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MODELING MONTHLY INFLATION RATE VOLATILITY IN KENYA USING ARCH TYPE FAMILY MODELS

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DECLARATION

I declare that this research project is my own original work and that to the best of my knowledge has not been submitted before to any other institution for examination:

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DEDICATION

This Research Project is dedicated to my wife Faith Moraa and my daughter Joy Adrienne for the support to the completion of the project.

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LIST OF ABBREVIATIONS

ACF	-Autocorrelation Function
AIC	-Akaike Information Criterion
AR	- Autoregressive
ARCH	-Autoregressive Conditional Heteroscedasticity
BIC	-Bayesian Information Criterion
CBK	-Central Bank of Kenya
CEE	- Central and Eastern Europe
GARCH	-Generalized Autoregressive Conditional Heteroscedasticity
HQIC	-Hannan-Quinn Information Criterion
KNBS	-Kenya National Bureau of Statistics
PACF	-Partial Autocorrelation Function
SIC	-Shwartz Information Criterion
UCL	- Upper Class Limit
LCL	- Lower Class Limit

LIST OF SYMBOLS

ε_t	-Error
y_t	-Time series observation
y_t^2	-Squared observation
ψ_t	-Set of observation
r_t	-Return
μ	-Constant part of return
$\alpha_0, \alpha_1, \beta$	-Model parameters
z_t	-Identically and Independent Standard Normal Random Variable
σ_t	-Standard deviation
σ_t^2	-Conditional variance
σ	-volatility estimate
b_i	-Volatility coefficient
I_i	-Inflation rate

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ABSTRACT

The purpose of this study describes financial time series modelling with special application to modelling inflation data for Kenya. Specifically the theory of time series is modelled and applied to the inflation data spanning from January 1985 to April 2016 obtained from the Kenya National Bureau of Statistics. The arch type family models were fitted and forecast to the data because data was characterized by variation in variance and mean. The outcome of the study revealed that the ARCH –family type models, particularly, the EGARCH (1, 1) with generalized error distribution (GED) was the best in modelling and forecasting Kenya's monthly rates of inflation. The study recommends that governments, policy makers interested in modelling and forecasting monthly rates of inflation should take into consideration Heteroscedastic models since it captures the volatilities in the monthly rates of inflation.

CHAPTER ONE

INTRODUCTION OF THE STUDY

1.0 Background of the Study

In recent years, rising inflation has become one of the major economic challenges facing most countries in the world especially developing countries like Kenya. The rise in the prices in an economy is referred to as inflation (Webster, 2000).

When there is an increase in inflation the effects of the citizens vary as some people will lose confidence with state of the currency since the currency depreciates. It leads to demand for high wages in the economy and for companies to overcome the wage increases they will increase the prices of goods and services so as to continue making profits in offering their services.

According to Schotman and Sweitzer(2000) they argued that rising inflation is a major cause of fear to investors since it reduces return for every investment.. Stable inflation plays a significant role to attract foreign investors since they are no fears for a decrease in the return of their investments (Suleman and Sarpong, 2012).

Bailey (1956) furthermore stated that the costs associated with unanticipated inflation are the distributive effects from creditors to debtors, increasing uncertainty affecting consumption, savings, and borrowing and investment decisions.

The maintenance of price stability is one of the macroeconomic challenges that the Kenyan government has been facing since its independence which is now 54 years ago. Inflation Rate in Kenya averaged 10.44 percent from 2005 until 2016, reaching an all-time high of 45.98% in 1993, 31.50% in May of 2008 and a record low of -0.1 in 1964 (KNBS, 2016).

Having achieved single digit inflation, Kenya will need to consider how best to manage monetary policy in a low moderate inflation environment. However, in the last few months in 2016, the country can be seen to be wining the fight against inflation as inflation has been kept at single digits. This creates a gap in the field of the research so that the pattern of inflation rate can be known by the investors, government and economists so as to plan well in budgeting.

1.2 Inflation rate volatility

According to Abdalla (2011), volatility refers to the spread of all unlikely outcomes of an uncertain variable. Inflation rate fluctuations have received much attention because it has an influence on the economy, international trade, investment analysis, profitability and risk management among others. For most financial time series, Bollerslev's GARCH (1, 1) model has been found to be sufficient for modelling inflation rate volatility.

Historical volatility is the realized volatility over a given time period and can be obtained whereby, with a series of past inflation rates, the standard deviation of the daily price changes can be calculated and these can then be extended to annual volatility. A financial time series,say (Y_t), is often a serially uncorrelated sequence with zero mean, even as it exhibits volatility clustering, suggesting that the conditional variance of (Y_t) given past returns is not constant.

According to Kamal et al (2012), many financial crises such as those of Latin America, Southeast Asia and Russian economies stemmed from sudden and unexpected oscillation of inflation rate thus highlighting the importance of measurement of inflation rate volatility, it's forecasting and behavior. Inflation rate movements' impact on volume and value of foreign trade and investment in that inflation rate volatility tends to affect imports and exports which further influence a country's balance of payments.

Over the last few years, modelling volatility of a financial time series has become an important area and has gained a great deal of attention from academics, researchers and others.

The Arch type models like GARCH cannot account for leverage effect, however they account for volatility clustering and leptokurtosis in a series, this necessitated to develop new and extended models over GARCH that resulted into new models GARCH-M, EGARCH, TGARCH and PGARCH.

1.3 Statement of the Problem

Due to the fact that inflation levels affect all other sectors of the economy especially business transactions, it is important to be able to forecast or estimate the value of inflation in the future so that such values are incorporated in decisions affecting all these other sectors. Inflation rate volatility affects policy makers as well as investors hence the need to study volatility pattern which can aid in financial decision making.

The CBK's primary objective is to formulate and implement monetary policies. Depending on the direction a government wants its economy to make, they may create monetary policies that either appreciate or depreciate their currency. Policy makers essentially rely on volatility estimations so as to enable them make decisions on what direction the currency should take. This indicates a gap in literature or information on the relative performance of these models in the context of developing countries and poses a challenge as to which of these models is the optimal choice for modelling and forecasting economic and financial data (in particular inflation rates) for developing countries.

This study therefore intends to model monthly inflation in Kenya using the ARCH-type models and to choose the most appropriate model suitable for inflation modelling and forecasting in Kenya.

1.4 Objectives of Study

1.4.1 General Objective

The main objective of the study was to model the monthly inflation rate volatility using ARCH type family models.

1.4.2 Specific Objectives

The specific objectives of the study were:

i.) Fitting the ARCH type model to the inflation returns.

ii.) Analyzing the adequacy of the fitted ARCH type family models.

iii.) Generation of a ten year forecast on the inflation rate returns

1.5 Significance of Study

It will facilitate investigation of inflation rate risk which can be an indicator of vulnerability in the economy. This will enable the government manage the volatility, at least in the short run as well as provide reliable models for policy makers to help them anticipate possible vulnerabilities of financial markets and the economy and to analyze and forecast volatility that can guide the central bank to intervene in the market when the need arises. Further, it will bring about an understanding of the behavior of inflation rates as well as an attempt to explain the sources of these movements as well as fluctuations.

