150	6mm by 6mm square section steel
9	1500	150	1	4	150	6mm by 6mm square section steel

The miniatures structures were either 1 bay by 1bay or 1 bay in one direction and several bays in the other direction as shown in Table 3.1. It was not desirable to work with miniature structures with more than 1 bay in both x and z direction since such structures would be too stiff to achieve a measurable deflection given the equipment used. The miniature structures were properly bolted on reinforced concrete bases to avoid movement of the bottom most joints.

Figure 3.3 Sample of unbraced miniature structures

Figure 3.3 above shows an unbraced structure. In an unbraced structure all the structural members are either horizontal or vertical. In a braced structure, some members are at an angle between 0 and 90 degrees to the horizontal.

Figure 3.3a-A sample of a braced miniature structure

3.3Getting the Relationship Between Natural Frequencyand Various Parameters

The output of the theoretical method utilized the computed frequencies of vibration for a sample of models of varying dimensions. The experimental method utilized values of deflection at the centre of mass at a given time for each model due to loading. The results were used to estimate the experimental frequencies of vibration (F).

The data obtained was analysed using a graphical method. The natural frequencies obtained by the two methods described above for a given model horizontal length were plotted against the model height on a logarithmic scale. Moreover the frequencies were plotted against the vertical member stiffnesses, horizontal member stiffness and number of bays for a given model height and length on a logarithmic scale. The model height was the dimension between the base and top of each miniature structure. The model length was the horizontal dimensions of the structure. The number of bays was the number of beam spans in a given direction for the structures. The vertical members stiffnesses were values of I/L, where I was the Second Moment of Area and L was length of member, for vertical members. The horizontal members stiffnesses were values of I/L, where I was the Second Moment of Area and L was length of member, for horizontal members. The intercepts of the plotted curves on the vertical axis, and gradient of the graphs were used recorded. These intercepts and gradients were used to determine the relationship between the natural frequency of vibration, height of structure, members stiffnesses and number of bays. The results obtained by the two methods were compared. The results prediction based on the literature was that the frequency of vibration would decrease as the as the height of the structure increased.

CHAPTER 4: RESULTS

4.1 Theoretical Results

The theoretical results are based on natural frequencies of vibration from the developed software (“Structuresoft"). The theoretical Results are tabulated below

4.1.1 Theoretical Results for 1 bay models –unbraced

The natural frequencies for 1 bay models, which were unbraced, were calculated using the mentioned software. The natural frequency results obtained are listed in table 4.1.1 below.

Table 4.1.1-Theoretical results for unbraced 1 bay models

Structure No	Number of Storeys	Model Height-mm	Length of each member -mm	Member type and size	Mass-KG	Deflection at joint of initial force (top)-mm	Frequency Calculated with software-Hertz	Frequency adjusted for Constant mass (of 2.76Kg)-Hertz
1	14	2100	150	6mm by 6mm square	4.83	0.48	1.05	1.40
2	12	1800	150	6mm by 6mm square	4.14	0.24	1.55	1.89
3	11	1650	150	6mm by 6mm square	3.80	0.63	1.03	1.21
4	10	1500	150	6mm by 6mm Square	3.45	0.31	1.55	1.73
5	8	1200	150	6mm by 6mm Square	2.76	0.18	2.26	2.26

It was found that for unbraced structures natural frequency generally decreased as height increased as shown in Figure 4.1.1

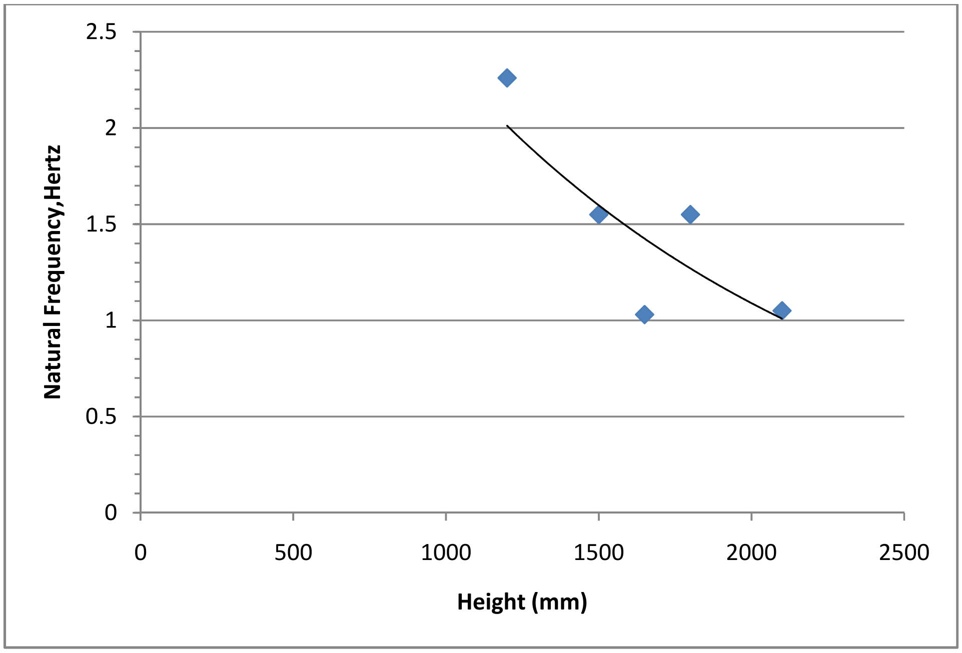


Figure 4.1.1- Natural Frequency, F, versus, Height, H, for 1 bay unbraced (theoretical)

The frequency was adjusted to maintain miniature structure mass as 2.76KG as shown in the last column of table 4.1.1. The mass was made constant in order to study the effect of height without influence of change of mass. However the mass of any of the other structures could have been chosen for this purpose. The values were plotted against miniature structure height as shown in Figure 4.1.1b.

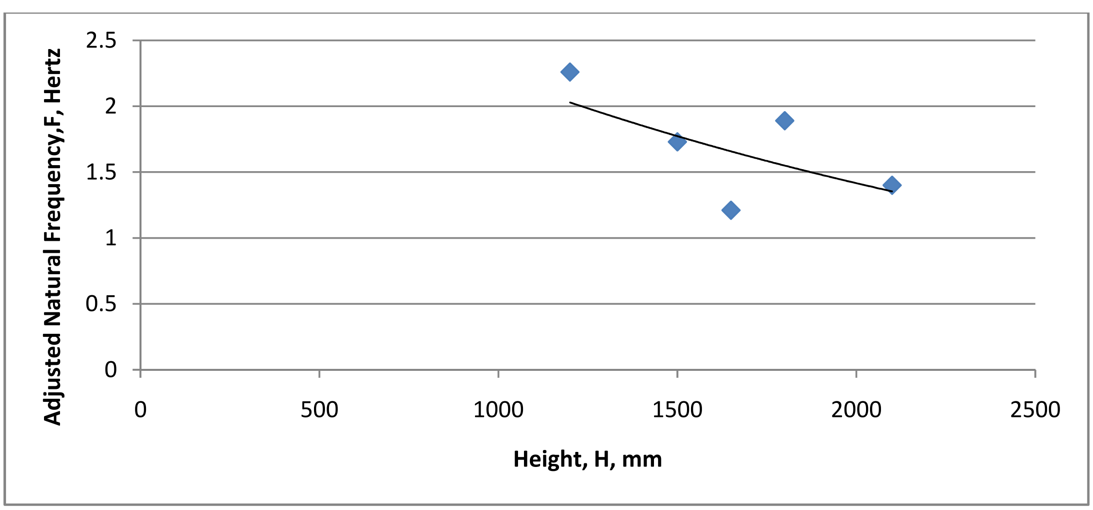


Figure 4.1.1b –Adjusted Frequency against Miniature Structure Height for unbraced structures-Theoretical

4.1.2 Theoretical Results for braced 1 bay models

The natural frequencies for 1 bay models, which were braced diagonally at each storey, were calculated using "Structuresoft". The natural frequency results obtained are listed in table 4.1.2 below.

Table 4.1.2-Theoretical results for braced 1 bay models

Structure No.	**No. of Floors**	**Model Height (mm)**	**Length of each member (mm)**	**Member type and size**	**Mass (Kg)**	**Deflection at joint of initial load (top)-mm**	**Frequency Calculated with software-Hertz**	**Frequency Adjusted for constant mass-Hertz**
1	14	2100	150	6mm by 6mm Square	8.26	0.117	1.62	2.47
2	12	1800	150	6mm by 6mm Square	7.07	0.122	1.71	2.42
3	10	1500	150	6mm by 6mm Square	5.90	0.12	1.86	2.40
4	8	1200	150	6mm by 6mm Square	4.72	$2.16*10^{-2}$	4.96	5.73
5	6	900	150	6mm by 6mm square	3.54	$1.06*10^{-2}$	8.18	8.18

The natural frequency of vibration decreased as height increased for braced structures as shown in Figure 4.1.2

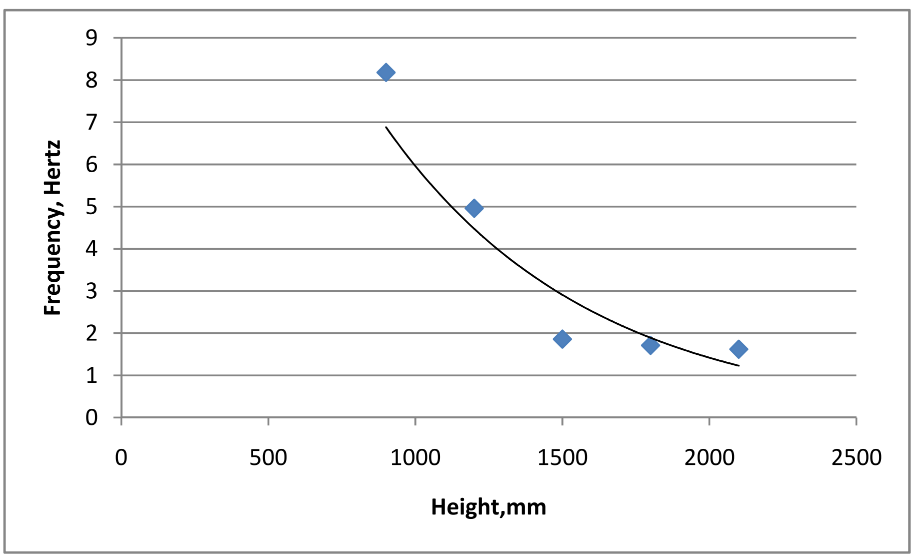


 Figure 4.1.2 Theoretical frequency,F, versus Height for braced structures

The Frequencies were adjusted for constant mass as shown in last column of table 4.1.2. The adjusted Frequencies were plotted against height of miniature structure as shown in Figure 4.1.2b

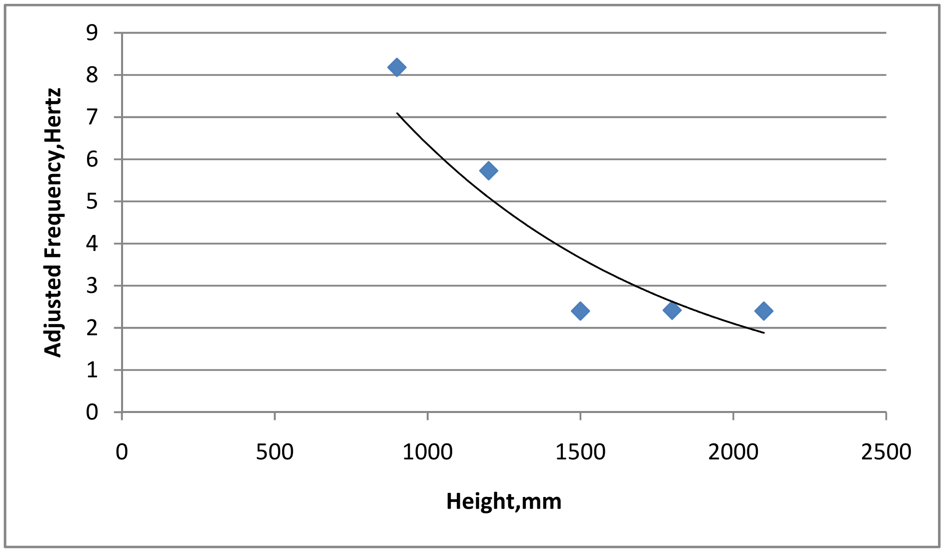


Figure 4.1.2b- Theoretical Adjusted frequencies against Height for braced structures

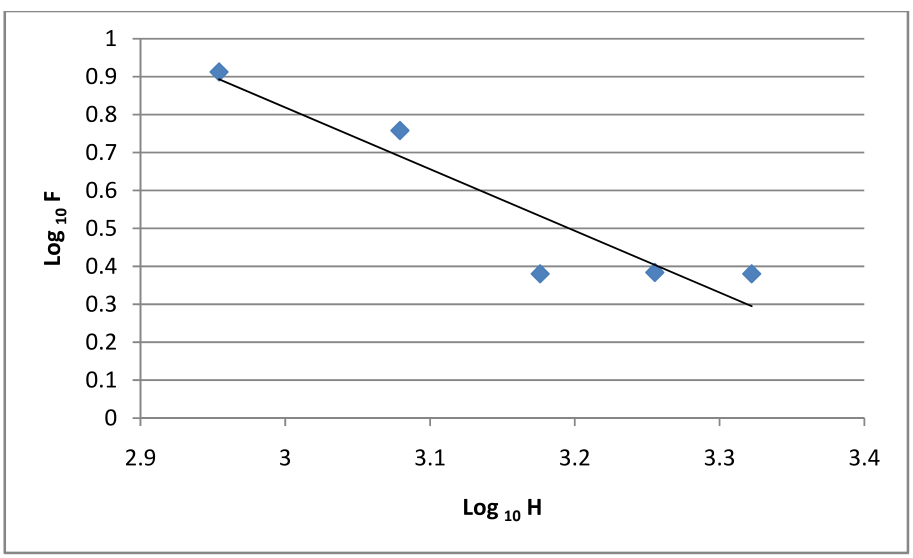


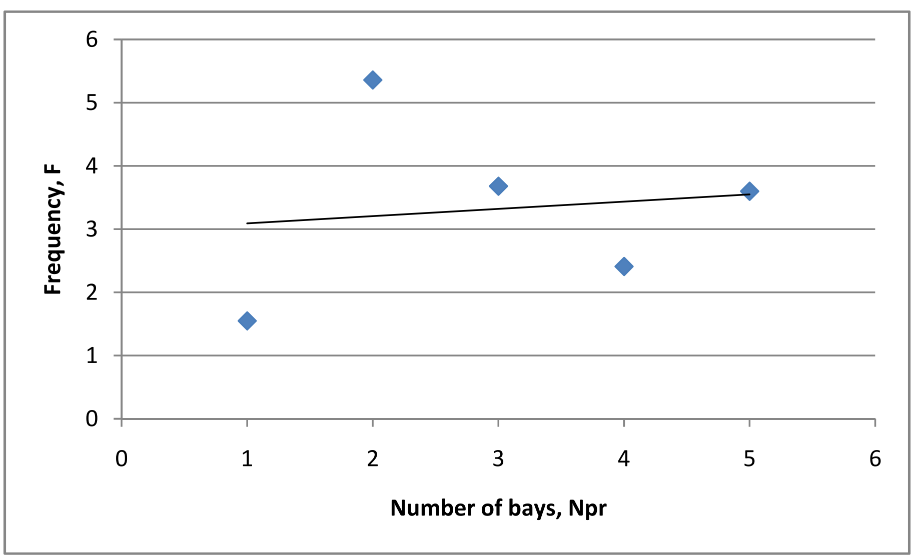
Figure 4.1.2-Theoretical Log $_{10}$ (F) vs Log $_{10}$ (H) for braced 1 bay structures

4.1.3 Theoretical Results for 1 bay, 2bay, 3bay, 4bay,5bay models in the stiffer direction

The natural frequencies for 1 bay, 2bay, 3bay and 4 bay models, which were unbraced, were computed for stiffer direction using "Structuresoft". The results are listed in table 4.1.3 below.

Table 4.1.3-Theoretical results for 1bay, 2bay, 3bay and 4bay in the stiffer direction

Structure No	Model Height-mm	Number of Bays	Length of each member-mm	Member type and size	Mass-Kg	Deflection at initial force joint (top)-mm	Frequency Calculated with software-Hertz	Adjusted Frequency for constant Mass of 3.45Kg, Hertz
1	1500	1	150	6mm by 6mm Square	3.45	0.31	1.55	1.55
2	1500	2	150	6mm by 6mm Square	5.62	$1.56*10^{-2}$	5.36	6.84
3	1500	3	150	6mm by 6mm Square	7.96	$2.39*10^{-2}$	3.68	5.58
4	1500	4	150	6mm by 6mm Square	8.94	$3.64*10^{-2}$	2.41	3.88
5	1500	5	150	6mm by 6mm Square	12.1	0.52	3.6	6.7



The natural frequency was plotted against the number of bays as shown in Figure 4.1.3. There was a general increase in frequency for motion in the stiffer direction as the number of bays increased as shown by the trend line generated by the Excel software.

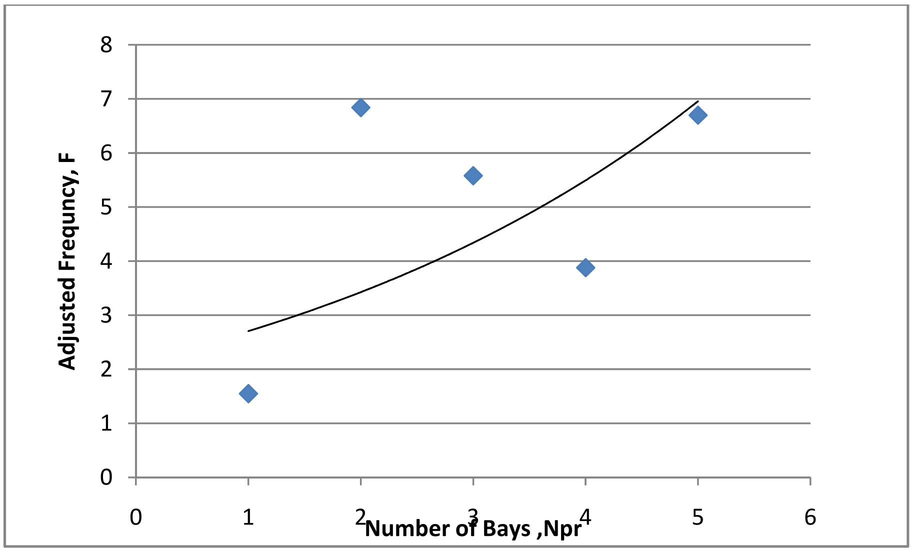


Figure 4.1.3b Adjusted Theoretical Frequencies against Number of Bays (Stiffer Direction Motion)

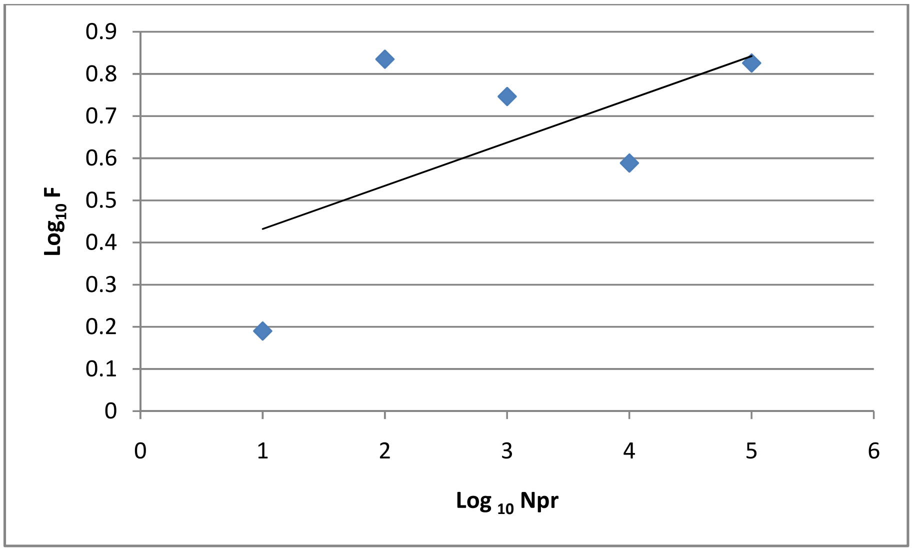


Figure 4.1.3c-Theoretical log $_{10}$ F against Log $_{10}$ Npr in stiffer directionmotion

The relationship between Log $_{10}$ F and Log $_{10}$ N where the number of bays increased and motion is parallel to the stiffer direction is shown in Figure 4.1.3c

4.1.4 Theoretical Results for 1 bay,2bay,3bay, 4bay,5baymodels - Less stiff direction motion

The natural frequencies for 1 bay, 2bay, 3bay and 4 bay models, which were unbraced, were calculated for the less stiff direction motion using the mentioned software. The natural frequency results obtained are listed in table 4.1.4 below.