They can use this information to make decisions on future investments from the observed patterns of inflation rate volatility considering that investors' confidence to invest in a particular country is inversely related to high volatilities in inflation rates. The Government will also be able to manage the inflation rate volatility at least in the short run. Kenya as a developing country would benefit greatly from this kind of research which hasn't been explored much in spite of the fact that inflation rate fluctuations have a huge contribution to macro - economic variables like interest rate and exchange rate.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

Dynamic nature of inflation behavior is an accepted phenomenon and all participants in the financial markets include regulators, professionals and academics have consensus about it. The answer to this question, because of the great number of involved variable, is not an easy task and up to now there is no consensus about it. However researchers in quest of answer to this question have investigated the inflation volatility from different angles. Therefore this chapter discusses various research done related to modelling inflation rate volatility.

2.1 Review of Previous Studies

Engle (1982) noted that for conventional econometric models, the conditional variance did not depend upon its own past. He thus proposed the ARCH model which was able to capture the idea that today's variance does depend on its past as well as the non-constant nature of the one period forecast variance. He also found that the parameter had to satisfy the non-negativity constraints as well as some stationary conditions.

Both the ARCH and GARCH models of Engle (1982) and Bollerslev (1986) could not tell how the variance of return was influenced differently by positive and negative news, Nelson (1991) suggested the EGARCH which worked better.

Ngailo and Massawe (2014) used monthly inflation data observations from Tanzania and considered the GARCH approach in modelling inflation rates for eleven years from 2000 to 2011. After performing all the diagnostic checks on Jarque bera test on kurtosis and the stationarity using Augmented ducker fuller test. They found out that the inflation returns volatility works better with the class of GARCH(1,1).

In Ghana, Oteng-Abayie and Doe (2013) concluded that inflation uncertainty in any economy raises inflation. They used GARCH models in investigating the connection between inflation and the uncertainty of inflation for 23 years from 1984- 2011.

Jere and Siyanga (2016) did a study in the inflation rate volatility in Zambia for five years from 2010 to 2014 and also did a forecast for one year using exponential smoothing and forecasted using ARIMA. The ARIMA ((12), 1, 0) was the best fit for forecasting.

Faisal (2012) examine the volatility of inflation rate in Bangladesh using time series GARCH model. He used monthly inflation rates spanning the period 1990-2011. According to them, the main objective of an inflation rate policy is to determine an appropriate inflation rate and ensure its stability and over the years, efforts put by the Government to achieve this have not yielded positive results. He thus sought to build a forecasting model that would adequately capture the volatility of inflation rate return series using GARCH model and the outcome of his research was to assist the government to manage the exposure of the inflation rate volatility in the short run, inform investors on future behavior of inflation rates thus helping them in decision making and help end users of volatility models such as importers, exporters, etc.

The effects of good and bad news on volatility in the Indian stock markets using asymmetric ARCH models during the global financial crises of 2008-2009 was investigated by Goudarzi and Ramanaraynan (2011). The asymmetric volatility models considered were the EGARCH and TGARCH models and the BSE 500 stock index was used as a proxy to the Indian stock market. The study found out that the BSE 500 return series reacted to good news and bad news asymmetrically. The EGARCH (1,1) and TGARCH (1,1) models were estimated for the BSE 500 stock returns series using the robust method of Bollerslev-Wooldridge's quasi-maximum likelihood estimation (QMLE) assuming the Gaussian standard normal distribution.

Awogbemi and Oluwaseyi (2011) results showed that ARCH and GARCH models are better models because they give lower values of AIC and BIC as compared to the conventional Box and Jenkins ARMA models for inflation in Nigeria. The researchers also observed that since volatility seems to persist in all the commodity items, people who expect a rise in the rate of inflation (the 'bullish crowd') will be highly favored in the market of the said commodity items.
Ocran (2007) on stylized facts about Ghana's inflation experience indicated that since Ghana's exit from the West African Currency Board soon after independence, inflation management has been ineffective despite two decades of vigorous reforms.

Jiang (2011) believed that it was worthy to investigate the inflation and inflation uncertainty relationship in China as it is commonly believed that one possible channel that inflation imposes significant economic costs is through its effect on inflation uncertainty. He addressed the relationship of inflation and its uncertainty in China's urban and rural areas separately given the huge urban-rural gaps In conclusion he said that EGARCH(1,1) was the best in studying the inflation arte volatility in China.

CHAPTER THREE

RESEARCH METHODOLOGY

3.0 Introduction

It introduces the non-stationary models used in measuring volatility.

$$r_t = \log \frac{I_t}{I_{t-1}} \tag{3.0}$$

Where:

r_t - Inflation return rate.

I_t - Current inflation rate

I_{t-1}- Previous inflation rate.

3.1 Financial Time series Models

The stylized characteristics of financial time series data include: almost zero correlation, absolute/squared data exhibit high correlation and excess kurtosis/ heavy tailed distribution.

3.2: The ARCH Model

Suppose $Y_1, Y_2, ..., Y_t$ are the time series observations and let ψ_t be the set of y_t up to time t, including y_t for $t \leq 0$. The process $\{y_t\}$ is an Autoregressive Conditional Heteroscedastic process of order p, ARCH(p), if:

$$r_t = \mu + y_t$$

$$y_t = \sigma_t \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2) \tag{3.1}$$

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j y_{t-j}^2 \tag{3.2}$$

With $\alpha_0 \geq 0$, $\alpha_j \geq 0$ and $\sum_{j=1}^{p} \alpha_j < 1$, as the ARCH model parameter limits. The conditions stated guarantees that the conditional variance be positive

Properties of ARCH(p) Model

The Mean;

From equation 3.1, the conditional expectation and variance of x_t is:

$E(y_t) = 0$ since the expectation of ε_t is 0.

The Second Moment or Variance;

$$E(y_t^2) = E(\sigma_t^2 \varepsilon_t^2) = E(\sigma_t^2) \tag{3.3}$$

Since $\sigma^2 = 1$ following a standard normal distribution of ε_t.

$$E(\sigma_t^2) = \alpha_0 + \alpha_j \sum_{j=1}^{p} E(y_{t-1}^2)$$

Given $E(\sigma_t^2) = E(y_{t-1}^2)$ under stationarity assumption,

$$E(\sigma_t^2) = \frac{\alpha_0}{1 - \sum_{j=1}^{p} \alpha_j} \tag{3.4}$$

For ARCH(1), the variance is given by;

$$E(\sigma_t^2) = \frac{\alpha_0}{1-\alpha_1} \tag{3.5}$$

The Kurtosis;

First, the forth moment of the time series is obtained,

$$E(y_t^4) = E\{(\sigma_t^2)^2 \varepsilon_t^4\} = E\{(\sigma_t^2)^2\}E(\varepsilon_t^4)$$

$$= 3E\{(\sigma_t^2)^2\} \tag{3.6}$$

$$E\{(\sigma_t^2)^2\} = E\{(\alpha_0 + \sum_{j=1}^{p} \alpha_j y_{t-1}^2)^2\}$$

$$= \alpha_0^2 + 2\alpha_0 \sum_{j=1}^{p} \alpha_j E(y_{t-1}^2) + \sum_{j=1}^{p} \alpha_j^2 E(y_{t-1}^4)$$

Substituting equation (3.6), we have;

$$E(y_t^4) = 3\{\alpha_0^2 + 2\alpha_0 \sum_{j=1}^{p} \alpha_j E(y_{t-1}^2) + \sum_{j=1}^{p} \alpha_j^2 E(y_{t-1}^4)\}$$

Under stationarity, $E(y_{t-1}^2) = E(\sigma_t^2) = \dfrac{\alpha_0}{1-\sum_{j=1}^{p} \alpha_j}$ and $E(y_t^4) = E(y_{t-1}^4)$

$$E(y_t^4) = 3\frac{\alpha_0^2 (1 + \sum_{j=1}^{p} \alpha_j)}{(1 - \sum_{j=1}^{p} \alpha_j)(1 - 3\sum_{j=1}^{p} \alpha_j^2)} \tag{3.7}$$

The Kurtosis is given by;

$$\text{K}(y) = \frac{E(y_t^4)}{\{E(y_t^2)\}^2}$$

Substituting equations (3.5) and (3.7), we get;

$$\text{K(y)} = 3\frac{(1+\sum_{j=1}^{p}\alpha_j)(1-\sum_{j=1}^{p}\alpha_j)}{1-3\sum_{j=1}^{p}\alpha_j^2} \quad (3.8)$$

Therefore, the kurtosis is

$$K = 3\frac{1-\sum_{j=1}^{p}\alpha_j^2}{1-3\sum_{j=1}^{p}\alpha_j^2} \quad (3.9)$$

When j=1, we get ARCH(1), then the Kurtosis of ARCH(1) is;

$$K = 3\frac{1-\alpha_1^2}{1-3\alpha_1^2} \quad (3.10)$$

Which is strictly greater than 3 unless $\alpha_1 = 0$. The kurtosis for a normally distributed random variable Z is 3. Thus, the kurtosis of y_t is greater than the kurtosis of a normal distribution, and the distribution of y_t has a heavier tail than the normal distribution, when $\alpha_1 > 1$.

3.1.1: Fitting Procedure for ARCH model

There are two steps in model fitting:

Step1: Plotting the return series and analyzing the autocorrelation function (ACF) and the partial autocorrelation function (PACF).

.We check for correlation in the return series by performing the autocorrelation function to compute and display the sample ACF of the returns and by plotting the partial correlation functions.

The ACF definition (Auto correlation function)

A time series $\{X_t\}$ has mean function $\mu_{t=E\ [X_t]}$

The auto correlation function (ACF) is

$$\rho x(h) = \frac{\gamma x(h)}{\gamma x(0)} = corr(X_{t+h,}X_t)$$

PACF Definition (Partial Autocorrelation Function) of the k-th order is defined as:

- $P_{kk} = \varphi_{kk} = corr\left(X_t - P(X_t | X_{t+1,\dots,}X_{t+k-1}), X_{t+k} - P\left(X_{t+k} | X_{t+1,\dots,}X_{t+k-1}\right)\right)$

Step2: Performing preliminary tests, such as ARCH effect test or the Q-test.

We can quantify the preceding qualitative checks for correlation using formal hypothesis checks, like Ljung-Box-Pierce Q-test and Engle's ARCH test. By performing a Ljung-Box-Pierce Q-test, we can verify, at least approximately, the presence of any significant correlation in the returns when tested for up to 20 lags of the ACF at the 0.05 level of significance.

Weakness of ARCH model

Despite ARCH model able to capture the characteristics of financial time series data, it has some weaknesses that may make GARCH model better. These weaknesses include; ARCH treats positive and negative returns in the same way (by past square returns), it is very restrictive in parameters, it does not provide any new insight for understanding financial time series, it often over-predicts the volatility, because it respond slowly to large shocks and volatility from it persists for relatively short amount of times unless p is large.

3.3: The GARCH Model

Although the ARCH model has a basic form, one of its characteristics is that it requires many parameters to de scribe appropriately the volatility process of an asset return. Thus, alternative models must be further searched, one of them being the one developed by Bollerslev (1986) who proposed a useful extension known as the generalized ARCH.

As against the ARCH model, the Generalized Autoregressive Centralized Heteroskedastic Model (GARCH) has only three parameters that allow for an infinite number of squared roots to influence the current conditional variance. This feature allows GARCH to be more parsimonious than ARCH model, which feature explains the wide preference for use in practice, as against ARCH.

While ARCH incorporates the feature of autocorrelation observed in return volatility of most financial assets, GARCH improves ARCH by adding a more general feature of conditional heteroscedasticity. Simple models - low values of parameters p and q in GARCH (p,q) - are frequently used for modeling the volatility of financial returns; these models generate good estimates with few parameters.

The process Y_t is a GARCH (p,q) if:

$$r_t = \mu + y_t$$

$$y_t = \sigma_t \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \ldots\ldots + \alpha_p y_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots\ldots + \beta_q \sigma_{t-q}^2$$

$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (3.11)$$

Where q>0, $p \geq o, \alpha_0 > 0, \alpha_i \geq 0$ for i=1,2,…….,p, $\beta_j \geq 0$ for j=1,……..q are the parameter limits. Again these conditions are needed to guarantee that the conditional variance $\sigma_t^2 > 0$.

Properties of GARCH (p,q)

The mean;

From equation (3.1), the conditional expectation and variance of x_t is:

$E(y_t) = 0$, since the expectation of ε_t is 0.