Table 4.1.4-Theoretical results for 1bay, 2bay, 3bay , 4bay& 5bays in the less stiff direction

Structure No	Model Height-mm	Number of Bays	Length of each member-mm	Member type and size	Mass-KG	Deflection at initial force joint (top)-mm	Frequency Calculated with software-Hertz	Frequency Adjusted for constant Mass-Hertz
1	1500	1	150	6mm by 6mm Square	3.45	0.31	1.55	1.55
2	1500	2	150	6mm by 6mm Square	5.62	7.69*10-2	2.40	3.05
3	1500	3	150	6mm by 6mm Square	7.96	0.106	1.73	2.64
4	1500	4	150	6mm by 6mm Square	9.94	0.23	1.05	1.79
5	1500	5	150	6mm by 6mm Square	12.1	1.0	0.46	0.86

The frequency of vibration was plotted against the number of bays as shown in Figure 4.1.4. There was a general decrease in the natural frequency of vibration in the less stiff direction as the number of bays increased as shown in the trend line generated by Excel software.

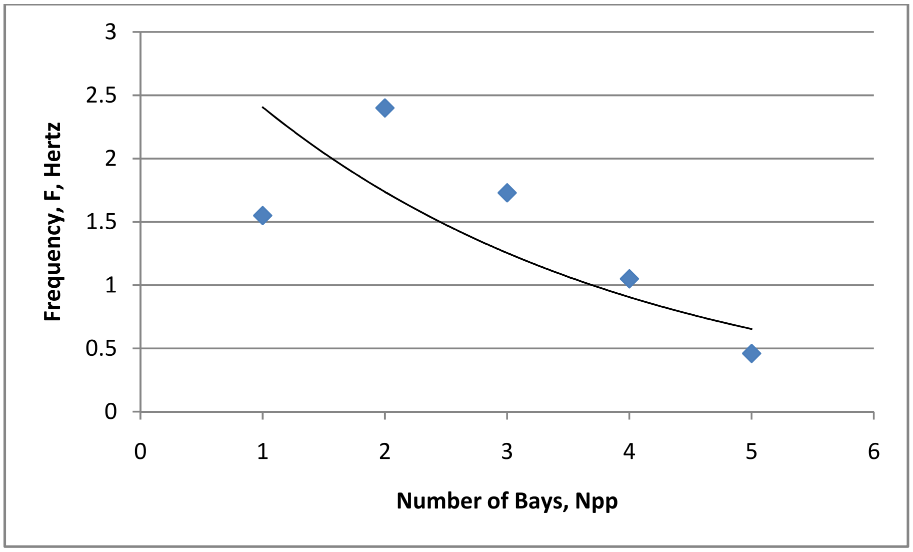


Figure 4.1.4 –Natural frequency against number of bays, Npp, in direction transverse to motion (theoretical).

The natural frequencies were adjusted for constant miniature structure mass as shown in the last column of Table 4.1.4. The adjusted frequencies were plotted against number of bays as shown in Figure 4.1.4b.

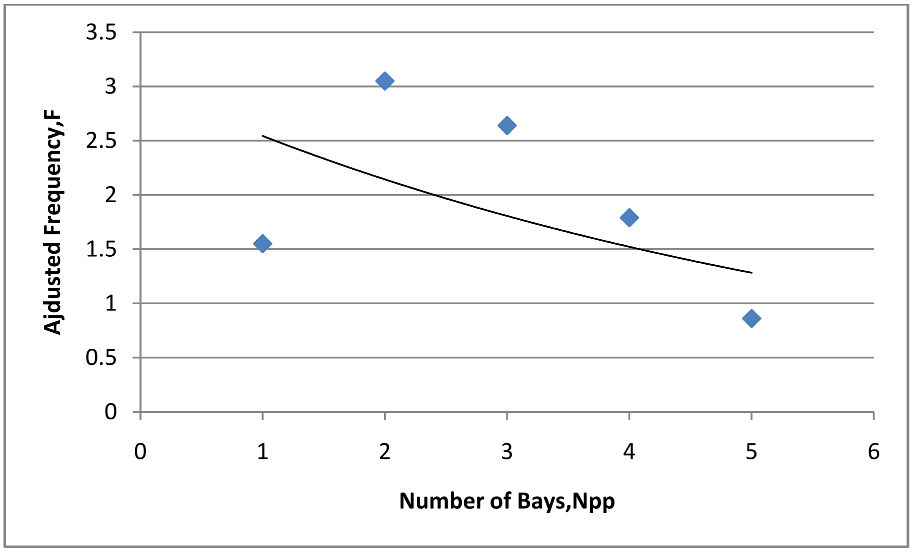


Figure 4.1.4b-Adjusted Frequencies against the number of bays in direction transverse to motion-Theoretical

The relationship between Log $_{10}$ (F) and Log $_{10}$ (Npp) where the number of bays increased and motion is parallel to less stiff direction is shown in Figure 4.1.4c

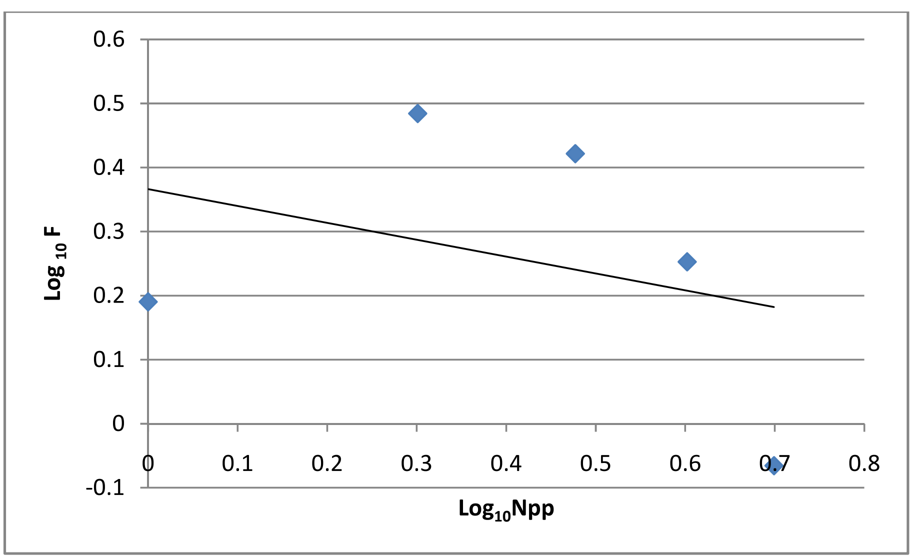


Figure 4.1.4c-Log $_{10}$ F against log $_{10}$ Npp as number of bays increase in one direction-Motion parallel to less stiff direction-Theoretical

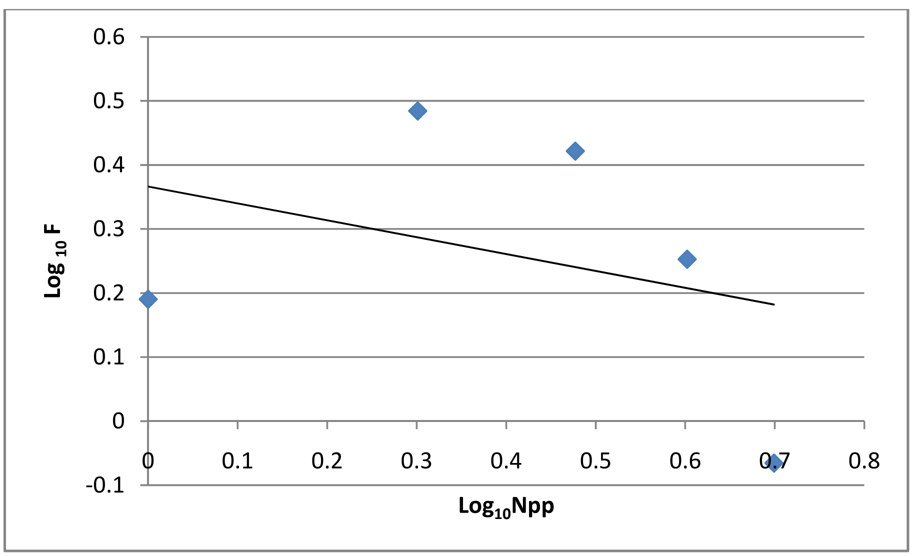


Figure 4.1.4c-Log $_{10}$F against log $_{10}$ Npp as number of bays increase in one direction-Motion parallel to less stiff direction-Theoretical

4.1.5 Theoretical Results for 1 bay models with same overall height but varying stiffness of vertical members

The natural frequencies for 1 bay models, with the same overall height but different storey were calculated using the mentioned software. All members in the miniature structure were 6mm square. The natural frequency results obtained are listed in table 4.1.5 below.

Table 4.1.5-Theoretical Results for 1bay models as vertical member stiffnesses varied

Model no.	Number of storeys	Model height -mm	Length of vertical members-mm	Mass -Kg	Deflection at joint of initial force	Frequency Calculated with software-Hertz	Frequency Modified for constant mass of 3.45Kg, F-Hertz	Column stiffness ratio (I/L),Sv – (mm^3)
1	8	1500	187.5	3.28		1.30	1.27	0.58
2	10	1500	150	3.45	0.49	1.55	1.55	0.72
3	15	1500	100	4.32	$5.4*10^{-2}$	3.27	3.66	1.08
4	20	1500	75	5.18	$3.6*10^{-2}$	3.69	4.52	1.44
5	26	1500	57.7	6.96	$6.51*10^{-3}$	7.47	10.6	1.87

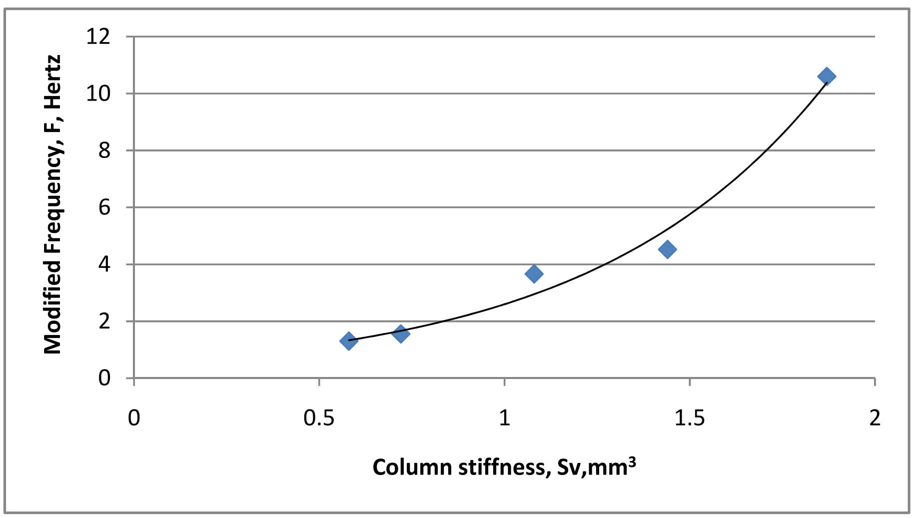


Figure 4.1.5-Theoretical frequencies against stiffness of columns

The natural frequency of vibration was plotted against the miniature structure stiffness ratio (I/L) for a fixed overall structure height as shown in Figure 4.1.5. The frequency increased as the stiffness increased.

4.1.6 Theoretical Results for 1 bay models as stiffness of horizontal members varies parallel to the direction of the initial force.

The natural frequencies for 1 bay models, with the same overall height but different stiffnesses for horizontal members parallel to direction of force was calculated using the mentioned software. All members were 6mm square. The vertical members and member perpendicular to motion were 150mm long. The natural frequency results obtained are listed in table 4.1.6 below.

Table 4.1.6-Theoretical results as horizontal members stiffnesses varied parallel to initial force

Model no.	No. of storeys	Model Height (mm)	No. of Bays	Horizontal member length 8-	Deflection at joint of initial force(top)	Frequency Calculated with software-(Hertz)	Mass (Kg)	Frequency Modified for Constant Mass of 2.53KG,F-Hertz	Horizontal member stiffness ratio (I/L)-mm3,Spl
1	8	1200	1	100	0.44	1.51	2.53	1.51	1.08
2	8	1200	1	150	0.18	2.24	2.76	2.34	0.72
3	8	1200	1	200	$9.15*10^{-2}$	3.03	3.00	3.30	0.54
4	8	1200	1	250	$2.88*10^{-2}$	5.20	3.23	5.87	0.43
4	8	1200	1	300	$1.1*10^{-2}$	8.1	3.49	9.51	0.36

The natural frequency of vibration increased as the horizontal member stiffness decreased parallel to direction of motion as shown in Figure 4.1.6.

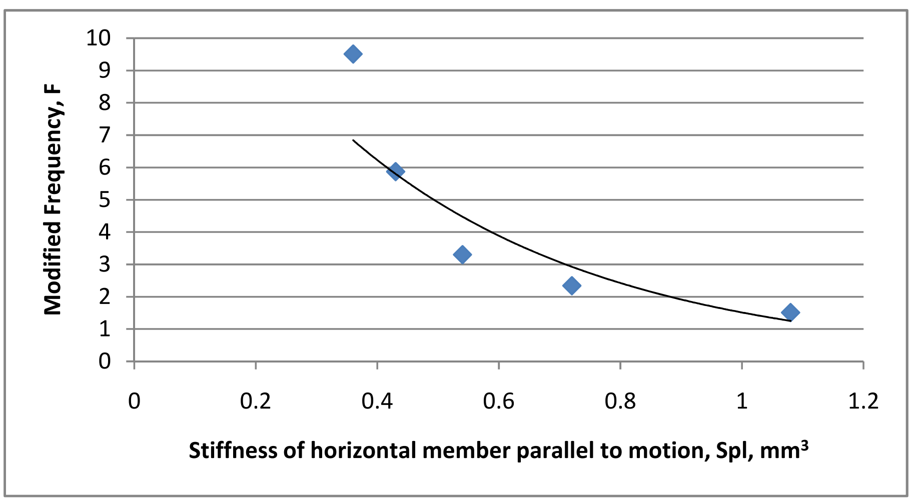


Figure 4.1.6 Frequency F, against Stiffness, Spl, of horizontal member parallel to motion-Theoretical

4.1.7 Theoretical Results for 1 bay models as stiffness of horizontal members varies perpendicular to the motion.

The natural frequencies for 1 bay models, with the same overall height but different stiffnesses for horizontal members perpendicular to direction of initial force was calculated using the mentioned software. All members were 6mm square. The vertical members and member parallel to motion were 150mm long. The natural frequency results obtained are listed in table 4.1.7 below.

Table 4.1.7-Natural frequencies for 1bay models (1200mm high) as stiffness of horizontal members varies perpendicular to motion-theoretical

Model no.	Number of storeys	Number of Bays	Length of horizontal members perpendicular to initial force-mm	Mass (Kg)	Deflection at joint of initial force(top)	Frequency Calculated with software - Hertz	Frequency modified for constant mass of 2.53Kg, F-Hertz	Stiffness of member perpendicular to motion, Spp
1	8	1	100	2.53	0.19	2.28	2.28	1.08
2	8	1	150	2.76	0.18	2.24	2.14	0.72
3	8	1	250	3.22	0.325	1.55	1.38	0.54
4	8	1	300	3.46	0.167	2.09	1.79	0.43
5	8	1	350	3.68		1.3	1.57	0.31

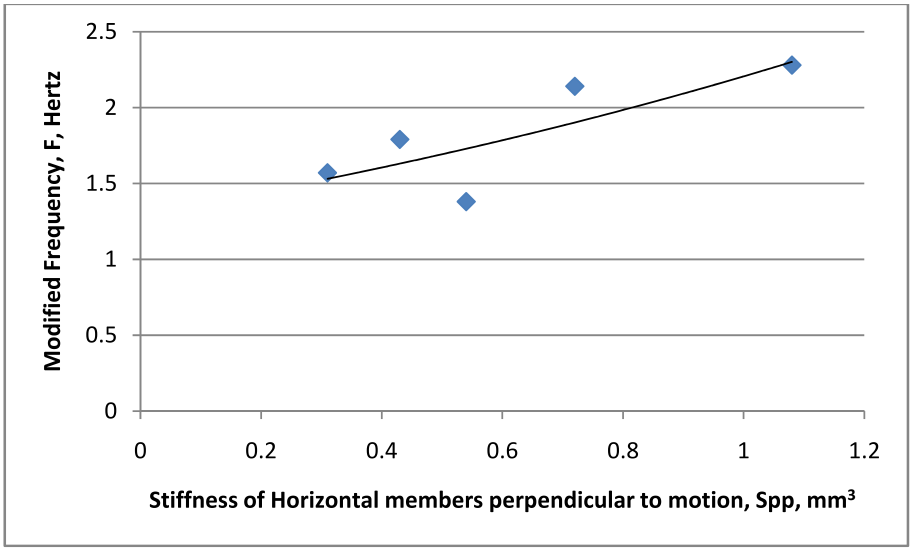


Figure 4.1.7-Theoretical Natural Frequency against the horizontal member stiffness perpendicular to the direction of motion

The natural frequency generally increased as the stiffness of the horizontal members perpendicular motion increased as shown in Figure 4.1.7.

4.1.8 Theoretical Results for 1 bay models with varying scale factors

The natural frequencies for 1 bay models with varying scale factors were calculated using the mentioned software. The natural frequency results obtained are listed in table 4.1.8 below.

Table 4.1.8-Natural frequencies for 1bay models with varying scale factors

``1	No. of storeys	Model Height (mm)	No. of Bays	Length of horizontal members (mm)	Length of Vertical member (mm)	Member type and size	Mass (Kg)	Deflection at joint of initial force (top)-mm	Frequency Calculated with software-Hertz
1	6	900	1	150	150	6mm by 6mm Square	1.58	0.16	2.25
2	6	1200	1	200	200	6mm by 6mm Square	2.76	0.22	1.43
3	6	1500	1	250	250	6mm by 6mm Square	3.45	0.50	0.86
4	6	1800	1	300	300	6mm by 6mm Square	4.15	0.88	0.59
5	6	2100	1	350	350	6mm by 6mm Square	4.84	1.35	0.44

Table 4.1.8b –Miniatures Structure Scale Factor, Sc, Vs Frequency, F

Model No.	Model Height	Scale Factor, compared to model of height 900mm	Mass (Kg)	Frequency	Frequency adjusted for Constant Mass of 1.58Kg, F-(Hertz)
1	900	1	1.58	2.25	2.25
2	1200	1.33	2.1	1.43	1.65
3	1500	1.67	3.45	0.86	1.27
4	1800	2.00	4.15	0.59	0.96
5	2100	2.33	4.84	0.44	0.77

The natural frequencies of vibration increased as the scale factor increased as shown in Figure 4.1.8.

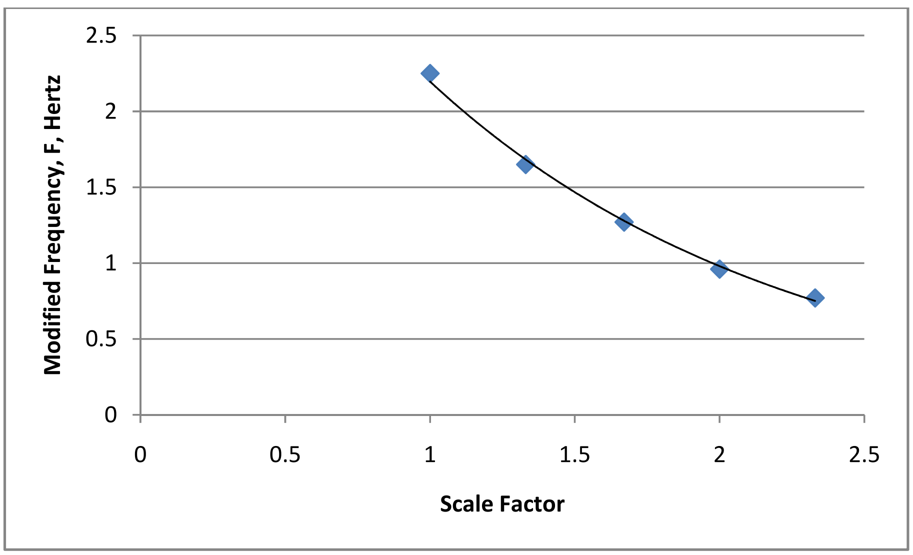


Figure 4.1.8-Frequency against Scale Factor

The relationship between Log $_{10}$ (F) and Log $_{10}$ (Sc) where F is the frequency and Sc is the scale factor is shown in Figure 4.1.8b.