The Second Moment or Variance;

$$E(y_t^2) = E(\sigma_t^2 \varepsilon_t^2) = E(\sigma_t^2) \qquad (3.12)$$

$$E(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i E(y_{t-i}^2) + \sum_{j=1}^{q} \beta_j E(\sigma_{t-j}^2) \qquad (3.13)$$

Given $E(\sigma_t^2) = E(y_{t-1}^2) = E(\sigma_{t-j}^2)$ under stationarity assumption,

$$E(\sigma_t^2) = \frac{\alpha_0}{1 - (\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j)} \qquad (3.14)$$

For GARCH (1, 1)

$$E(\sigma_t^2) = \frac{\alpha_0}{1-(\alpha_1+\beta_1)} \quad (3.15)$$

The Kurtosis;

First the forth moment of the time series is obtained;

$$E(y_t^4) = E\{(\sigma_t^2)^2 \varepsilon_t^4\} = E\{(\sigma_t^2)^2\}E(\varepsilon_t^4) = 3E\{(\sigma_t^2)^2\}$$

But

$$E\{(\sigma_t^2)^2\} = E\{(\alpha_0 + \sum_{i=1}^{p}\alpha_i y_{t-i}^2 + \sum_{j=1}^{q}\beta_j \sigma_{t-j}^2)^2\}$$

$$= \alpha_0^2 + 2\alpha_0 \sum_{i=1}^{p}\alpha_i E(y_{t-i}^2) + 2\alpha_0 \sum_{j-1}^{q}\beta_j E(\sigma_{t-j}^2) + \sum_{i=1}^{p}\alpha_i^2 E(y_{t-i}^4) + \sum_{j=1}^{q}\beta_j^2 E[(\sigma_{t-j}^2)^2] + 2\sum_{i=1}^{p}\sum_{j=1}^{q}\alpha_i\beta_j E(y_{t-i}^2\sigma_{t-j}^2)$$

When i = j = 1, we get GARCH (1, 1)

$$E\{(\sigma_t^2)^2\} = \alpha_0^2 + \alpha_1^2 E(y_{t-1}^4) + \beta_1^2 E\{(\sigma_{t-1}^2)^2\} + 2\alpha_1\beta_1 E(y_{t-1}^2\sigma_{t-1}^2) + 2\alpha_0\alpha_1 E(y_{t-1}^2) + 2\alpha_0\beta_1 E(\sigma_{t-1}^2)$$

$$= \alpha_0^2 + (3\alpha_1^2 + 2\alpha_1\beta_1 + \beta_1^2)E\{(\sigma_{t-1}^2)^2\} + 2\alpha_0(\alpha_1+\beta_1)E(\sigma_{t-1}^2) \quad (3.16)$$

Assuming the process is stationary, $E\{(\sigma_t^2)^2\} = E\{(\sigma_{t-1}^2)^2\}$

Hence

$$E\{(\sigma_t^2)^2\} = \frac{\alpha_0^2 + 2\alpha_0(\alpha_1+\beta_1)E(\sigma_{t-1}^2)}{1-3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2}$$

$$= \frac{\alpha_0^2 + 2\alpha_0^2(\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)(1-3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2)}$$

$$E(y_t^4) = 3E\{(\sigma_t^2)^2\}$$

$$K \quad = 3\frac{\alpha_0^2 + 2\alpha_0^2(\alpha_1 + \beta_1)}{(1-\alpha_1-\beta_1)(1-3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2)} \tag{3.17}$$

The Kurtosis is given by;

$$K \quad = \frac{E(y_t^4)}{\{E(y_t^2)\}^2}$$

Substituting equation (3.15) and equation (3.17), we get;

$$K \quad = 3\frac{1-(\alpha_1+\beta_1)^2}{1-3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2} \tag{3.18}$$

Which is strictly greater than 3 unless $\alpha_1 = 0$

The same fitting procedure is applicable for a general GARCH (p,q).

3.3 GARCH Model Extensions

There are some aspects of the model which can be improved so that it can better capture the characteristics and dynamics of a particular time series in leverage effects, volatility clustering and leptokurtosis are commonly observed in financial time series.

3.3.1 Symmetric GARCH

This property is seldom(not often) in accordance with empirical results where a leverage effect often is present, i.e., volatility increases more after negative return shocks than after positive return shocks of the same magnitude (“bad news” generates higher volatility more than “good news” lowers the volatility).

3.3.2 Asymmetric GARCH Measurement

While ARCH/GARCH models provide a venue for modeling conditional heteroscedastic volatility with a Normal or Non-normal error distributions, these models assume that positive and negative shocks have the same effect on volatility because it depends on the square of previous shocks.

In practice, the price of financial assets often reacts more pronouncedly to "bad" news than "good" news. Such a phenomenon leads to a so called *leverage effect*, as first noted by Brooks (2008). The term "leverage" stems from the empirical observation that the volatility (conditional variance) of a stock tends to increase when its returns are negative.

3.4 The E-GARCH

To overcome some weaknesses of the GARCH model in handling financial time series Nelson(1991) introduced EGARCH.

Another way of making σ_t^2 non-negative is by making ln(σ_t^2 linear in some function of time and lagged $_t$'s. This formulation leads to the asymmetric GARCH model, Exponential GARCH of Nelson(1991):

Exponential GARCH (EGARCH) proposed by Nelson (1991) gives a leverage effects and asymmetry in its equation. In the EGARCH model the specification for the conditional covariance is given by the following form:

$$\ln\left(\delta^2{}_t\right) = \alpha_0 + \sum_{j=1}^{q} \beta_j \ln\left(\delta_{t-j}\right) + \sum_{i=1}^{p} \alpha_i \left| \frac{u_{t-i}}{\sqrt{\delta_{t-i}}} \right| + \sum_{k=1}^{r} \gamma_k \frac{u_{t-k}}{\sqrt{\delta_{t-k}}} \qquad (3.19)$$

Two advantages stated in Brooks (2008) for the pure GARCH specification; by using $\ln\left(\delta^2{}_t\right)$ even if the parameters are negative, will be positive and asymmetries are allowed for under the EGARCH formulation.

Where:

γ_k -leverage effects

If $\gamma_k < 0$ - leverage effect exist

$\gamma_k \neq 0$ asymmetric impact.

Nelson notes, "to accommodate the asymmetric relation between stock returns and volatility changes, the value of $g(z_t)$ must be a function of both the magnitude and the sign of z_t". This leads to the following representation:

$$g(z_t) = \theta_1 z_t + \theta_2 [|z_t| - E(|z_t|)] \qquad (3.20)$$

Sign effect **Magnitude effect**

Thus, $g(z_t)$ allows the conditional varianceσ_t^2 to respond asymmetrically to rises and falls in stock price.In contrast to the GARCH models, the EGARCH models do not have any restrictions on the parameters in the model.

3.5 Glosten-Jagannathan-Runkle (GJR) model:

To capture the leverage effect, Glosten, Jaganathan, and Runkle (1993) show how to allow good news and bad news to have different effects on volatility by using y_t^2 as a threshold. A GJRGARCH(1,1) model can be generally expressed as:

$$\delta_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \gamma_i u_{t-i}^2 d_{t-1} + \sum_{i=1}^{p} \beta_j \delta_{t-j} \qquad (3.21)$$

where d_{t-1} is a dummy variable that is:

$$d_{t-1} = \begin{cases} 1 \text{ if } u_{t-1} < 0, & \text{bad news} \\ 0 \text{ if } u_{t-1} \geq 0, & \text{goodnews} \end{cases} \qquad (3.22)$$

In the model, effect of good news is shown by α_i, while bad news shows their impact by $\alpha + \gamma$. In addition if $\gamma \neq 0$ news impact is asymmetric and $\gamma > 0$ leverage effect exists.