4.2 Experimental Results

The experimental results were based on data-logger readings. The vibration periods of the sine waves generated by the vibrating miniature structures were measured in seconds. The periods were used to calculate the frequencies. The photos showing sample data-logger readings on vibrations are presented in Figures 4.2a, 4.2b and 4.2c below:

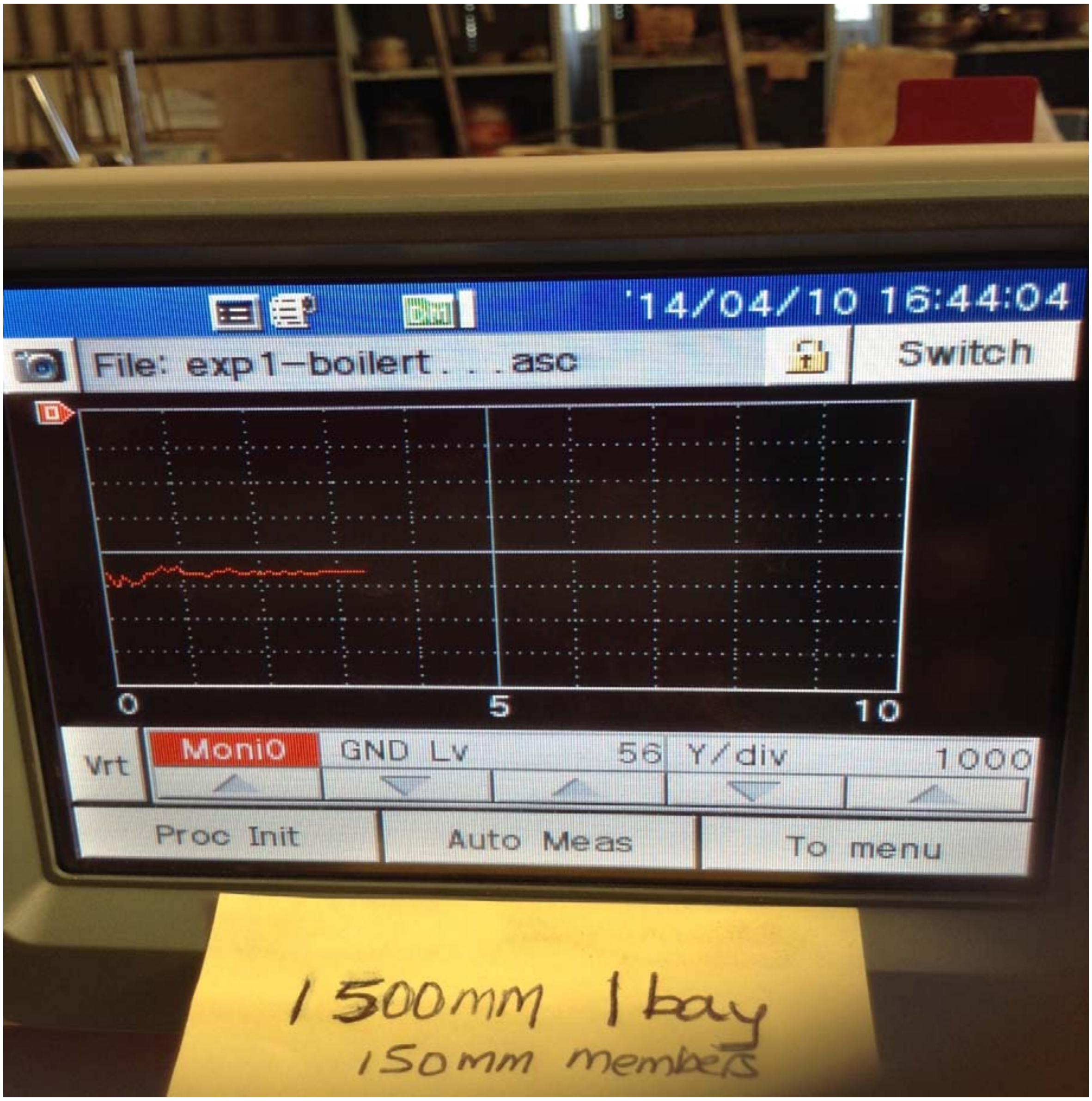


Figure 4.2a-Vibration result for 1500 High Unbraced Model with 1 bayas seen on Data Logger Monitor

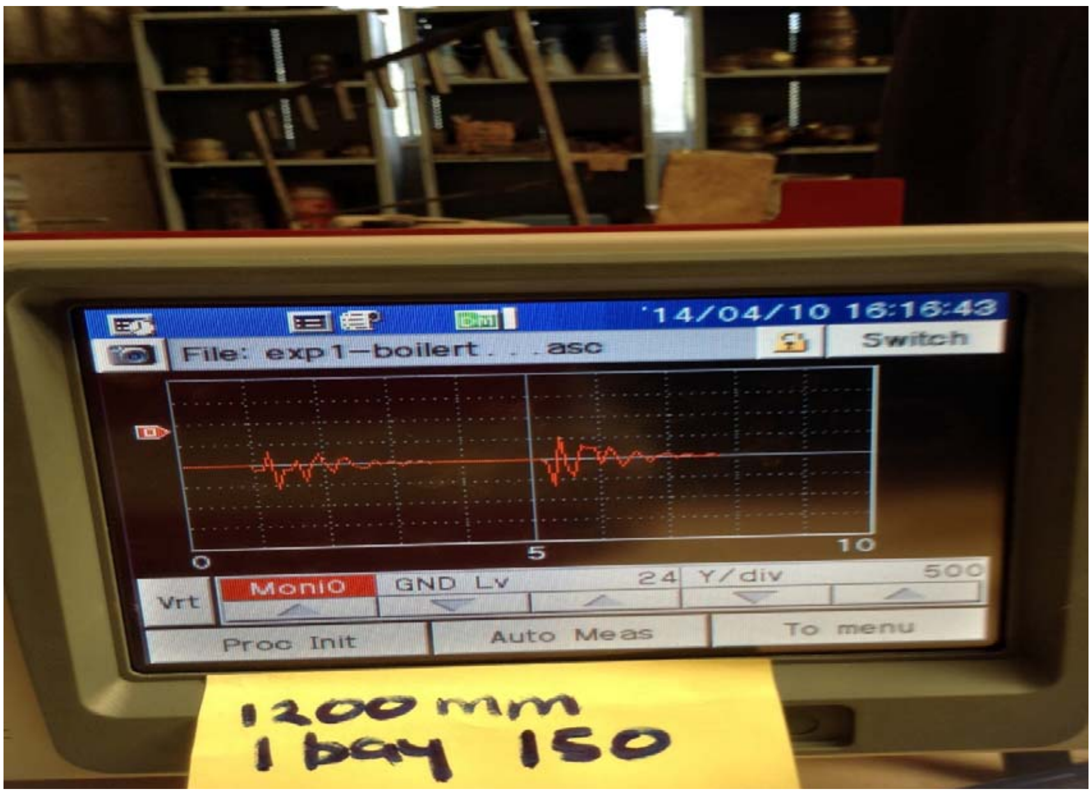


Figure 4.2b-Vibration Results for 1200mm High Unbraced model

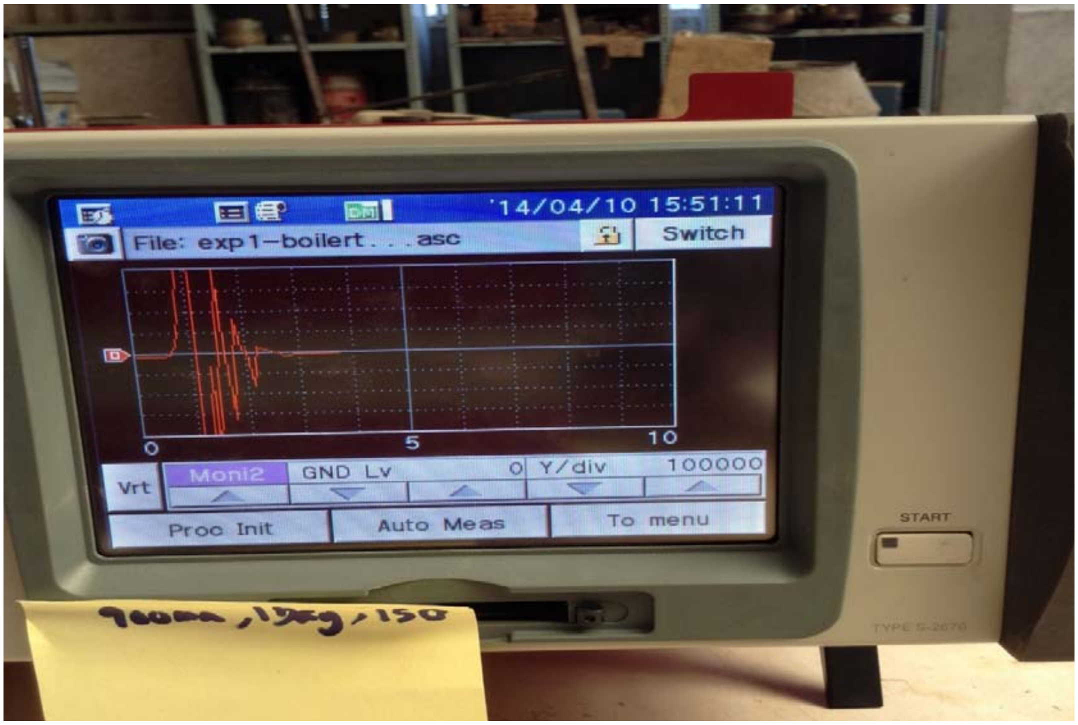


Figure 4.2c-Vibration Results for 900mm High Unbracedmodel structure with 1 bay

4.2.1 Experimental Results for unbraced 1 bay models

The experimental results for unbraced 1 bay models are presented in tables below:

Table 4.2.1-Values of natural frequencies against the structure height-experimental case-unbraced

Model No.	Model Height (mm) (H)	Number of Bays	Mean Value of frequency	Standard deviation of frequencies	Mass (Kg)	Frequency modified for constant mass, F	Log10 H	Log10F
1	1800	1	1.13	0.047	4.14	1.60	3.26	0.053
2	1500	1	1.22	0.087	3.45	1.58	3.17	0.087
3	1200	1	2.02	0.037	2.76	2.33	3.08	0.305
4	900	1	2.45	0.05	2.07	2.45	2.95	0.39

The frequency of vibration decreased as the height of the structure increased as shown in Figure 4.2.1.

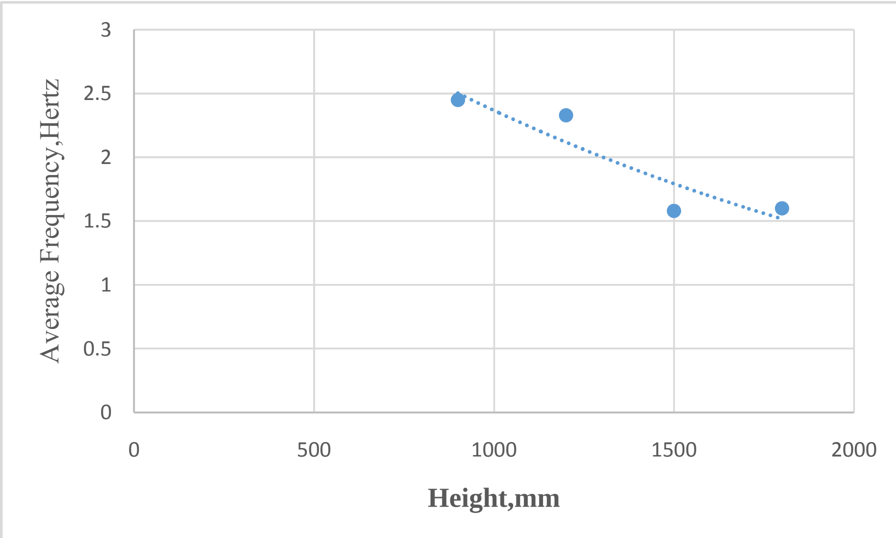


Figure 4.2.1. Average Frequency,F, against Miniature structure height, H-Experimental(Unbraced)

4.2.2 Experimental Results for braced 1 bay models

Table 4.2.2-Values of natural frequencies against the structure height-experimental case-braced

Model No.	**Model Height (mm) H**	**No. of Bays**	**Mean Value of frequency**	**Standard deviation of frequencies**	**Mass (Kg)**	**Frequency modified for constant mass of 4.72Kg, F**	**$Log_{10}H$**	**$Log_{10}F$**
1	2100	1	2.67	0.215	8.25	3.53	3.32	0.55
2	1800	1	1.33	0.047	7.07	1.63	3.25	0.21
2	1500	1	3.77	0.063	5.9	4.21	3.17	0.62
3	1200	1	4.06	0.38	4.72	4.06	3.08	0.61

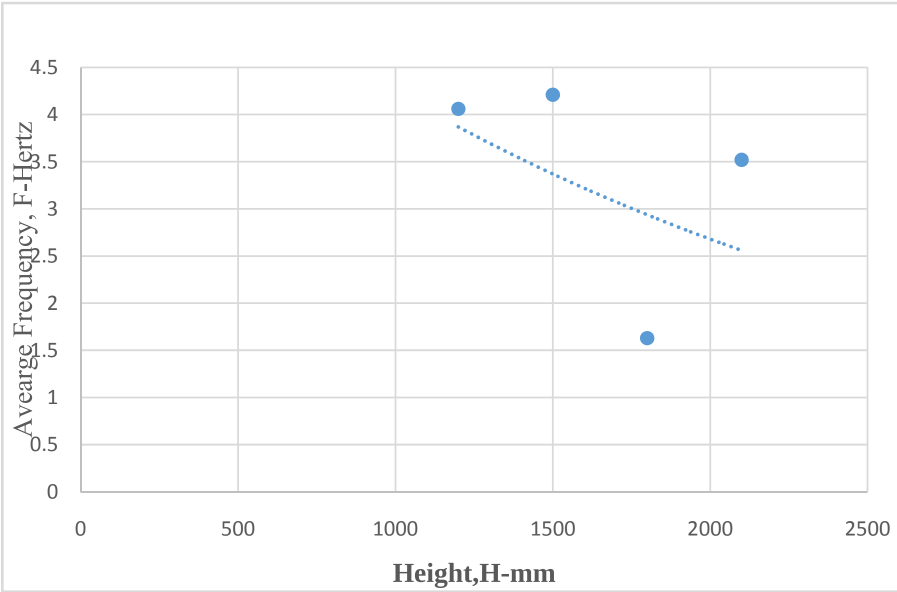


Figure 4.2.2-Average Frequency versus Height of miniature structure, H, braced case-Experimental.

The frequency of vibration decreased as the height of the miniature structure increased as shown in Figure 4.2.2.

4.2.3-Experimental Results for models with more than 1bay where vibration is parallel to longer side.

The Results of Natural frequency where horizontal motion is parallel to the longer side are presented in Table 4.2.3

Table 4.2.3-Values of natural frequencies against the number of bays for motion parallel to stiffer direction

Model No.	Model Height (mm) H	Number of Bays	Mean Value of frequency (F)	Standard deviation of frequencies	Mass (Kg)	Frequency adjusted for Constant mass of 3.45KG,F	$Log_{10}H$	$Log_{10}F$
1	1500	1	1.22	0.087	3.45	1.22	0	0.086
2	1500	2	3.0 7	0.12	5.62	3.92	0.301	0.593
3	1500	3	3.43	0.07	7.96	5.20	0.477	0.718
4	1500	4	4.52	0.07	9.94	7.67	0.0602	0.88

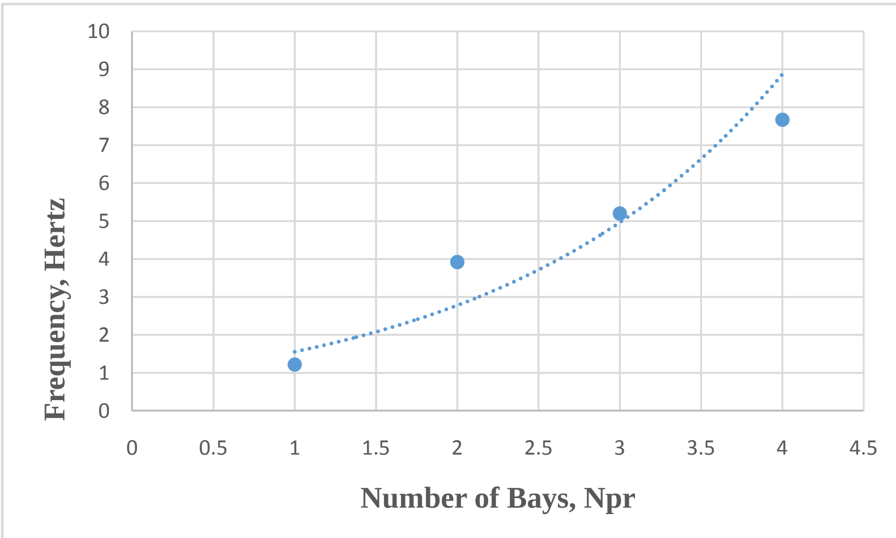


Figure 4.2.3-Frequency,F, against number of Bays, Npr in stiffer direction motion - experimental

4.2.4 Experimental Results for models with more than 1bay where vibration is perpendicular to longer side.

In this set of experiments the number of bays were increased in one direction (stiffer direction) while the number bays remained 1 in number in the less stiff direction.

Table 4.2.4. Values of Natural Frequencies against number of bays for motion perpendicular to stiffer direction.

Model No.	Model Height (mm) (H)	No. of bays	Mean value of frequency (F)	Standard deviation of frequency	Mass (Kg)	Frequency modified for Constant Mass of 3.45KG, F	$Log_{10}Npp$	$Log_{10}F$
1	1500	1	1.22	0.087	3.45	1.22	0	0.086
2	1500	2	3.1	0.14	5.62	3.96	0.301	0.49
3	1500	3	2.5	0.47	7.96	3.8	0.477	0.39
4	1500	4	0.87	0.047	9.94	1.48	0.0602	-0.06

The natural frequency of vibration, before modification to make mass constant, generally decreased as the number of bays increased as shown in Figure 4.2.4.

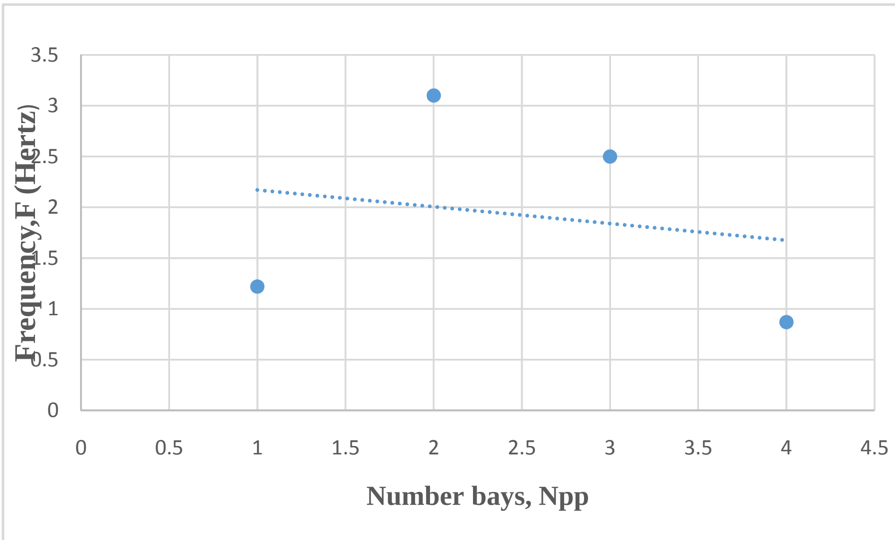


Figure 4.2.4-Natural Frequency against number of Bays increase Transverse to Motion -Experimental

The frequency, after modification for constant mass, generally increased as number of bays increased as shown in Figure 4.2.4b.