For the satisfaction of non-negativity condition coefficients would be $\alpha_0 > 0$, $\alpha_i > 0$, $\beta \geq 0$ and $\alpha_i + \gamma_i \geq 0$. That is the model is still acceptable, even if $\gamma_i < 0$, provided that $\alpha_i + \gamma_i \geq 0$.

3.6 Forecast of Conditional Variance in GARCH model

The formula used to calculate the multi-step ahead forecasts of the conditional variance for the GARCH(1,1)model is obtained as illustrated. For such a model, the variance equation is

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (3.23)$$

Denote the forecast origin by n and the forecast horizon by h Let F_n be the information set available at time n. For $h = 1$, the 1-step ahead forecast of the conditional variance is simply

$$E(\sigma_{n+1}^2 \mid F_n) = E(\alpha_0 + \alpha_1 y_n^2 + \beta_1 \sigma_n^2 \mid F_n)$$

$$= \alpha_0 + \alpha_1 y_n^2 + \beta_1 \sigma_n^2 \qquad (3.24)$$

For $h = 2$, by using the assumption that Z_t 's are i.i.d. $N(0,1)$, we have

$$E(\sigma_{n+2}^2 \mid F_n) = E(\alpha_0 + \alpha_1 y_{n+1}^2 + \beta_1 \sigma_{n+1}^2 \mid F_n)$$

$$= \alpha_0[1 + (\alpha_1 + \beta_1)] + \alpha_1(\alpha_1 + \beta_1) y_n^2 + \beta_1(\alpha_1 + \beta_1)\sigma_n^2$$

$$= \alpha_0 + (\alpha_1 + \beta_1) * E(\sigma_{n+1}^2 \mid F_t) \qquad (3.25)$$

By the same argument, it is easily seen that for $h = j$, the j-step ahead forecast of the conditional variance of the GARCH(1,1) model is

$$E(\sigma_{n+j}^2 \mid F_n) = \alpha_0 \sum_{k=o}^{j-1} (\alpha_1 + \beta_1)^k + \alpha_1(\alpha_1 + \beta_1)^{j-1} y_n^2 + \beta_1(\alpha_1 + \beta_1)^{j-1} \sigma_n^2$$

$$= \alpha_0 + (\alpha_1 + \beta_1) * E(\sigma_{n+j-1}^2 \mid F_t) \qquad (3.26)$$

Therefore, the forecasts of the conditional variances of an GARCH (1,1) model can be computed recursively.

3.7 Conditional Error distributions

3.7.1 Normal distribution

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty$$

3.7.2 Student t distribution

When v→∞ the distribution converges to a standard normal**.**

$$f(x) = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\sqrt{\upsilon\pi\Gamma\left(\frac{\upsilon}{2}\right)}\left(1+\frac{x^2}{\upsilon}\right)^{\frac{\upsilon+1}{2}}}.$$

3.7.3 Generalized error distribution

$$f(x) = \frac{\lambda . s}{2.\Gamma\left(\frac{1}{s}\right)} . \exp(-\lambda^s . [x-\mu]^s)$$

Where:

$\lambda -$ Scale parameter

$\mu -$ location parameter

$\Gamma(\mathrm{Z}) -$ Euler Function

$\mathrm{s} -$ Shape Parameter

3.8 Model selection criteria

Selection criteria assess whether a fitted model offers an optimal balance between the goodness-of-fit and parsimony. The most comon model selection criteria such as the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Schwarz information criterion (SIC), Hannan Quinn Criterion (HQ) and Loglikelihood (LL)were used as bases for selection criteria.

- AIC = 2log (maximum likelihood) + 2k where k = p + q + 1 if the model contains an intercept or a constant term and k = p + q.
- BIC= - 2log (L) + 2(m)
- HQ=-2log(L) + 2m log (log n)
- SIC=-2log(L) + (m + m log n)

Where, n and m are number of observations (sample size) and parameter in the model respectively and log l is the ℓoglikelihood. The desirable model is one that minimizes the AIC, the BIC, the HQ, SIC and LL.

CHAPTER FOUR

DATA ANALYSIS AND RESULTS

4.0 Introduction

This section reviews the analysis and interpretation of the monthly inflation rate of Kenya from January 1985 to April 2016.

4.1 Data

Secondary data consisting of year-on-year inflation data for each month from January 1985 to April 2016 was used in this study.. www.knbs.or.ke. Inflation rate is derived from the consumer price index has shown:

$$I_t = \left(\frac{P_t}{P_{t-1}} - 1 \right) * 100\%$$

Where:

P_t - The current prices

P_{t-1} - The previous prices

The inflation rate returns is:

$$r_t = \log \frac{I_t}{I_{t-1}}$$

4.2 EXPLORATORY DATA ANALYSIS.

The time plot of monthly inflation rates is as shown:

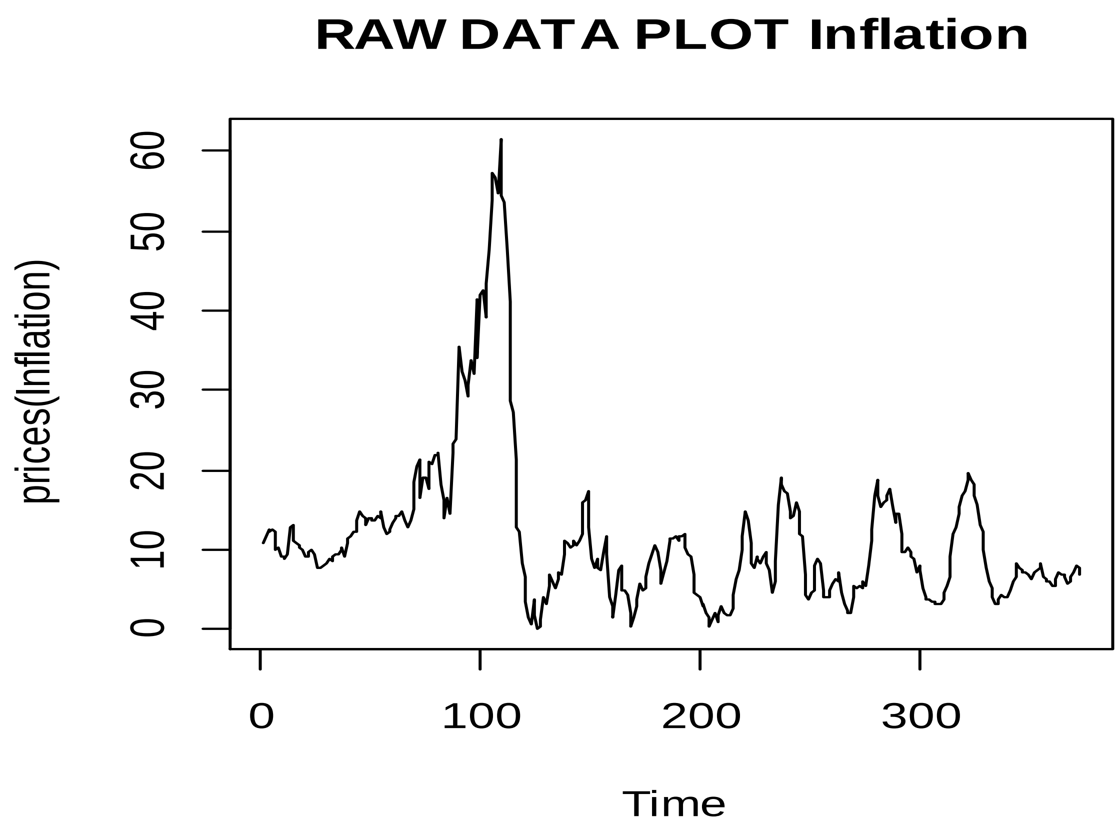

Figure 4.1: The plot of raw inflation data

The inflation rate trend can be clearly seen observed as the rate rises gradually. Clustering is observed. The volatility characteristics of financial time series data can be clearly seen from the rise drop of the inflation rate

4.3 INFLATION RATE RETURNS DATA

4.3.1 Simple and log Returns time series plot

The time plot for Log returns is as shown in figure 4.2;

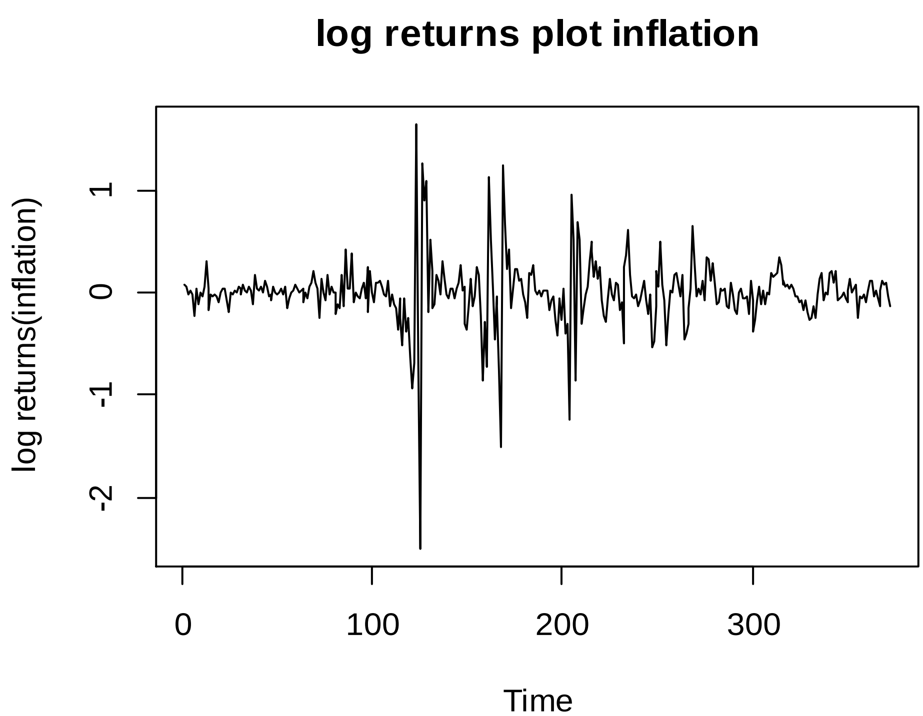

Figure 4.2 Log returns

From the log returns plots of the returns, volatility clustering can be clearly observed. (i.e where a large changes are followed by other large changes of either sign and small changes are followed by small changes). The mean reverting (returns tend to remain around a certain value) property can also be seen clearly where the returns revolve around zero.

4.3.2 Simple and Log returns Descriptive Statistics

Table 4.1 Simple and log Returns Descriptive Statistics

	Simple Returns	Log Returns
Nobs	376	376
1 Quartile	-0.08624	-0.090191
3 Quartile	0.096058	0.09172
Mean	0.049766	-0.002058
Sum	18.71214	-0.773666
SE Mean	0.02099	0.016332
LCL Mean	0.008494	-0.034171
UCL Mean	0.091039	0.030056
Variance	0.165655	0.100289
Stdev	0.407007	0.316685
Skewness	4.911781	0.881051
Kurtosis	37.91698	15.20271

Based on the results of the basic statistics of the data, the mean of the simple and log a return 0.049766 and -0.002058 is close to zero. The values of the kurtosis are 37.91698 and 15.20271 which are greater than 3 hence the data exhibits excess kurtosis showing heaving tail distribution. Similarly, the values of the skewness are 4.911781and 0.881051which are greater than 0 (zero).

4.3.3 THE ACF AND PACF PLOTS

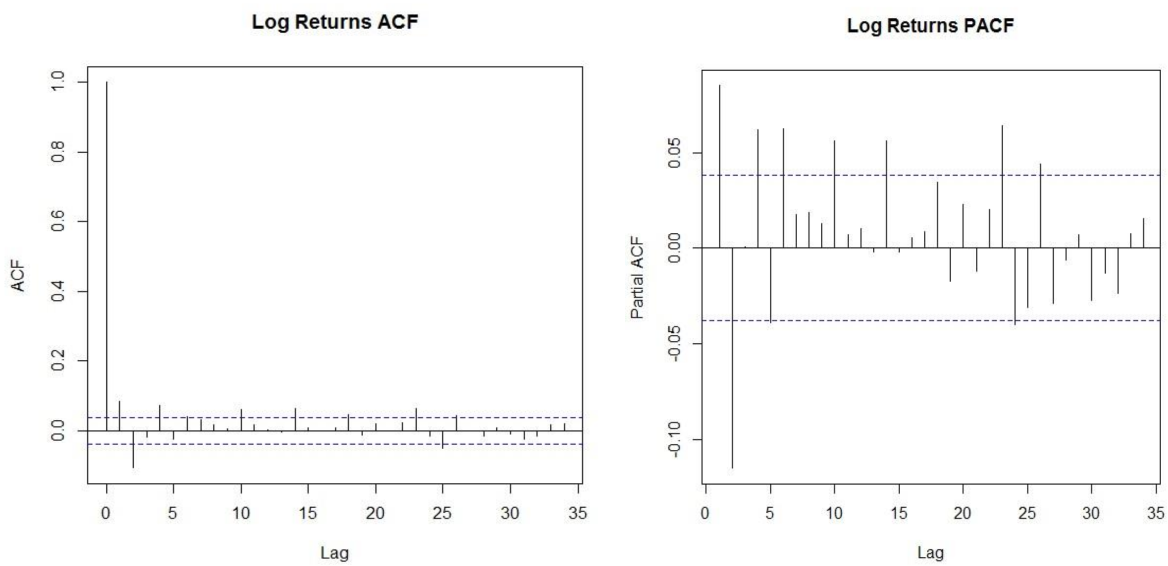

Figure 4.3: Plot of log returns for PACF and ACF

The PACF and ACF show 7 and 8 significant lags which demonstrates long term dependence.