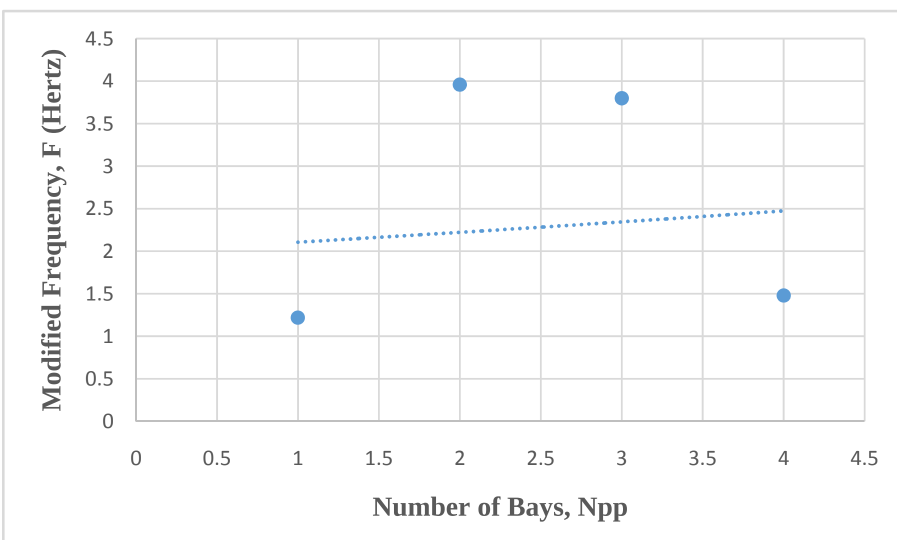


Figure 4.2.4b-Modified Frequency, F, against the number of Bays, Npp

4.2.5 Experimental Results for miniature structures with varying column stiffnesses

Table 4.2.5-Values of natural frequencies against the column stiffnesses for 1.5m high models

Model No.	Model Height (mm) (H)	No. of Bays	Mean Value of frequency (F)	Standard deviation of frequencies	Mass (Kg)	Frequency Modified for Constant mass of Kg, F	Flexural stiffness ratio (I/L), S- in mm^3	$Log_{10}S$	$Log_{10}F$
1	1500	1	1.22	0.087	3.45	1.22	0.72	-0.143	0.086
2	1500	1	3.07	0.13	4.32	3.44	1.08	0.033	0.297
3	1500	1	3.43	0.094	5.18	4.20	1.44	0.150	0.467
4	1500	1	4.52	0.22	6.96	6.42	1.87	0.27	0.687

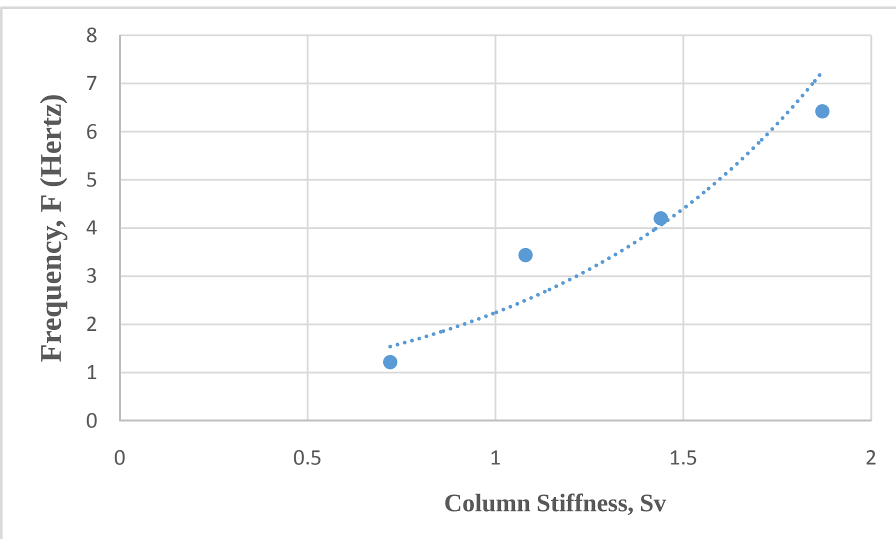


Figure 4.2.5-Modified Frequency, F, against Stiffness of columns (Sv)-Experimental

The frequency increased as the stiffness of columns increased as shown in Figure 4.2.5

4.2.6 Experimental Results for models where the horizontal member stiffness varies parallel to the direction of motion.

The natural frequencies of miniature structures where the horizontal member stiffnesses parallel to the direction of motion were varying were measured. The model where the members were all 150mm long was used as a control and the members parallel to motion were varied in length. The number of storeys in the miniature structures were maintained as eight (8).

In the experimental case, the values of vibration frequency as horizontal member stiffness parallel to motion varied is presented in table 4.2.6.

Table 4.2.6-Summary of effect of changing stiffness of horizontal members parallel to motion

Length of member parallel to motion	Stiffness ratio (I/L), Spl-mm^3	Average frequency, F	Standard deviation	Mass (Kg)	Frequency modified for Constant mass of 2.53Kg	Log_{10}Spl	Log_{10}F
100	1.08	1.73	0.05	2.53	1.73	0.033	0.24
150	0.72	1.22	0.087	2.76	1.27	-0.143	0.11
200	0.54	2.0	0.09	3.00	2.18	-0.268	0.34
250	0.43	3.68	0.41	3.23	4.16	-0.365	0.62

The natural Frequency of vibration decreased as the stiffness of horizontal members parallel to motion increased as shown in Figure 4.2.6.

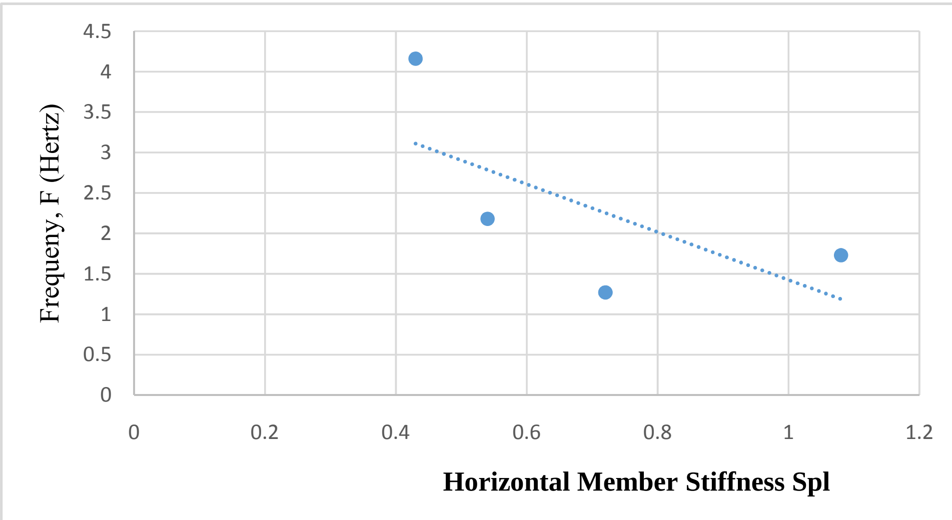


Figure 4.2.6 Frequency F, against Horizontal member stiffness parallel to motion

4.2.7 Experimental Results where the horizontal member stiffness varied perpendicular to the direction of motion.

The natural frequencies of miniature structures where the horizontal member stiffnesses perpendicular to the direction of motion were varying were measured. The model where the members were all 150mm long was used as a control and the members perpendicular to motion were varied in length. The number of storeys in the miniature structures were maintained as eight (8). In the experimental case, the values of vibration frequencies as horizontal member stiffness perpendicular to motion changed are presented in table 4.2.7.

Table 4.2.7-Summary of effect of changing stiffness of horizontal members perpendicular to motion

Length of member parallel to motion	Stiffness, Spp-mm^3	Average frequency, F	Standard deviation	Mass (Kg)	Frequency Modified for Constant Mass of 2.53Kg	$Log_{10}Spp$	$Log_{10}F$
100	1.08	2.1	0.13	2.53	2.1	0.033	0.32
150	0.72	1.1	0.087	2.76	1.15	-0.143	0.060
200	0.54	1.5	0.06	3.00	1.63	-0.268	0.213
250	0.43	1.4	0.08	3.23	1.58	-0.365	0.20

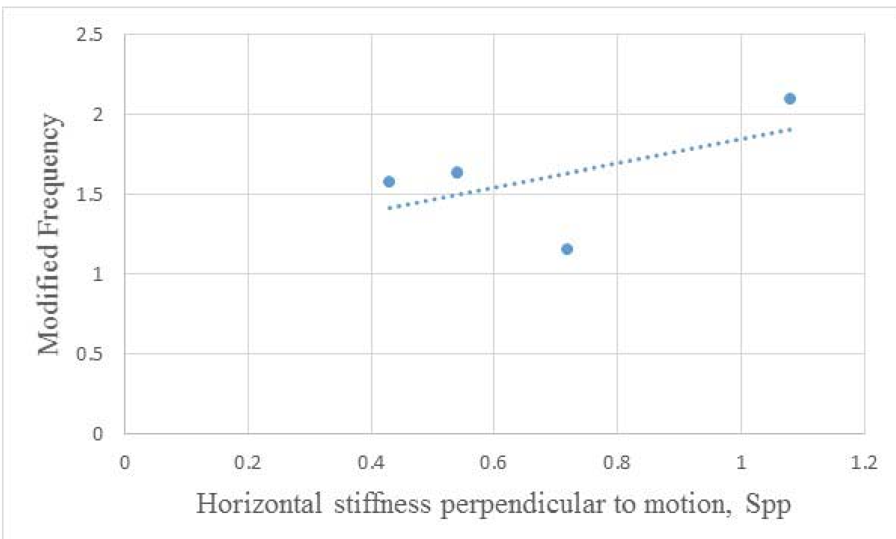


Figure 4.2.7-Experimental Natural frequency against Horizontal Members Stiffness Perpendicular to motion

In the experimental case, the natural frequency of vibration increased as the stiffness of the horizontal members perpendicular to motion increased as in Figure 4.2.7.

4.2.8 Experimental Results for Natural frequency of models which differ by scale factor.

Miniature structures which differed in scale and were of 6 storey each where tested for natural frequency. The models had the following member configuration:

(i) 150mm member lengths each 6mm square in section
(ii) 200mm member lengths each 6mm square in section
(iii) 250 mm member lengths each 6mm square in section
(iv) 300 mm member lengths each 6mm square in section

In the experimental case, the values of vibration frequency as scale of models changed are presented in table 4.2.8.

Table 4.2.8-Effect on frequency due to scale change

Member length (mm)	**Average frequency, F**	**Standard Deviation of frequency**	**Scale**	**Mass (Kg)**	**Frequency Modified for Constant mass of 1.58 Kg**	**$Log_{10}Sc$**	**$Log_{10}F$**
150	2.45	0.05	1	1.58	2.45	0	0.39
200	2.07	0.1	1.33	2.1	2.38|	0.12	0.32
250	1.2	0.11	1.67	3.45	1.77	0.22	0.08
300	0.9	0.17	2	4.15	1.46	0.30	-0.046

The Frequency was found to decrease as the scale factor increased as shown in Figure 5.1.8.

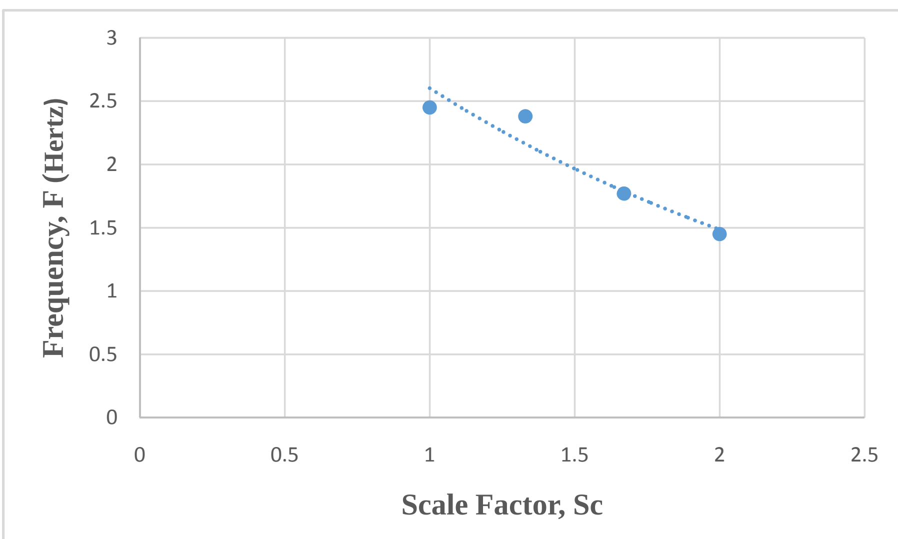


Figure 5.1.8-Frequency, F, against Scale Factor,Sc

CHAPTER5:DISCUSSION

The results obtained experimentally and theoretically,which were presented in the previous chapter, are discussed in this chapter.

5.1 Discussion of Relationship between Frequency and various parameters

5.1.1 NaturalFrequency against Height for Unbraced single bay frames

The relationship between the natural frequency and height of unbraced single bay structures in the theoretical case in shown in Figure 5.1.1.

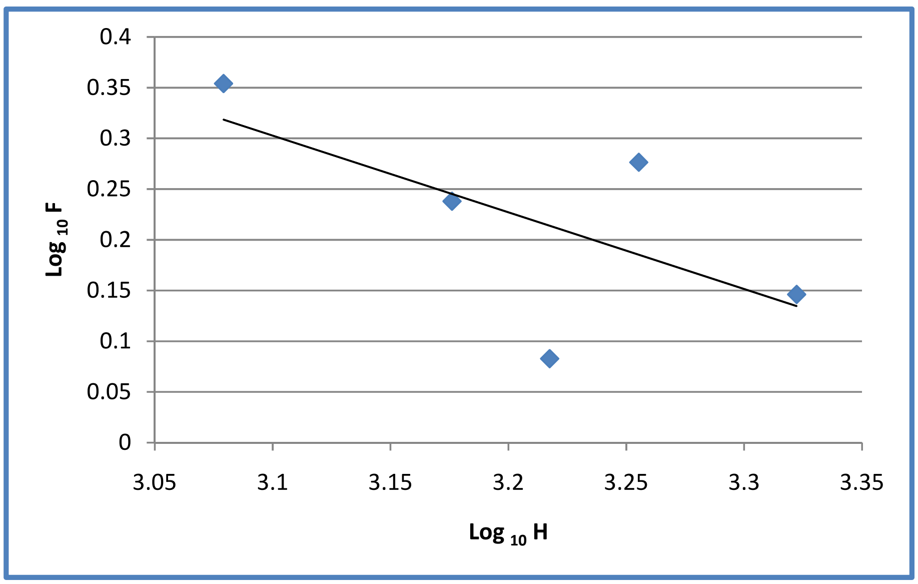


Figure 5.1.1 -$Log_{10}F$ against $Log_{10}H$ for unbraced 1 bay miniature structures-Theoretical

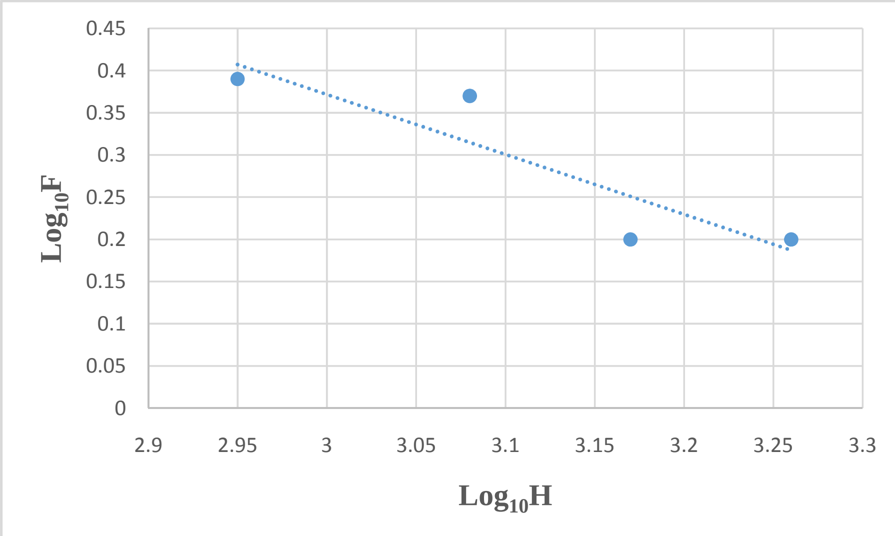


Figure 5.1.1b-Log_{10}(F) Versus Log_{10}(H) for unbraced miniature structures-experimental

In the theoretical case $Log_{10}F=C_{10}-0.72Log_{10}H$based on Figure 5.1.1. Therefore in the theoretical case $F=C_2/H^{0.72}$ with r^2 value of 0.42. In the experimental case the relationship formula based on the graph in Figure 5.1.1b is:log_{10} (F) $=C_1-0.75log_{10}$ (H) where C_1 is a constant. Therefore frequency, $F=C_2/H^{0.75}$ with r^2 value of 0.95 where C_2 is a constant and H is the height of the structure. This is valid for given stiffness of members and given number of bays and given mass of structure. The findings in both approachesare in line with the finding (Nilesh and Desai, 2012) that the natural frequency decreased with increase in number of storeys.

5.1.2 Braced single bay frames

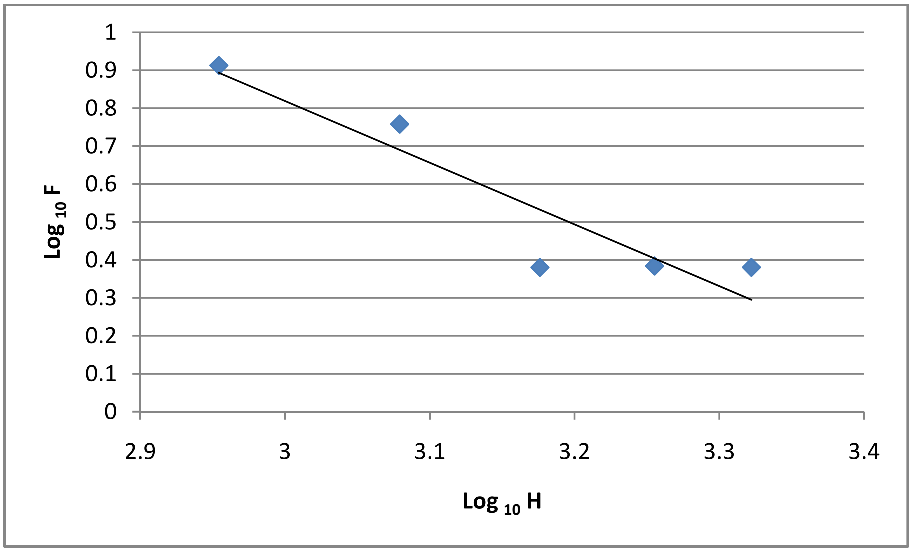


Figure 5.1.2-Theoretical Log $_{10}$ (F) vs Log $_{10}$ (H) for braced 1 bay structures

In the theoretical,based on Figure 5.1.2, $Log_{10}F=C_3-1.53Log_{10}H$ where C_3 is a constant. Therefore in the theoretical case $F=C_4/H^{1.53}$ with r^2 value of 0.44where $C_4$is a constant. Similarly, in the experimental case it was found that log_{10} (F) $=C_3-0.8log_{10}$ (H) where C_3 is a constant from Figure 5.1.2b. Therefore for braced case $F=C_4/H^{0.8}$ with r^2 value of 0.19 where C_4 is a constant. The findings in both approachesare in line with the finding (Nilesh and Desai, 2012) that the natural frequency decreased with increase in number of storeys.

5.1.3 Increase in number of bays (N) Parallel to direction of vibration

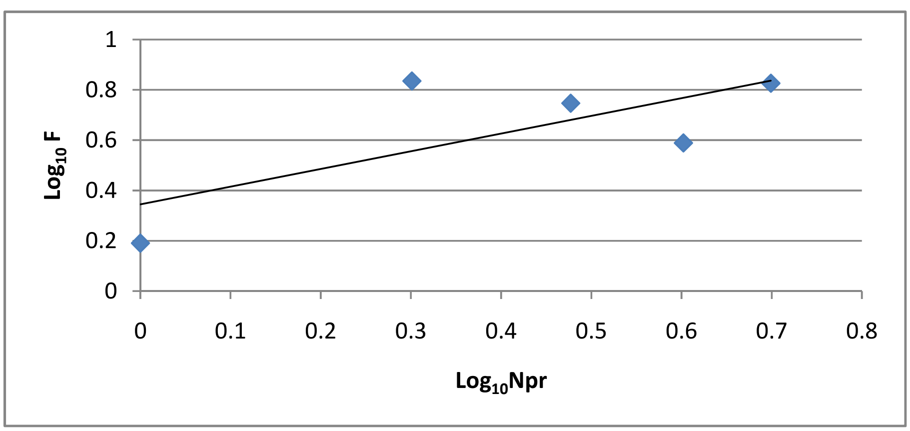


Figure 5.1.3-Theoretical log $_{10}$ F against Log $_{10}$ Npr as Bays increase in the direction motion

In the theoretical case the relationship between Log $_{10}$ F and Log $_{10}$ N where the number of bays increased and motion is parallel to the stiffer direction is shown in Figure 5.1.3. The relationship for the experimental case is shown in Figure 5.1.3b.

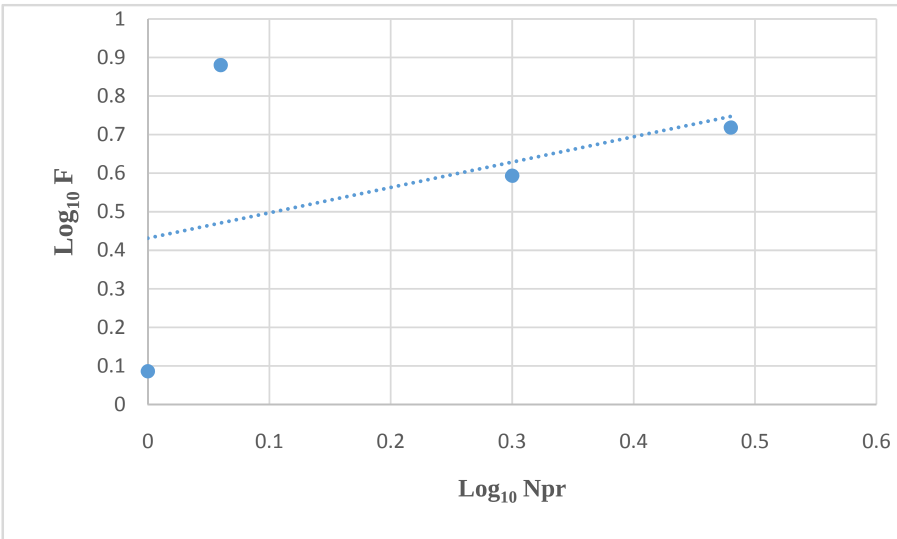


Figure 5.1.3b- Log_{10} (Frequency,F) VsLog_{10} (Number of bays,Npr), as Bays increase in the Direction of Motion-Experimental

In the theoretical case, Log10F=C9+0.68Log10Npr. Therefore F=C10*Npr0.68 with r2 value of 0.23 based on Figure 5.1.3. In the experimental case, based on the graph in Figure 5.1.3b, log 10(F) =C9+0.68 log10 (Npr) with r2value of 0.19 where C9 is a constant. This means that in the experimental case F=C10*(Npr0.68) where C10 is a constant. The findings in both approaches are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration increases as the number of bays in the direction of motion increase.

5.1.4 Increase in Number of bays (Npp) perpendicular to direction of vibration.

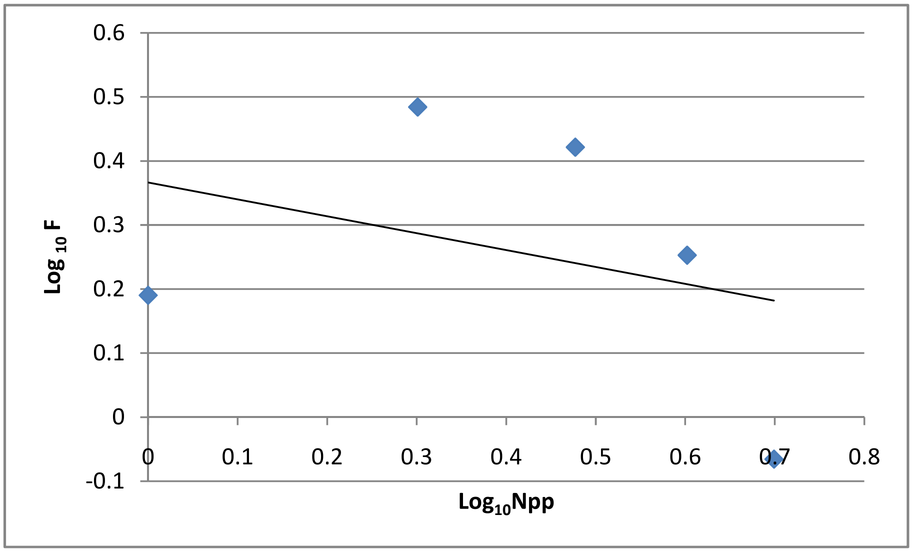


Figure 5.1.4-Log $_{10}$F against log $_{10}$ Npp as number of bays increase in the direction Transverse to Motion -Theoretical

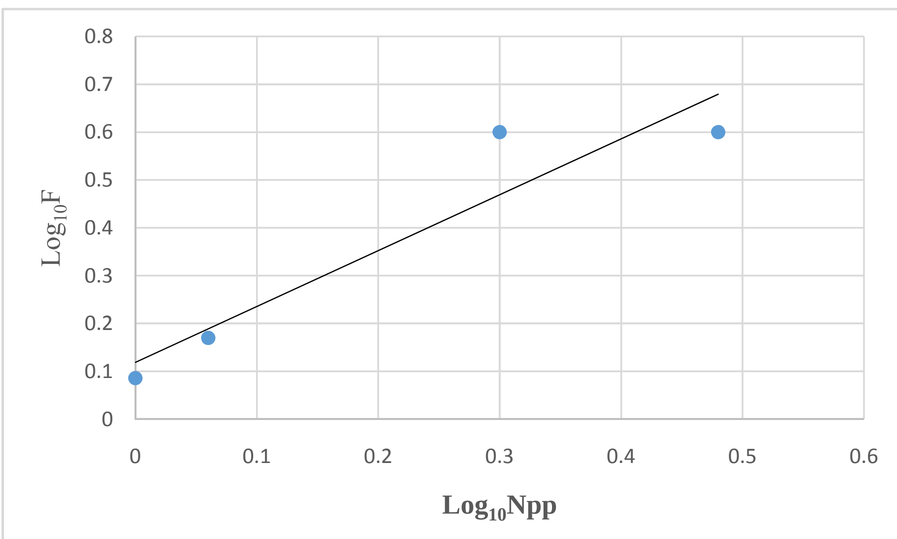


Figure 5.1.4b-Log$_{10}$ (F) against Log$_{10}$ (Npp) for increase in bays in direction perpendicular to motion-Experimental

In the theoretical Log10F=C5-0.28Log10Npp based on Figure 5.1.4 where C5 is a constant. Therefore in this case F=C6/Npp0.28 where C6 is a constant with r2 value of 0.997.In the experimental case, based on Figure 5.1.4b, the relationship between Log10F and Log10Npp is: Log10 (F) =C5+1.17log10 (Npp) where C5 is a constant. Therefore for motion in less stiff direction experimental case, F=C6*Npp1.17 with r2 value of 0.92. The findings in the theoretical approach are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration decreases as the number of bays in the direction transverse to motion increase. The findings in the experimental approach are not in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration decreases as the number of bays in the direction transverse to motion increase. This differencemay due to errors introduced due to damping in the experimental approach.

5.1.5 Change of Natural frequency as the Column Stiffness Increases

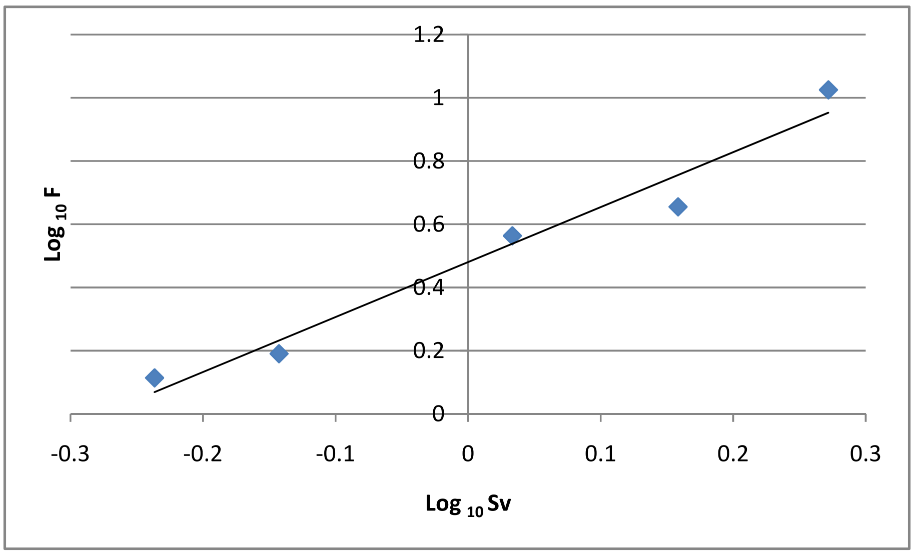


Figure 5.1.5 Log$_{10}$ (F) against Log$_{10}$ (Sv) -Theoretical

In the theoretical case, the relationship between log$_{10}$ (F) and log$_{10}$(Sv) is shown in Figure 5.1.5 where F is the frequency and Sv is the column stiffness ratio (I/L).

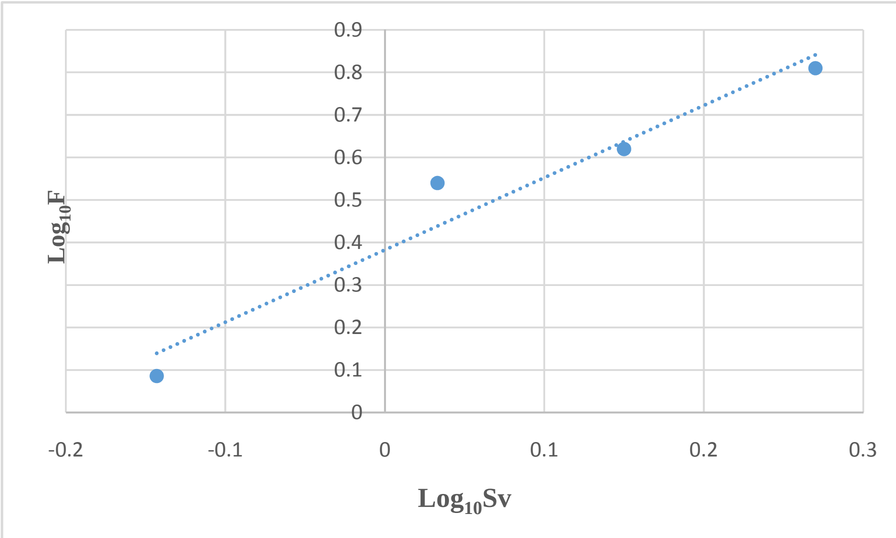


Figure 5.1.5b- Log 10(Frequency,F) against log10 (Column stiffness, Sv)-Experimental

In the theoretical case $Log_{10}F=C_{10}+1.4Log_{10}Sv$ based on Figure 5.1.5 where C_{10} is a constant.Therefore in the theoretical case $F=C_{11}*Sv^{1.4}$ with r^2 value of 0.94.In the experimental case the relationship between Log_{10} (F) and Log_{10} (S) is $log_{10}(F) =C_{10}+1.7Log_{10} (Sv)$ where C_{10} is a constant as shown in Figure 5.1.5b. Therefore in experimental case $F=C_{11}*Sv^{1.7}$ with r^2 value of 0.92 where C_{11} is a constant. The results in both approaches are in line with the literature with gives frequency, *f*, is given by $f=W/2\pi=(1/2\pi)(\sqrt{(Ko/m)}$ as shown in equation 2.37 since as column stiffnesses increase the overall structure stiffness, Ko, increases.

5.1.6 Change in horizontal stiffness parallel to motion

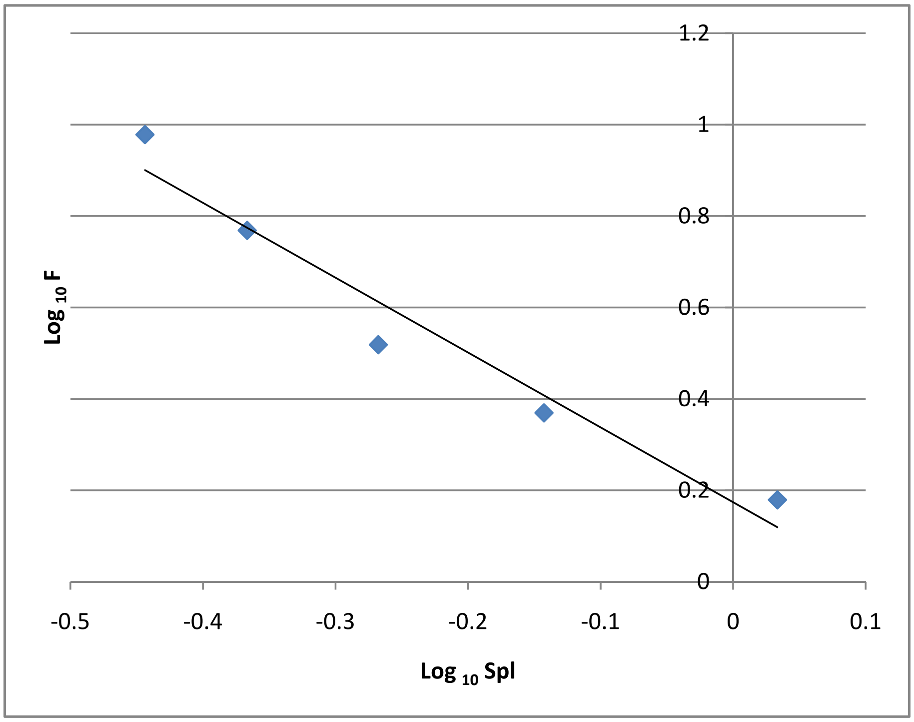


Figure 5.1.6-Theoretical Log_{10} (F) against log_{10} (Spl) where stiffness of horizontal members varies parallel to horizontal deflection

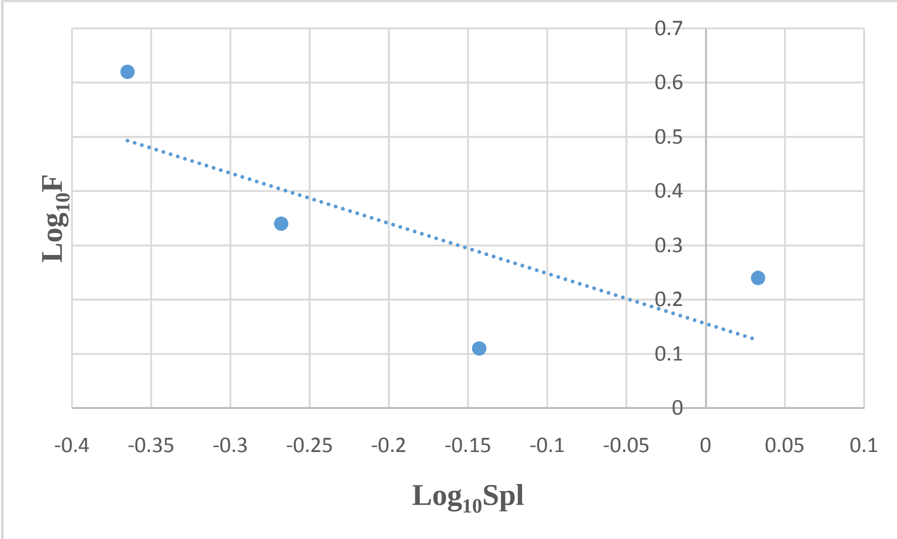


Figure 5.1.6b-Experimental Log_{10}(F) VS Log_{10}(Spl) for motion parallel to change in horizontal member stiffness

In the theoretical case Log10F=C15-1.59Log10Spl where C15 is constant base on Figure 5.1.6. Therefore, in the theoretical case F=C16/(Spl1.59) with r2 value of 0.96. In the experimental case the relationship between log10 (F) and Log10 (Spl) is shown in Figure 5.1.6b.The relationship is: log10(F)=C15-1.83*Log10(Spl) with r2 value of 0.58 where C15 is a constant. Therefore F=C16/((Spl)1.83). The findings in both approaches are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration increases as the stiffnesses of horizontal members in the direction of motion increase.

5.1.7 Change Horizontal Member Stiffness Perpendicular to motion

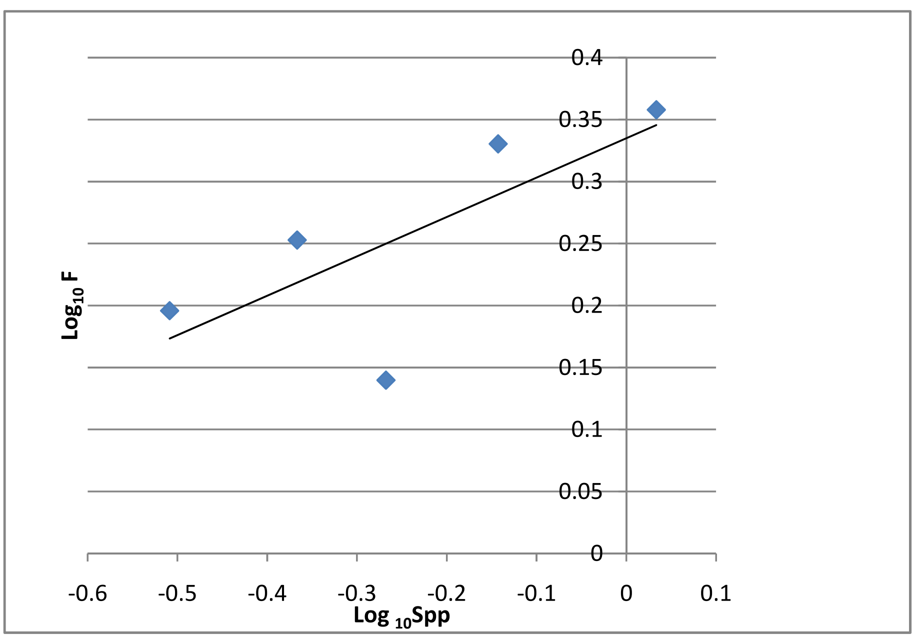


Figure 5.1.7-Theoretical Log_{10}(F) against Log_{10}(Spp) where stiffness of horizontal member stiffness varies perpendicular to motion.