4.3.4 ARCH Effect Tests

Table 4.2: Ljung box test for Log Returns at different lags.

Test statistic	15.9591	18.80741	21.86465
Parameter	3	5	7
P-value	0.001156088	0.00208751	0.002680528

From the Ljung box test for log returns, the p-values and the values of the test statistic at different lags suggests that the ARCH effects are significant. The resulting p-values and the values of the test statistic from the Ljung box test for squared log returns at different lags suggests that the ARCH effects are significant.

Tests for auto correlation in the inflation rate series indicate that there is auto correlation in the series as well as in the squared series (Figure 4.4) thus The ARCH model was considered appropriate.

4.5 GARCH MODEL.

4.5.1 GARCH QQ Distribution Plots

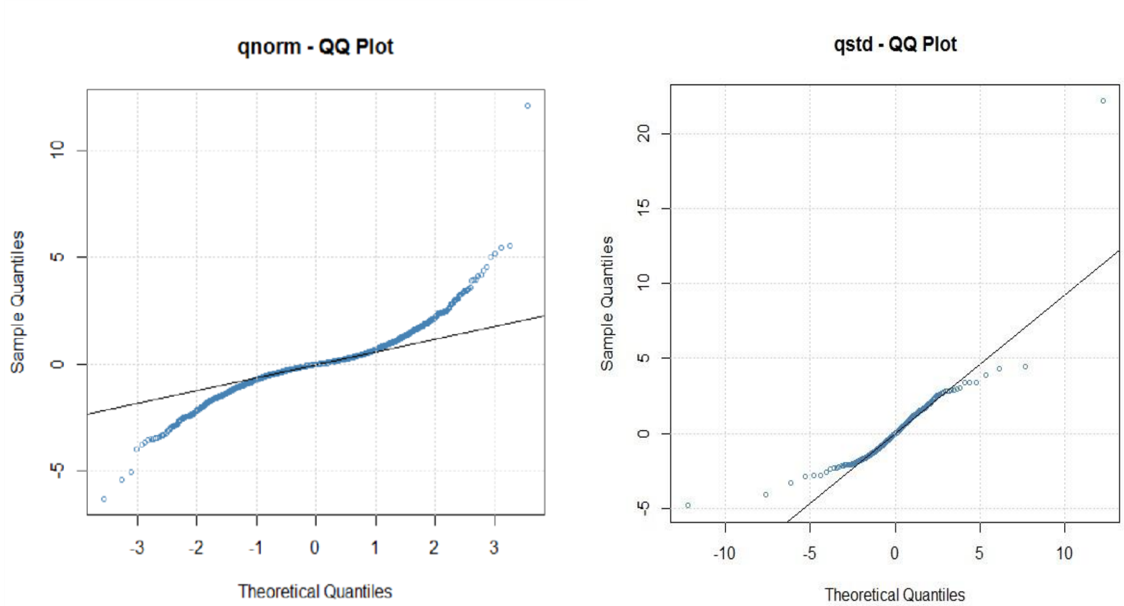

Figure 4.4 The QQ plot of GARCH (1,1) model with distribution.

This is clearly indicated by the failure of the data to be linear at the tails suggesting a more heavily tailed distribution for the residuals since norm QQ is a poor fit. The std – QQ plot however seems to have a relatively fair fit with student-t distribution being the residual distribution.

Table 4.3: The AIC and Log Likelihood distributions of GARCH

MODEL	RESIDUAL DISTRIBUTION	AIC	LOG LIKELIHOOD
GARCH (1,1)	Normal	-0.541131	102.9011
	Student-t	-0.9002348	107.7234
	Generalized Error	-0.901778	107.677
GARCH (1,2)	Normal	-0.544731	102.9586
	Student-t	-0.901070	107.6776
	Generalized Error	-0.900034	107.6812
GARCH (2,1)	Normal	-0.540380	102.9011
	Student-t	-0.900948	107.676
	Generalized Error	-0.900032	107.6813
GARCH (2,2)	Normal	-0.543974	102.9586
	Student-t	-0.900957	107.6811
	Generalized Error	-0.900012	107.6792

Based on (AIC) and LL value on the results in table 4.4, GARCH (1,1) appears to be the model that best fits the data with GED.