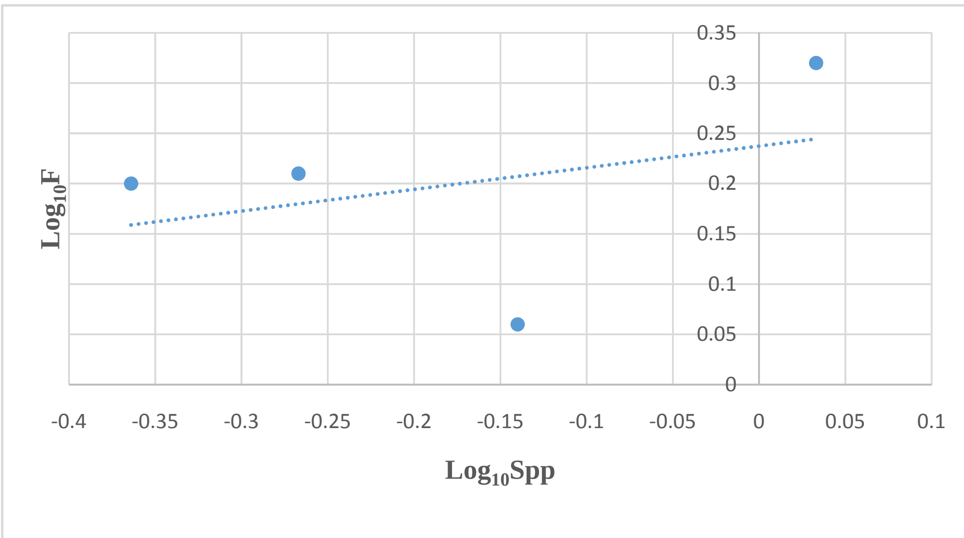


Figure 5.1.7b-Experimental Log $_{10}$(Frequency, F) against Log$_{10}$ (Stiffness, Spp)

In the theoretical case Log10F=C18+0.33Log10Spp where C18 is a constant based on Figure 5.1.7.Therefore in the theoretical case F=C19*spp0.33 with r2 value of 0.6. In the experimental case the relationship between Log 10(F) and log 10(Spp) is shown in Figure5.1.7b. The relationship is: Log10 (F) =C18+0.23*Log10 (Spp) where C18 is a constant. Therefore in the experimental case F=C19*Spp0.23 with r2 value of 0.12. The findings in both approaches are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration decreases as the stiffnesses of horizontal members in the direction transverse to motion increase.

5.1.8 Change in scale of miniature structure

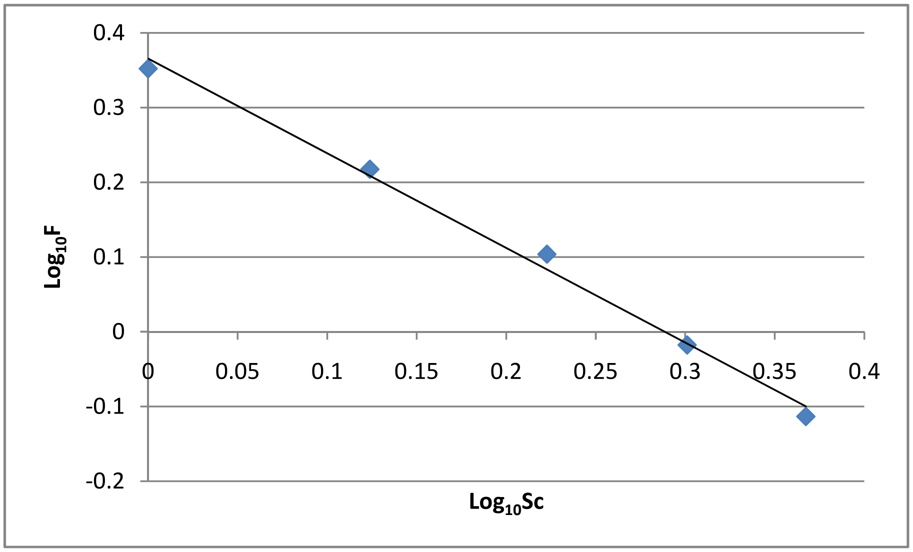


Figure 5.1.8-Theoretical Log $_{10}$ (F) against Log $_{10}$ (Sc)

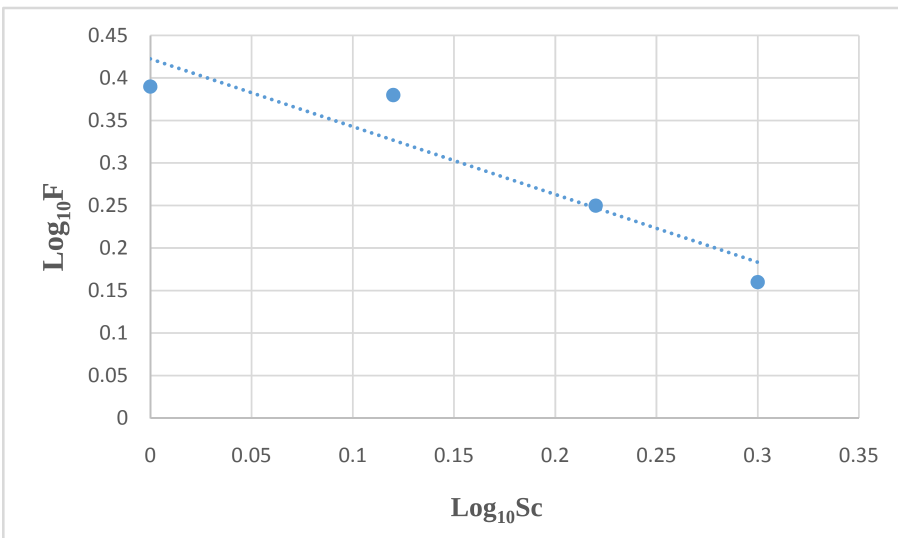


Figure 5.1.8b- Log $_{10}$(Frequency,F)Vs Log$_{10}$(Scale ,Sc)

In the theoretical case, based on Figure 5.1.8, $Log_{10}F=C_{20}-1.24Log_{10}Sc$ where C_{20} is a constant. Therefore in the theoretical case $F=C_{21}/(Sc^{1.24})$ with r^2value of 0.99.In the experimental case, the relationship between $log_{10}(F)$ and $Log_{10}(Sc)$ is shown in Figure 5.1.8b. The relationship is: $Log_{10}(F)=C_{20}-0.9Log_{10}(Sc)$ where C_{20} is a constant. Therefore in the experimental case for varying scale models, $F=C_{21}/(Sc^{0.9})$ with r^2 value of 0.98 where C_{21} is a constant. The results in both approaches are line with the literature with gives frequency, f, is given by $f=W/2\pi=(1/2\pi)(\sqrt{(Ko/m)})$as shown in equation 2.37 since as column stiffnesses increase the overall structure stiffness, Ko, increases. When the scale increases the stiffnesses decrease and therefore the frequency of vibration is expected to decrease.

5.2 Relationship between the Experimental and Theoretical Values

The theoretical values of natural frequencies had the same trend of variation with number of bays and stiffness as the experimental. However the experimental values were higher in most cases.

5.2.1 Relationship between Theoretical and Experimental results for unbraced 1 bay miniature structures as Height increased.

The experimental and theoretical values of natural frequencies for 1bay unbraced frames are presented in Table 5.2.1. The values in the tables are based on the trend lines in the various graphs presented earlier in this thesis.

Table 5.2.1 Comparison of Theoretical and Experimental Natural Frequencies for Unbraced 1 bay miniature structures

Height of miniature structure	1800	1500	1200	900
Theoretical Natural Frequency (Hertz)	1.6	1.8	2.1	2.6
Experimental Natural Frequency (Hertz)	1.5	1.75	2.2	2.4
Percentage variation	-6.25%	-2.7%	4.8%	-7.7%

The relationship is shown graphically below.

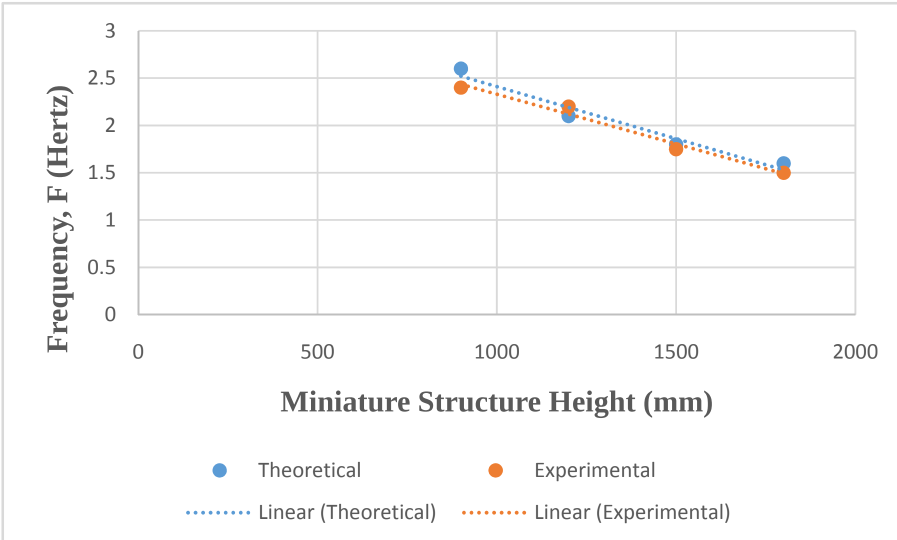


Figure 5.2.1-Comparison of Theoretical and Experimental frequencies for unbraced 1 bay structures as Height increased.

The experimental values of natural frequencies for 1 bay unbraced frames were lower than the theoretical values as shown in Figure 5.2.1. The difference is mainly due to errors which are discussed in section 5.3. However, the frequencies for experimental case are expected to be lower than the theoretical ones due to damping. There was positive correlation between the two sets of values. The correlation coefficient was 0.99. The findings in both approaches are in line with the finding (Nilesh and Desai, 2012) that the natural frequency decreased with increase in the number of storeys.

5.2.2Relationship between Theoretical and Experimental results for braced 1 bay miniature structures.

The theoretical and experimental values of natural frequencies for 1bay braced frames are presented in Table 5.2.2. The values are based on the trend lines.

Table 5.2.2 Comparison of Theoretical and Experimental Natural Frequencies For braced 1 bay miniature structures

Height of miniature structure	1800	1500	1200	900
Theoretical Natural Frequency (Hertz)	1.5	2.4	3.5	5.0
Experimental Natural Frequency (Hertz)	2.9	3.3	3.8	4.5
Percentage Variation	93%	37.5%	4.8%	-3.7%

The relationship is shown graphically below

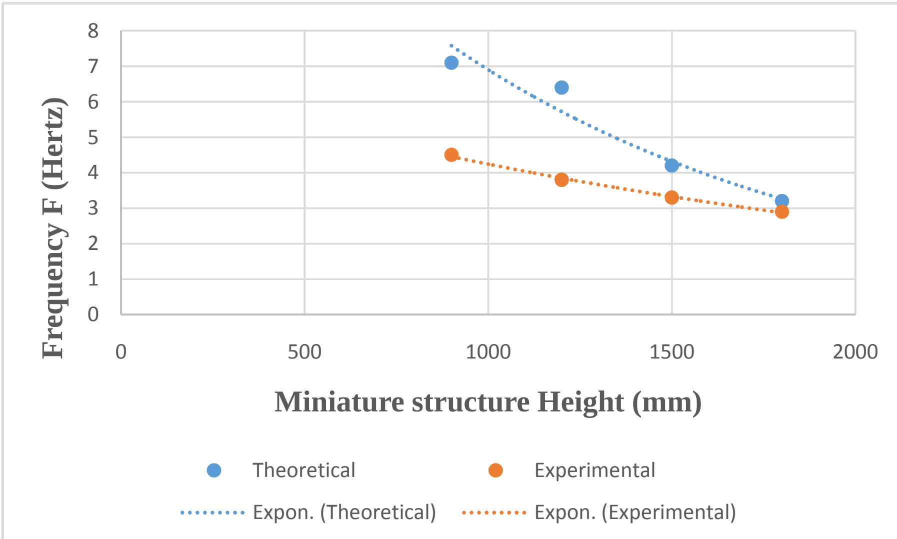


Figure 5.2.2-Comparison of Theoretical and Experimental frequencies for braced 1 bay structures

The experimental values of natural frequencies for 1 bay braced frames were lower than the theoretical values as shown in Figure 5.2.2. The difference is mainly due to errors which are discussed in section 5.3. However, the frequencies for the experimental case frequencies are expected to be lower than the theoretical ones due to damping. There was positive correlation between the two sets of values. The correlation coefficient was 0.99. The findings in both approaches are in line with the finding (Nilesh and Desai, 2012) that the natural frequency decreased with increase in number of storeys.

5.2.3Relationship between Theoretical and Experimental results as number of bays increased in the direction of motion.

The experimental and theoretical values of natural frequencies as number of bays increased in the direction of motion are presented in Table 5.2.3. The values are based on the trend lines.

Table 5.2.3 Comparison of Theoretical and Experimental Natural Frequencies as Number of Bays increased in the direction of motion.

Number of Bays	1	2	3	4
Theoretical Natural Frequency (Hertz)	1.8	2.3	3.2	4
Experimental Natural Frequency (Hertz)	1.2	2.7	5	8.7
Percentage Variation	-50%	14%	36%	54%

The experimental valuesof vibration frequency were mostly lower than the theoretical values as the number bays increased in the direction of motion as shown in Figure 5.2.3. The difference is mainly due to errors which are discussed in section 5.3. However the experimental case frequencies are expected to be lower than the theoretical ones due to damping. There was positive correlation between the two sets of values. The correlation coefficient was 0.99. The findings in both approaches cases are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration increases as the number of bays in the direction of motion increase.

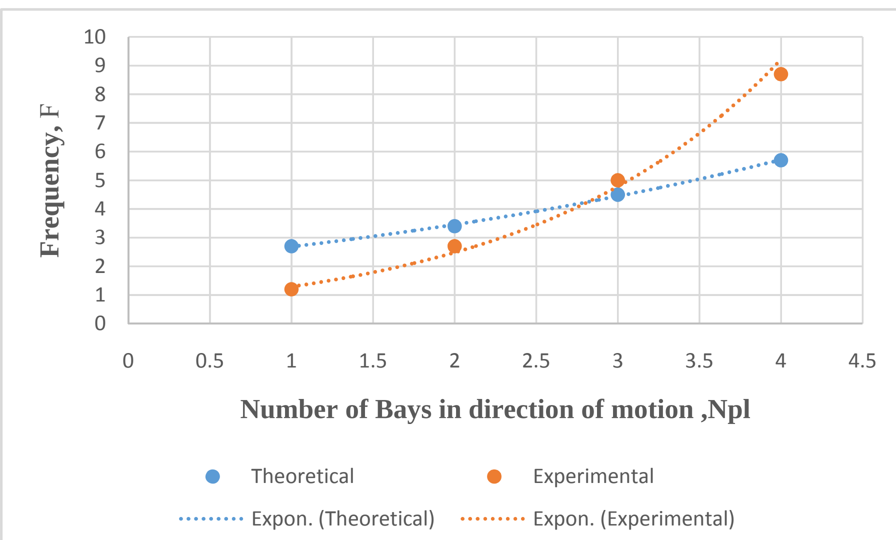


Figure 5.2.3-Comparison of Theoretical and Experimental frequencies as number of bays increased in the direction of motion.

5.2.4 Relationship between Theoretical and Experimental results as number of bays increased Transverse to direction of motion.

The experimental and theoretical values of natural frequencies as the number of bays increased in one direction are presented in Table 5.2.4. The values are based on the trend lines.

Table 5.2.4 Comparison of Theoretical and Experimental Natural Frequencies as Number of Bays increased Transverse to Motion

Number of Bays	1	2	3	4
Theoretical Natural Frequency (Hertz)	2.4	1.75	1.25	1
Experimental Natural Frequency (Hertz)	2.3	2	1.8	1.7
Percentage Variation	-42%	14.3%	4.4%	70%

The experimental values of natural frequencies, as number bays increased in one direction, were generally lower than the theoretical values as shown in Figure 5.2.4. The difference is mainly due to errors which are discussed in section 5.3. However, the experimental case frequencies are expected to be lower than the theoretical ones due to damping. There was positive correlation between the two sets of values. The correlation coefficient was 0.99. However after modifying the frequencies for constant mass there was a negative correlation. The modified frequency decreased as the number of bays increased in the direction transverse to motion in the theoretical approach. The modified frequency increased as the number of bays increased in the direction transverse to motion in the experimental approach. The findings in the theoretical approach are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration decreases as the number of bays in the direction transverse to motion increase. The findings in the experimental approach are not in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration decreases as the number of bays in the direction transverse to motion increase. This difference may due to errors in the experimental approach.

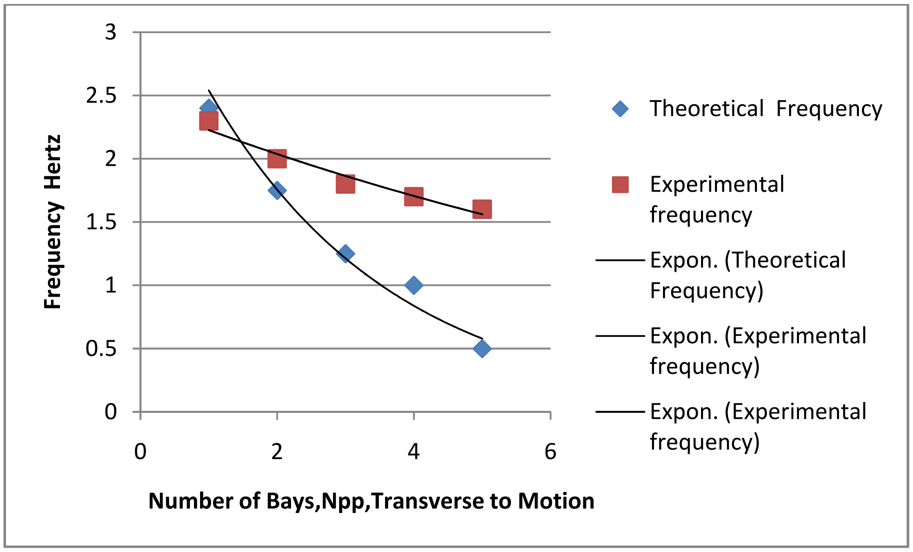


Figure 5.2.4-Comparison of Theoretical and Experimental frequencies as number of bays increased Transverse to Motion (before modifying for constant mass).

5.2.5 Relationship between Theoretical and Experimental results as stiffness of columns increased

The values of natural frequencies obtained theoretically and values of natural frequencies obtained by experiment as column stiffness increased are presented in Table 5.2.5. The values are based on the trend lines.

Table 5.2.5 Comparison of Theoretical and Experimental Natural Frequencies as Stiffness of columns increased.

Stiffness of Columns	0.72	1.08	1.44	1.87
Theoretical Natural Frequency (Hertz)	1.55	3	5.2	10.2
Experimental Natural Frequency (Hertz)	1.5	3.4	4.2	7.2
Percentage Variation	-3.2%	13.3%	-19.2%	-29.4%

The experimental values of natural frequencies were lower than the theoretical values as shown in Figure 5.2.5. The difference is mainly due to errors which are discussed in section

5.3. However, the experimental case frequencies are expected to be lower than the theoretical ones due to damping. There was positive correlation between the two sets of values. The correlation coefficient was 0.98.The results in both approaches are in line with the literature with gives frequency, f, is given by $f = W/2\pi = (1/2\pi)(\sqrt{(Ko\ /m})$ as shown in equation 2.37 since as column stiffnesses increase the overall structure stiffness, Ko, increases.

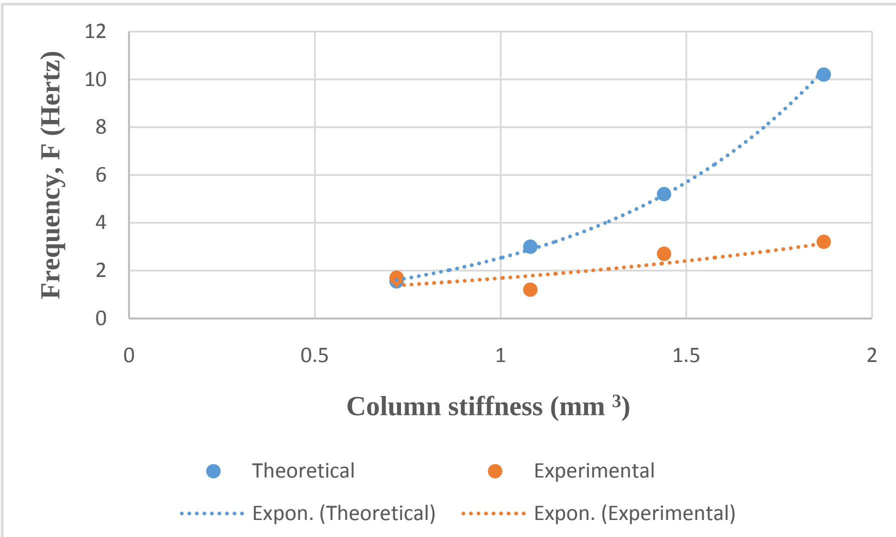


Figure 5.2.5-Comparison of Theoretical and Experimental frequencies as column stiffness increased for unbraced miniature structures.

5.2.6 Relationship between Theoretical and Experimental results as stiffness of horizontal increased parallel to initial force direction

The experimental and theoretical values of natural frequencies as horizontal member stiffnesses increased parallel to initial force direction are presented in Table 5.2.6. The values are based on the trend lines.

Table 5.2.6 Comparison of Theoretical and Experimental Natural Frequencies as stiffnesses of horizontal member parallel to initial force increased.

Stiffness of horizontal Members parallel to initial force direction, Spl (mm^3)	1.08	0.72	0.54	0.43
Theoretical Natural Frequency (Hertz)	1.4	2.7	3.8	4.7
Experimental Natural Frequency (Hertz)	1.7	1.2	2.7	3.2
Percentage Variation	21.4%	-55.5%	-28.9%	-31.9%

The relationship is shown graphically below.

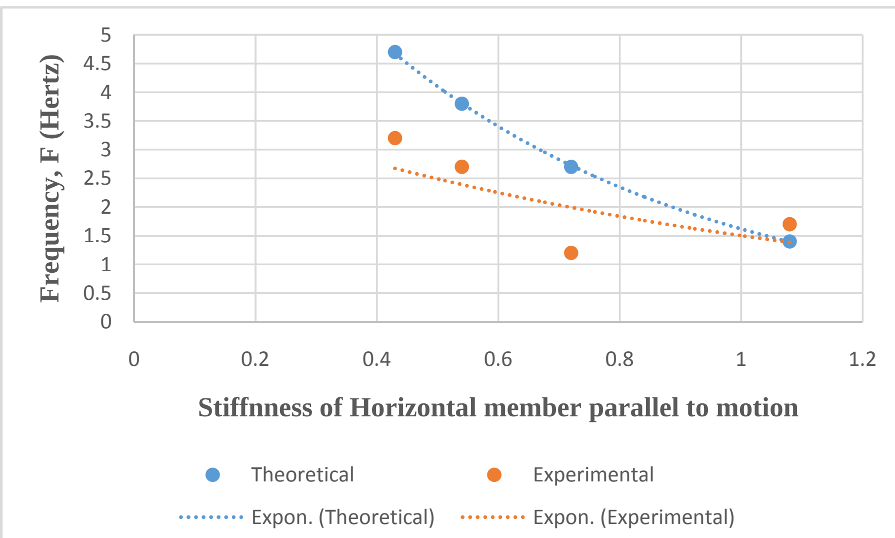


Figure 5.2.6-Comparison of Theoretical and Experimental frequencies as stiffnesses for horizontal members parallel to initial force direction increased.

The experimental values of natural frequencies were lower than the theoretical ones as shown in Figure 5.2.6. The difference is mainly due to errors which are discussed in section 5.3. However, the experimental case frequencies are expected to be lower than the theoretical ones due to damping. There was positive correlation between the two sets of values. The correlation coefficient was 0.88. The findings in both approaches are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration increases as the stiffnesses of horizontal members in the direction of motion increase.

5.2.7 Relationship between Theoretical and Experimental results as stiffness of horizontal increased perpendicular to initial force direction

The experimental and theoretical values of natural frequencies as horizontal member stiffnesses increased perpendicular to initial force direction are presented in Table 5.2.7. The values are based on the trend lines.

Table 5.2.7 Comparison of Theoretical and Experimental Natural Frequencies as stiffnesses of horizontal members perpendicular to initial force increased.

Stiffness of horizontal Members perpendicular to initial force direction, Spp (mm^3)	1.08	0.72	0.54	0.43
Theoretical Natural Frequency (Hertz)	2.3	1.8	1.75	1.6
Experimental Natural Frequency (Hertz)	1.8	1.6	1.5	1.35
Percentage Variation	-21.7%	-11.1%	-14.3%	-15.6%

The experimental values of natural frequencies were lower than the theoretical ones as shown in Figure 5.2.7. The difference is mainly due to errors which are discussed in section 5.3. However, the experimental case frequencies are expected to be lower than the theoretical ones due to damping. There was positive correlation between the two sets of values. The correlation coefficient was 0.97. The findings in both approaches are in line with the finding (Anwar and Hossain, 1996) that the frequency of vibration decreases as the stiffnesses of horizontal members in the direction transverse to motion increase.

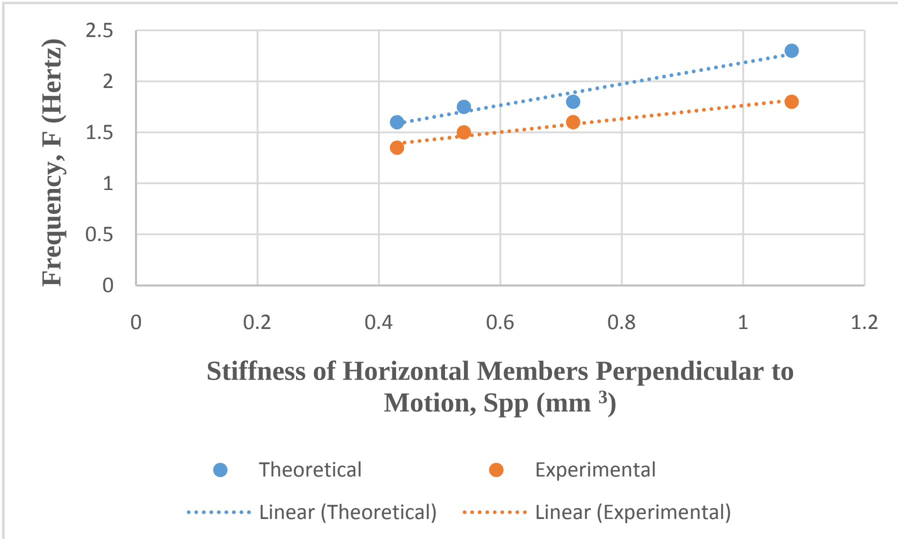


Figure 5.2.7-Comparison of Theoretical and Experimental frequencies as horizontal stiffnesses increased perpendicular to initial force direction.

5.2.8 Relationship between Theoretical and Experimental results as scale factor increased

The experimental and theoretical values of natural frequencies as scale factor of miniature structure increased are presented in Table 5.2.8. The values are based on the trend lines.

Table 5.2.8 Comparison of Theoretical and Experimental Natural Frequencies as scale factor increased.

Scale factor of miniature structure compared to 900mm high model (6floor)	1.33	1.67	2	2.33
Theoretical Natural Frequency (Hertz)	1.7	1.25	1	0.8
Experimental Natural Frequency (Hertz)	2.25	1.8	1.5	1.1
Percentage Variation	32.4%	44%	50%	37.5%

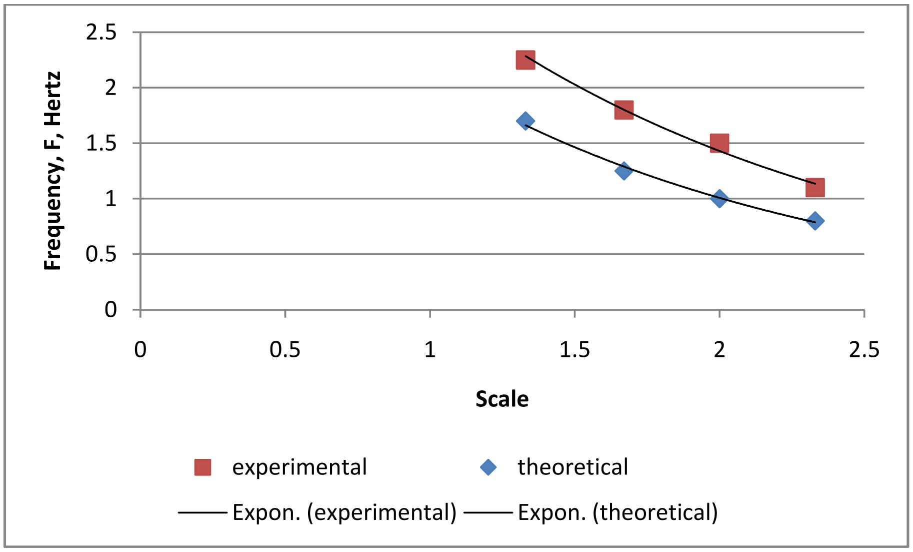


Figure 5.2.8:Comparison of Theoretical and Experimental frequencies as scale factor increased.

The experimental values of natural frequencies were lower than the theoretical values as shown in Figure 5.2.8. The difference is mainly due to errors which are discussed in section 5.3. There was positive correlation between the two sets of values. The correlation coefficient was 0.99.The results in both approaches are in line with the literature with gives frequency, f, is given by $f=W/2\pi=(1/2\pi)(\sqrt{(Ko}/m)$ as shown in equation 2.37 since as column stiffnesses increase the overall structure stiffness, Ko, increases. When the scale increases the stiffnesses decrease and therefore the frequency of vibration is expected to decrease.

Most of the r^2 values for theoretical case were better than the corresponding ones for experimental. Moreover, the experimental frequencies are reduced by damping. The theoretical values may have some data entry errors since a large quantity of data had to be entered. It is better to adopt a formula based on an average of the two methods.

5.3 Sources of Error

The sources of error in the experiments are the following.

(i) Due to errors in making miniature structures

(ii) Due to inaccuracies in reading the wave lengths from data logger

(iii) It was assumed that damping was zero but in reality there was some damping even from the air around and internal structural damping. The damping reduced experimental frequencies.

(iv) Data entry errors in the theoretical case as a large quantity of data had to be entered.

(v) Temperature variation errors

(vi) Voltage variation errors

There was about 10% margin of error due to wave length reading accuracy, 10% error due to voltage variation and a maximum possible error of 0.6% due to temperature variation. However temperatures did not vary greatly during the experiments. However these errors may not have been concurrent or additive.

5.4 Resulting Relationship

The following relationships were found:

(i) The relationship between natural frequency, F, and Height of structure, H, for given column and beam length or column and beam stiffness as: $F=C_2/H^{0.735}$ which is an average between the theoretical and experimental value.

(ii) The relationship between natural frequency, F, and number of bays, parallel to motion, Npr, as $F=C_{10}*(Npr^{0.74})$which is an average between the theoretical and experimental value.

(iii) The relationship between natural frequency, F, and number of bays, perpendicular to motion, Npp, as $F=C_6*Npp^{0.445}$which is an average between the theoretical and experimental value.

(iv) The relationship between natural frequency, F, and stiffness of vertical members, Sv, as $F=C_{11}*(Sv^{1.55})$ which is an average between the theoretical and experimental value.

(v) The relationship between natural frequency, F, and stiffness of horizontal members parallel to motion, Spl, as $F=C_{16}/(Spl^{1.71})$which is an average between the theoretical and experimental value.

(vi) The relationship between natural frequency, F, and stiffness of horizontal members perpendicular to motion, Spp, as $F=C_{19}*(Spp^{0.28})$which is an average between the theoretical and experimental value.

Therefore a comprehensive formula for natural frequency for unbraced structures is

$F=C*(1/H^{0.735})*(Npr^{0.74}))*(Npp^{0.445})*(Sv^{1.55})*(1/(Spl^{1.71}))*Spp^{0.28}$ where C is a constant depending on material type. This is equation is based on the theoretical model.

In the theoretical case by taking a particular case where there is 10 storeys unbraced model of 150mm long members, H=1500mm, Npr =1, Npp=1, Sv=0.72mm^3 Spl=0.72mm^3 , Spp=0.72mm^3 and frequency=1.73 Hertz, gives an approximate value of C=389 in the case of steel structures. In the experimental case taking the same parameters of a structure frequency=1.58 Hertz and C= 354. Therefore average value of C=371.The formula is in line with the existing literature on natural frequencies. However no formula has been given like this before.

CHAPTER 6 :CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

It can be concluded that there exists a relationship between the natural frequency of vibration and the height of a structure, the number of bays, and the stiffnesses of structural members.

The natural frequency of vibration varies as below:

- It decreases as the height of structure increases.
- It increases as the stiffness of columns increases.
- It increases as the number of bays increase in the direction of motion.
- It decreases as the number of bays increase in the direction transverse to motion.
- It decreases as the stiffnesses of horizontal members parallel to motion increase.
- It increases as the stiffnesses of horizontal members transverse to motion increase.
- It decreases as the scale factor increases.

The specific objectives were met as follows:

- The theoretical approach helped to predict the changes in natural frequencies as the parameters under study changed.
- The experimental approach helped to validate the theoretical approach.
- The relationship developed was between the natural frequency as the dependant variable and the height of structure, number of bays, and stiffnesses as the independent variables.

6.2Recommendations

There is need to conduct further studies for other modes of vibrations and more bays in both directions. There is also need to determine the K value for other materials other than steel. There is need to conduct further research using Finite Element Method where deflections at centre of mass are estimated more accurately. Moreover, data should be gathered of earthquake frequencies in various geographical regions to guide design against resonance of structures to earthquake vibrations.

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APPENDIX

Appendix 1:Code for enumerating the coordinates

The code for enumerating the coordinates due to non-foundation joints is presented below.

```
NCO = (NCO - NCO)
For i = 1 To NJ
If SC(i) = 0 Then
For k = 1 To 6
NCO = NCO + 1
CORDSOURCE(NCO) = i
If k = 1 Then CORDTYPE(NCO) = 1
If k = 2 Then CORDTYPE(NCO) = 2
If k = 3 Then CORDTYPE(NCO) = 3
If k = 4 Then CORDTYPE(NCO) = 4
If k = 5 Then CORDTYPE(NCO) = 5
If k = 6 Then CORDTYPE(NCO) = 6
Next k
End If
Next i
```

The code below is for enumerating additional coordinates due to release of foundation joints in x-direction. The code for release in y-direction and z-direction is done in a similar manner.

```
For J = 1 To NJ
If SC(J) = 1 And Not (RX(J) <> 1) Then
NCO = NCO + 1
CORDSOURCE(NCO) = J
CORDTYPE(NCO) = 1
End If
Next J
```

Appendix 2:Code for calculation of external loads at joints.

The code below is for input of external loads in x-direction (Coordinate Type 1) at joints. The external loads at the rest of the coordinates are input in a similar manner.

```
For CORD = 1 To NCO
For J = 1 To NJ
If CORDSOURCE(CORD) = J Then
If CORDTYPE(CORD) = 1 Then
EL(CORD) = APX(J)
End If
End If
Next J
```

Appendix 3:Code for calculation of internal loads at joints

The code below is for input of internal loads at joints due to axial loads along x-direction . This applied on coordinate type 1 at various joints.

```
For CORD = 1 To NCO
For M = 1 To NM
For J = 1 To NJ
If CORDSOURCE(CORD) = J Then
If CORDTYPE(CORD) = 1 Then
If LJ(M) = J Then
IL(CORD) = IL(CORD) + AMJ(M, J, 1)
End If
End If
End If
Next J
Next M
```

Next CORD

The code below is for input of internal loads at joints due to moment about z-axis. This is applied on coordinate type 3 at various joints.

For CORD = 1 To NCO

For M = 1 To NM

For J = 1 To NJ

If LJ(M) = J Then

If CORDSOURCE(CORD) = J Then

If CORDTYPE(CORD) = 3 Then

IL(CORD) = IL(CORD) + AMJ(M, J, 3)

End If

End If

End If

Next J

Next M

Appendix 4:Generation of stiffnesses

Let S1 = Sin (θxy): C1 = Cos (θxy): S2 = Sin ((θyz): C2 = Cos ((θyz): S3 = Sin ((θxz): C3 = Cos ((θ**xz)**

Consider a unit deflection in coordinate 1. The following stiffnesses are generated:

If Lxy(M) <> 0 Then KM(M, 1, 1) = (12 * EIz(M) * (S1) ^ 2) / (Lxy(M)) ^ 3 + EA(M) / Lxy(M) * ((C1) ^ 2)

If Lxz(M) <> 0 Then KM(M, 1, 1) = KM(M, 1, 1) + 12 * EIy(M) * (S3 ^ 2) / (Lxz(M)) ^ 3 + (EA(M) / Lxz(M)) * (C3 ^ 2)

If Lxy(M) <> 0 Then KM(M, 2, 1) = (-12 * EIz(M) * (S1 * C1) / (Lxy(M)) ^ 3 + (EA(M) * C1 * S1) / Lxy(M))

If Lxy(M) <> 0 Then KM(M, 3, 1) = -(6 * EIz(M) * S1) / ((Lxy(M) ^ 2))

If M = 1 Then

```
'ShowToUser "KM (1,1,1)" & KM(1,1,1)
'ShowToUser "KM (1,2,2)" & KM(1,2,2)
End If
If Lxz(M) <> 0 Then KM(M, 4, 1) = (-12 * EIy(M) * (S3 * C3) / ((Lxz(M)) ^ 3) + (EA(M) * C3 * S3) / Lxz(M))
If Lxz(M) <> 0 Then KM(M, 5, 1) = -(6 * EIy(M) * S3) / ((Lxz(M)) ^ 2)
KM(M, 6, 1) = 0
If Lxy(M) <> 0 Then KM(M, 7, 1) = (-12 * EIz(M) * (S1) ^ 2) / (Lxy(M)) ^ 3 - EA(M) / Lxy(M) * ((C1) ^ 2)
If Lxz(M) <> 0 Then KM(M, 7, 1) = KM(M, 7, 1) - 12 * EIy(M) * (S3 ^ 2) / ((Lxz(M)) ^ 3) - EA(M) * (C3 ^ 2) / (Lxz(M))
 If Lxy(M) <> 0 Then KM(M, 8, 1) = ((12 * EIz(M) * (S1 * C1) / (Lxy(M)) ^ 3) - (EA(M) * C1 * S1) / Lxy(M))
 If Lxy(M) <> 0 Then KM(M, 9, 1) = (6 * EIz(M) * S1) / ((Lxy(M)) ^ 2)
If Lxz(M) <> 0 Then KM(M, 10, 1) = (12 * EIy(M) * (S3 * C3) / ((Lxz(M)) ^ 3) - (EA(M) * C3 * S3) / Lxz(M))
If Lxz(M) <> 0 Then KM(M, 11, 1) = (6 * EIy(M) * S3) / ((Lxz(M)) ^ 2)
'Consider a unit displacement(rotation) in coordinate 3, the following stiffnesses are generated.
If Lxy(M) <> 0 Then KM(M, 1, 3) = -6 * EIz(M) / (Lxy(M)) ^ 2
If Lxy(M) <> 0 Then KM(M, 2, 3) = 6 * EIz(M) / (Lxy(M) ^ 2)
If Lxy(M) <> 0 Then KM(M, 3, 3) = 4 * EIz(M) / Lxy(M)
If Lxz(M) <> 0 Then KM(M, 3, 3) = KM(M, 3, 3) + GJz(M) * S3 / Lxz(M)
If Lyz(M) <> 0 Then KM(M, 3, 3) = KM(M, 3, 3) + GJz(M) * S2 / Lyz(M)
KM(M, 4, 3) = 0
KM(M, 5, 3) = 0
KM(M, 6, 3) = 0
If Lxy(M) <> 0 Then KM(M, 7, 3) = -6 * EIz(M) / (Lxy(M) ^ 2)
```

```
If Lxy(M) <> 0 Then KM(M, 8, 3) = 6 * EIz(M) / (Lxy(M) ^ 2)
If Lxy(M) <> 0 Then KM(M, 9, 3) = 2 * EIz(M) / Lxy(M)
KM(M, 10, 3) = 0
KM(M, 11, 3) = 0
KM(M, 12, 3) = 0
```

The rest of the stiffnesses are generated in a similar manner

Appendix 5:Code for evaluating the stiffness matrix elements

The code for summing the stiffnesses will be as below:

```
For CORD = 1 To NCO
For CORD2 = 1 To NCO
If LJ(M) = i Then
If RJ (M) = J Then
If (CORDTYPE(CORD) = 1 And CORDTYPE(CORD2) = 1) Then
If CORDSOURCE(CORD) = i Then
If CORDSOURCE(CORD2) = J Then
stiffcord(CORD, CORD2) = KM(M, 1, 7)
stiffcord(CORD2, CORD) = KM(M, 7, 1)
End If
End If
End If
End If
End If
Next CORD2
Next CORD
```

Stiffcord (CORD, CORD2) is the force in coordinate “CORD” due to unit deflection in coordinate” CORD2”. The value above is force caused in x-direction due to x-direction displacement on the right member joint. The rest of the stiffnesses are evaluated in a similar way.

Appendix6: Code for equilibrium equations and their solution

Equilibrium equations to be solved

The equations to be solved will be based on the relationship that at any coordinate, External load(EL) + Internal load(IL)+Sum of (stiffness times deflection)=0 for equilibrium.

Solution of the equations:

Solution of the resulting simultaneous equations is by Gaussian method forward elimination and backward substitution. The code is given below:

```
Public Sub Gausian eliminate ()

On Error Resume Next

For I1 = 1 To NCO

ELDASH(I1) = EL(I1) +IL(I1)

Next I1

For CORD = 1 To NCO

For CORD2 = 1 To NCO

STIFFDASH (CORD, CORD2) = stiffcord (CORD, CORD2)

Next CORD2

Next CORD

NCODASH = NCO - 1

For I1 = 1 To NCODASH

IHIGHER = I1 + 1

For J = I HIGHER To NCO

For COLL = I HIGHER To NCO
```

```
STIFFDASH(J, COLL) = STIFFDASH(J, COLL) - ((STIFFDASH (J, I1) / STIFFDASH(I1, I1)) * STIFFDASH(I1, COLL))
Next COLL
ELDASH(J) = ELDASH(J) - (STIFFDASH(J, I1) / STIFFDASH(I1, I1)) * ELDASH(I1)
CONTDIF:
Next J

Next I1
For i = 1 To NCO
STIFFDIFSUM(i) = 0
Next i
stiffoNCO = STIFFDASH(NCO, NCO)
ELDONCO = ELDASH(NCO)
If
 Not (ELDONCO <> 0) Then
DIFCORD(NCO) = 0
GoTo CONTDIF2
End If
DIFCORD(NCO) = ELDONCO / stiffoNCO
CONTDIF2:
For i = NCO To 2 Step -1
IDASH = i - 1
For J = NCO To i Step -1
JDASH = J - 1
STIFFO = STIFFDASH(i, i)
ELDO = ELDASH(i)
```

```
STIFFDIFSUM(IDASH) = STIFFDIFSUM(IDASH) + (STIFFDASH(IDASH, J)) * DIFCORD(J)
CONTDIF1:
stiffda = STIFFDASH(IDASH, IDASH)
If stiffda<> 0 Then
If J = i Then
ELDASHO = ELDASH(IDASH)
stiffda = STIFFDASH(IDASH, IDASH)
STIFFDIFFOSUM = STIFFDIFSUM(IDASH)
eldoCORD = (ELDASHO - STIFFDIFFOSUM)
'DIFCORD(IDASH) = eldoCORD / stiffda
End If
End If
Next J
If Not (eldoCORD<> 0) Then
DIFCORD(IDASH) = 0
GoTo contdif3
End If
DIFCORD(IDASH) = eldoCORD / stiffda
contdif3:
Next i
For CORD = 1 To NCO
ShowToUser "cord.... " & CORD & "DEFLECTION." & "" & "" &DIFCORD(CORD)
If CORDTYPE(CORD) = 1 Then ShowToUser "LINEAR IN X-DIRECTION"
If CORDTYPE(CORD) = 2 Then ShowToUser "LINEAR IN Y-DIRECTION"
If CORDTYPE(CORD) = 3 Then ShowToUser "ROTATION ABOUT   Z-AXIS"
If CORDTYPE(CORD) = 4 Then ShowToUser "LINEAR IN Z-DIRECTION"
```

```
If CORDTYPE(CORD) = 5 Then ShowToUser "ROTATION ABOUT IN Y-AXIS"
If CORDTYPE(CORD) = 6 Then ShowToUser "ROTATION ABOUT IN X-AXIS"

Next CORD
Exit Sub
End Sub
```

Appendix 7-Experimental Data

A. Experimental data for unbraced 1bay models

Table 8.2.1a-Experimental Results for 1 bay models of height 1800mm (unbraced)

Data no.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	1800	1	150	6mm Square	7/2/2015	1.1
2	1800	1	150	6mm Square	7/2/2014	1.2
3	1800	1	150	6mm Square	7/2/2015	1.1
4	1800	1	150	6mm Square	7/2/2014	1.1
5	1800	1	150	6mm Square	9/2/2015	1.2
6	1800	1	150	6mm Square	9/2/2015	1.1

Table 8.2.1b- Experimental Results for 1 bay models of height 1500mm (unbraced)

Data no.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	1500	1	150	6mm Square	5/7/2014	1.2
2	1500	1	150	6mm Square	5/7/2014	1.3
3	1500	1	150	6mm Square	5/7/2014	1.2
4	1500	1	150	6mm Square	5/7/2014	1.2
5	1500	1	150	6mm Square	5/7/2014	1.2
6	1500	1	150	6mm Square	5/7/2014	1.2

Table 8.2.1c-Experimental Results for 1 bay models of height 1200mm (unbraced)

Data no.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	1200	1	150	6mm Square	5/4/2014	2.0
2	1200	1	150	6mm Square	5/4/2014	2.1
3	1200	1	150	6mm Square	5/4/2014	2.0
4	1200	1	150	6mm Square	5/4/2014	2.0
5	1200	1	150	6mm Square	5/4/2014	2.0
6	1200	1	150	6mm Square	5/4/2014	2.0

Table 8.2.1d-Experimental Results for 1 bay models of height 900mm (unbraced)

Data no.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	900	1	150	6mm Square	5/7/2014	2.5
2	900	1	150	6mm Square	5/7/2014	2.4
3	900	1	150	6mm Square	5/4/2014	2.5
4	900	1	150	6mm Square	5/4/2014	2.5
5	900	1	150	6mm Square	5/4/2014	2.4
6	900	1	150	6mm Square	5/4/2014	2.4

B. Experimental data for braced 1 bay miniature structures

The experimental Results for braced 1 bay models are presented in the tables below:

Table 8.2.2a-Experimental Results for 1 bay models of height 2100mm (braced)

Data no.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	2100	1	150	6mm Square	23/4/2015	2.6
2	2100	1	150	6mm Square	23/4/2015	2.7
3	2100	1	150	6mm Square	24/4/2015	2.6
4	2100	1	150	6mm Square	24/4/2015	2.7
5	2100	1	150	6mm Square	25/4/2015	2.7
6	2100	1	150	6mm Square	25/4/2015	2.7

Table 8.2.2b- Experimental Results for 1 bay models of height 1800mm (braced)

Data no.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	1800	1	150	6mm Square	7/7/2015	1.3
2	1800	1	150	6mm Square	7/7/2015	1.4
3	1800	1	150	6mm Square	7/7/2015	1.4
4	1800	1	150	6mm Square	7/7/2015	1.3
5	1800	1	150	6mm Square	7/7/2015	1.3
6	1800	1	150	6mm Square	7/7/2015	1.3

Table 8.2.2c- Experimental Results for 1 bay models of height 1500mm (braced)

Data No.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	1500	1	150	6mm Square	9/10/2014	3.86
2	1500	1	150	6mm Square	9/10/2014	3.86
3	1500	1	150	6mm Square	17/10/204	3.75
4	1500	1	150	6mm Square	17/10/2014	3.75
5	1500	1	150	6mm Square	9/10/2014	3.71
6	1500	1	150	6mm Square	9/10/2014	3.71

Table 8.2.2d -1200 mm high Miniature Structure-braced

Data No.	Model Height (mm)	No. of bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	1200	1	150	6mm square	16/10/2014	3.3
2	1200	1	150	6mm square	16/10/2014	3.3
3	1200	1	150	6mm square	16/10/2014	3
4	1200	1	150	6mm square	16/10/2014	3
5	1200	1	150	6mm square	9/10/2014	3
6	1200	1	150	6mm square	9/10/2014	3

C.Experimental Data For increasing number of Bays in Direction of Motion

Table 8.2.3a Experimental Results for 2x1 bays models (vibration parallel to longer side)

Data No.	Model Height (mm)	No. of Bays	Length of member (mm)	Member type and size	Direction of vibration	Date	Measured Frequency
1	1500	2	150	6mm Square	Parallel to longer side	17/2/2015	2.9
2	1500	2	150	6mm Square	Parallel to longer side	17/2/2015	3.3
3	1500	2	150	6mm Square	Parallel to longer side	17/2/2015	3.1
4	1500	2	150	6mm Square	Parallel to longer side	18/2/2015	3.0
5	1500	2	150	6mm Square	Parallel to longer side	18/2/2015	3.1
6	1500	2	150	6mm Square	Parallel to longer side	18/2/2015	3.0

Table 8.2.3b-Experimental results for 10 storey model with 3x1 bays-(vibration parallel to longer side)-below

Data No.	Model Height (mm)	No. of Bays	Length of each member (mm)	Member type and size	Direction of vibration	Date	Measured Frequency
1	1500	3	150	6mm Square	Parallel to longer side	17/2/2015	3.4
2	1500	3	150	6mm Square	Parallel to longer side	17/2/2015	3.3
3	1500	3	150	6mm Square	Parallel to longer side	17/2/2015	3.5
4	1500	3	150	6mm Square	Parallel to longer side	18/2/2015	3.5
5	1500	3	150	6mm Square	Parallel to longer side	18/2/2015	3.4
6	1500	3	150	6mm Square	Parallel to longer side	18/2/2015	3.5

Table 8.2.3c-Experimental Frequencies for 10storey model with 4 bays-(vibration parallel to longer side)

Data No.	Model Height (mm)	No. of Bays	Length of each member (mm)	Member type and size	Direction of vibration	Date	Measured Frequency
1	1500	4	150	6mm Square	Parallel to longer side	17/2/2015	4.5

2	1500	4	150	6mm Square	Parallel to longer side	17/2/2015	4.6
3	1500	4	150	6mm Square	Parallel to longer side	18/2/2015	4.4
4	1500	4	150	6mm Square	Parallel to longer side	18/2/2015	4.6
5	1500	4	150	6mm Square	Parallel to longer side	19/2/2015	4.5
6	1500	4	150	6mm Square	Parallel to longer side	19/2/2015	4.5

D. Experimental Data for increasing number of Bays Perpendicular to motion

Table 8.2.4a- Experimental Frequencies for 10storey model with 2 bays-Less stiff direction motion(below)

Data No.	Model Height (mm)	No. of bays	Length of each Member (mm)	Member Type & Size	Direction of Vibration	Date	Measured Frequency (Hertz)
1	1500	2	150	6mm square	Perpendicular to longer side	17/2/2015	2.5
2	1500	2	150	6mm square	Perpendicular to longer side	17/2/2015	2.5
3	1500	2	150	6mm square	Perpendicular to longer side	17/2/2015	2.2
4	1500	2	150	6mm square	Perpendicular to longer side	17/2/2015	2.2
5	1500	2	150	6mm square	Perpendicular to longer side	19/2/2015	2.2
6	1500	2	150	6mm square	Perpendicular to longer side	19/2/2015	2.2

Table 8.2.4b- Experimental Frequencies for 10storey model with 3 bays-Less stiffdirection motion (below)

Data No.	Model Height (mm)	No. of Bays	Length of each Member (mm)	Member type and size	Direction of Vibration	Date	Measured Frequency (Hertz)
1	1500	3	150	6mm square	Perpendicular to longer side	17/2/2015	2.5
2	1500	3	150	6mm square	Perpendicular to longer side	17/2/2015	2.5

3	1500	3	150	6mm square	Perpendicular to longer side	17/2/2015	2.5
4	1500	3	150	6mm square	Perpendicular to longer side	17/2/2015	2.5
5	1500	3	150	6mm square	Perpendicular to longer side	18/2/2015	2.6
6	1500	3	150	6mm square	Perpendicular to longer side	18/2/2015	2.5

Table 8.2.4c-1500mm Miniatures Structure with 4x1 bays-Vibration in Less Stiff Direction

Data No.	Model Height (mm)	No. of Bays	Length of each member	Member Type and size	Direction of Vibration	Date	Measured Frequency
1	1500	4	150	6mm square	Perpendicular to longer side	19/2/2015	0.9
2	1500	4	150	6mm square	Perpendicular to longer side	19/2/2015	0.9
3	1500	4	150	6mm square	Perpendicular to longer side	18/2/2015	0.9
4	1500	4	150	6mm square	Perpendicular to longer side	18/2/2015	0.9
5	1500	4	150	6mm square	Perpendicular to longer side	18/2/2015	0.8
6	1500	4	150	6mm square	Perpendicular to longer side	18/2/2015	0.8

E.Experimental Data for Varying Column Stiffnesses

The experimental Results for miniature structures with varying column stiffnesses are presented in the tables below:

Table 8.2.5a Experimental Results for varying column stiffenesses-15 storey 100mm floor height

Data No.	Model Height (mm)	Number of floor	Floor to Floor Height (mm)	Member type and size	Date	Measured Frequency
1	1500	15	100	6mm Square	6/2/2015	2
2	1500	15	100	6mm square	6/2/2015	1.9
3	1500	15	100	6mm Square	7/2/2015	1.9
4	1500	15	100	6mm Square	7/2/2015	1.8
5	1500	15	100	6mm Square	9/2/2015	2.2
6	1500	15	100	6mm Square	9/2/2015	2.1

Table 8.2.5b-Experimental results for varying stiffness of columns 20 storey 75mm floor height

Data No.	Model Height (mm)	Number of Floors	Floor to Floor Height (mm)	Member type and size	Date	Measured Frequency
1	1500	20	75	6mm Square	6/2/2015	2.8
2	1500	20	75	6mm Square	6/2/2015	2.9
3	1500	20	75	6mm Square	7/2/2015	2.9
4	1500	20	75	6mm Square	7/2/2015	2.9
5	1500	20	100	6mm Square	9/2/2015	3.1
6	1500	20	100	6mm Square	9/2/2015	3.0

Table 8.2.5c-Experimental Results for varying column height 26 storey 57.7mm floor height case

Data No.	Model Height (mm)	Number of Floors	Floor to Floor Height (mm)	Member type and size	Date	Measured Frequency
1	1500	26	57.7	6mm Square	6/2/2015	4.4
2	1500	26	57.7	6mm Square	6/2/2015	4.9
3	1500	26	57.7	6mm Square	7/2/2015	5.0
4	1500	26	57.7	6mm Square	7/2/2015	4.8
5	1500	26	57.7	6mm Square	9/2/2015	5.1
6	1500	26	57.7	6mm Square	9/2/2015	4.9

F. Experimental Data for Varying horizontal member Stiffnesses parallel to Direction of Motion

Table 8.2.6a-Experimental finding where horizontal member parallel to motion was 100mm long and the rest were 150mm

Data No.	Model Height (mm)	No. of Floors	Floor to Floor Height (mm)	Member type and size	Length of Member Parallel to Motion (mm)	Date	Measured Frequency (Hertz)
1	1200	8	150	6mm Square	100	5/5/2015	1.7
2	1200	8	150	6mm Square	100	5/5/2015	1.8
3	1200	8	150	6mm Square	100	6/5/2015	1.8
4	1200	8	150	6mm Square	100	6/5/2015	1.7
5	1200	8	150	6mm Square	100	7/5/2015	1.7
6	1200	8	150	6mm Square	100	7/5/2015	1.7

Table 8.2.6b-Experimental finding where horizontal member parallel to motion was 200mm long and the rest were 150mm

Data No.	Model Height (mm)	No. of Floors	Floor to Floor Height (mm)	Member type and size	Length of Member Parallel to Motion (mm)	Date	Measured Frequency (Hertz)
1	1200	8	150	6mm Square	200	5/5/2015	2.1
2	1200	8	150	6mm Square	200	5/5/2015	2.0
3	1200	8	150	6mm Square	200	6/5/2015	2.2
4	1200	8	150	6mm Square	200	6/5/2015	2.0
5	1200	8	150	6mm Square	200	7/5/2015	2.0
6	1200	8	150	6mm Square	200	7/5/2015	2.0

Table 8.2.6c-Experimental finding where horizontal member parallel to motion was 250mm long and the rest were 150mm

Data No.	Model Height (mm)	Number of Floors	Floor to Floor Height (mm)	Member type and size	Length of Member Parallel to Motion (mm)	Date	Measured Frequency (Hertz)
1	1200	8	150	6mm Square	250	5/5/2015	4.1
2	1200	8	150	6mm Square	250	5/5/2015	4.0
3	1200	8	150	6mm Square	250	6/5/2015	4.2

4	1200	8	150	6mm Square	250	6/5/2015	4.0
5	1200	8	150	6mm Square	250	7/5/2015	4.1
6	1200	8	150	6mm Square	250	7/5/2015	4.1

G-Experimental Data for horizontal Momber Stiffnesses Perpendicular to Motion

Table 8.2.7a-Experimental finding where horizontal member perpendicular to motion was 100mm long and the rest were 150mm

Data No.	Model Height (mm)	Number of Floor	Floor to Floor Height (mm)	Member type and size	Length of Member Perpendicular To Motion (mm)	Date	Measured Frequency -Hertz
1	1200	8	150	6mm Square	100	5/5/2015	2.0
2	1200	8	150	6mm Square	100	5/5/2015	2.2
3	1200	8	150	6mm Square	100	6/5/2015	2.1
4	1200	8	150	6mm Square	100	6/5/2015	2.3
5	1200	8	150	6mm Square	100	7/5/2015	2.3
6	1200	8	150	6mm Square	100	7/5/2015	2.1

Table 8.2.7b-Experimental finding where horizontal member perpendicular to motion was 200mm long and the rest were 150mm

Data No.	Model Height (mm)	Number of Floors	Floor to Floor Height (mm)	Member type and size	Length of Member Perpendicular to Motion (mm)	Date	Measured Frequency (Hertz)
1	1200	8	150	6mm Square	200	5/5/2015	1.5
2	1200	8	150	6mm Square	200	5/5/2015	1.4
3	1200	8	150	6mm Square	200	6/5/2015	1.5
4	1200	8	150	6mm Square	200	6/5/2015	1.5
5	1200	8	150	6mm Square	200	7/5/2015	1.4
6	1200	8	150	6mm Square	200	7/5/2015	1.5

Table 8.2.7c-Experimental finding where horizontal member perpendicular to motion was 250mm long and the rest were 150mm

Data	Model	Number	Floor to	Member	Length of Member	Date	Measured

No.	Height (mm)	of Floors	Floor Height (mm)	type and size	Perpendicular to Motion (mm)		Frequency (Hertz)
1	1200	8	150	6mm Square	250	5/5/2015	1.4
2	1200	8	150	6mm Square	250	5/5/2015	1.3
3	1200	8	150	6mm Square	250	6/5/2015	1.4
4	1200	8	150	6mm Square	250	6/5/2015	1.3
5	1200	8	150	6mm Square	250	7/5/2015	1.4
6	1200	8	150	6mm Square	250	7/5/2015	1.3

G. Experimental Data where Scale Factor Changes for 1 bay 6 storey miniature structures

Table 8.2.8a-Experimental Results for 1 bay models of height 900mm (unbraced)

Data no.	Model Height (mm)	Number of Bays	Length of each member (mm)	Member type and size	Date	Measured Frequency
1	900	1	150	6mm Square	5/7/2014	2.5
2	900	1	150	6mm Square	5/7/2014	2.4
3	900	1	150	6mm Square	5/4/2014	2.5
4	900	1	150	6mm Square	5/4/2014	2.5
5	900	1	150	6mm Square	5/4/2014	2.4
6	900	1	150	6mm Square	5/4/2014	2.4

Table 8.2.8b-Results on miniature structure with 6storeys and member length 200mm

Length of Members (mm)	Number of storeys	Date of test	Frequency
200	6	6/5/2015	2.2
200	6	6/5/2015	2.1
200	6	7/5/2015	2.2
200	6	7/5/2015	2.1
200	6	8/5/2015	2.1
200	6	8/5/2015	2.2

Table 8.2.8c-Results on miniature structure with 6storeys and member length 250mm

Length of Members (mm)	Number of storeys	Date of test	Frequency
250	6	6/5/2015	1.1
250	6	6/5/2015	1.0
250	6	7/5/2015	1.2
250	6	7/5/2015	1.1
250	6	8/5/2015	1.1
250	6	8/5/2015	1.2

Table 8.2.8d-Results on miniature structure with 6storeys and member length 300mm

Length of Members (mm)	Number of Storeys	Date of test	Frequency
300	6	6/5/2015	0.8
300	6	6/5/2015	0.7
300	6	7/5/2015	0.7
300	6	7/5/2015	0.8
300	6	8/5/2015	0.7
300	6	8/5/2015	0.7