4.6 GJR GARCH MODEL.

4.6.1 GJR GARCH (1,1) distribution QQ plots.

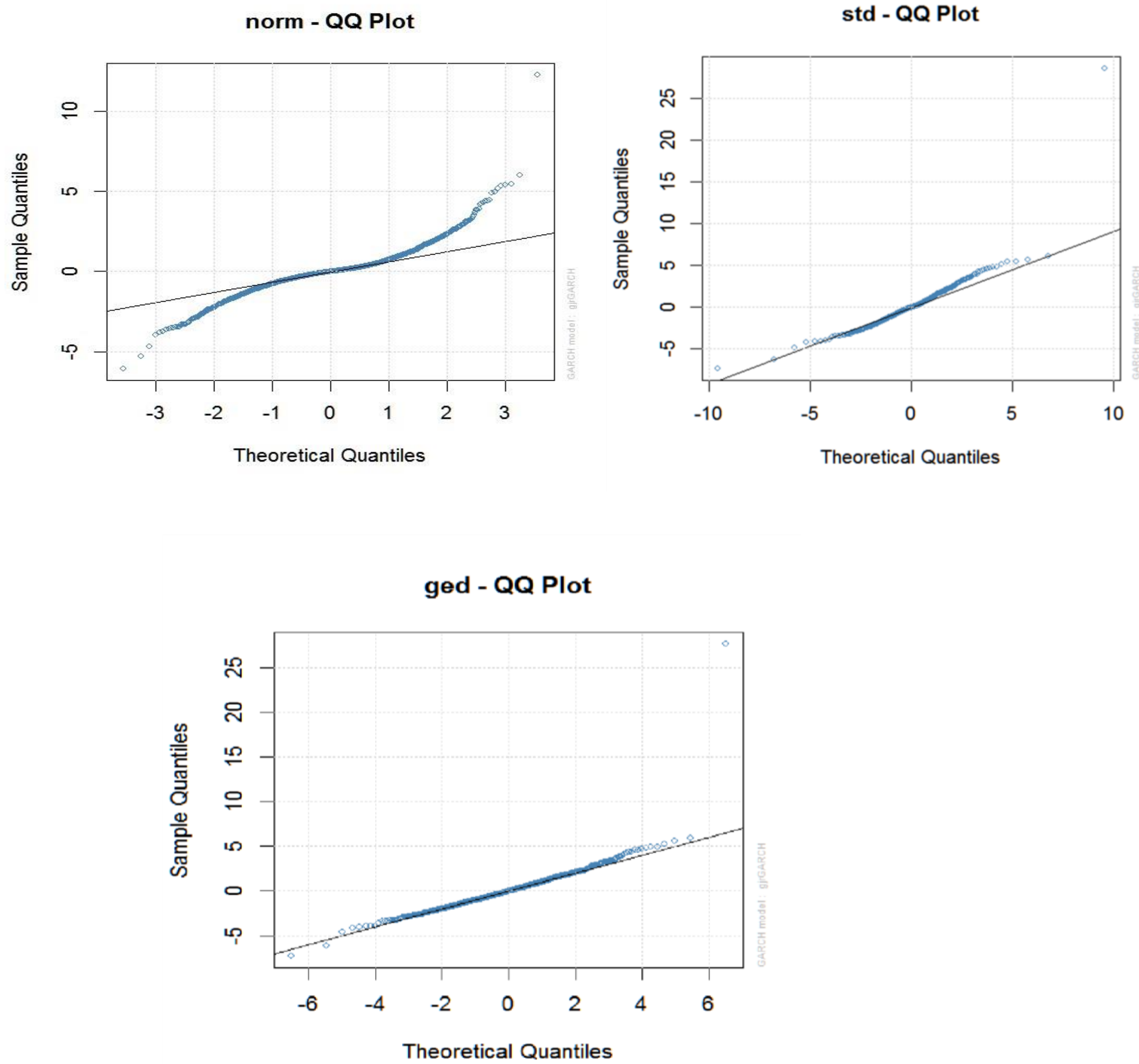

Figure 4.5 GJR-GARCH (1,1) and QQ plots.

The norm - QQ plot suggests that the data is a poor fit when the residual distribution is Normal, thus the data is possibly non-normal. This is clearly indicated by the failure of the data to be linear at the tails suggesting a more heavily tailed distribution for the residuals.

The std – QQ plot seems to have a relatively fair fit with student-t distribution being the residual distribution. A good fit linear fit is observed with GED.

4.6.2 GJR GARCH with Error distribution.

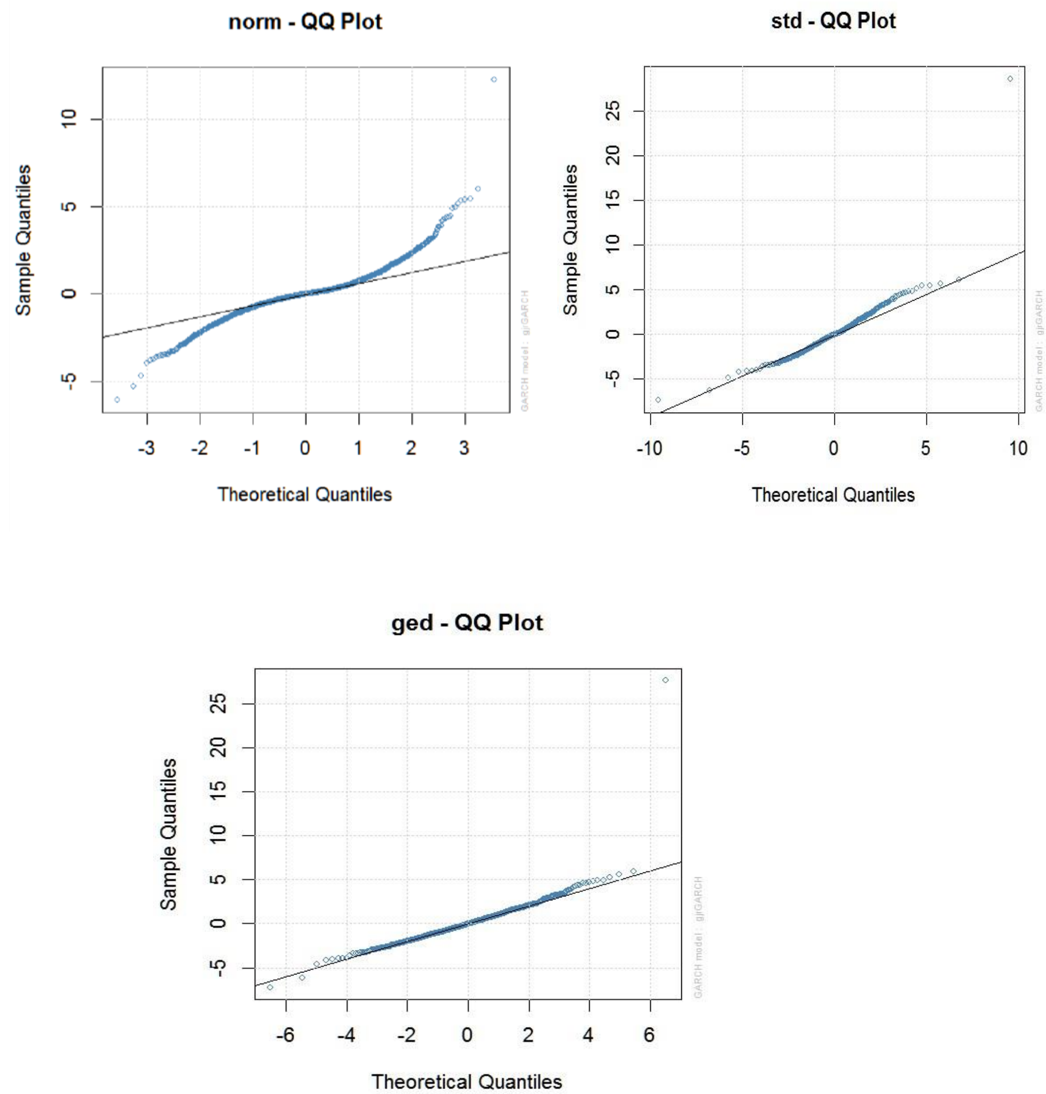

Figure 4.6: The QQ plot for GJR-GARCH with residual distribution.

The QQ plot for GJR-GARCH have similar properties to those of GJR-GARCH (1,1) for the Normal distribution, student-t & GED residual distribution. Best dataset observed with GED.

Table 4.4: The AIC and Log Likelihood for GJR-GARCH with residual distribution.

MODEL	ERROR DISTRIBUTION	AIC	LOG LIKELIHOOD
GJR-GARCH (1,1)	Normal Distribution.	-0.5541	103.0827
	Student-t Distribution	-0.8869	107.4905
	Generalized Error Distribution	**-0.90123**	**107.815**
GJR-GARCH (1,2)	Normal Distribution.	-0.5572	103.134
	Student-t Distribution	-0.8863	107.4926
	Generalized Error Distribution	-0.9109	107.8178
GJR-GARCH (2,1)	Normal Distribution.	-0.5547	103.1101
	Student-t Distribution	-0.8855	107.4919
	Generalized Error Distribution	-0.9100	107.8151
GJR-GARCH (2,2)	Normal Distribution.	-0.5560	103.1375
	Student-t Distribution	-0.8852	107.4975
	Generalized Error Distribution	-0.9095	107.8196

Based on AIC and LL value on the results on table 4.5, GJR-GARCH (1, 1) with generalized error distribution (GED) residual distribution appears to be the model that best fits the data.

4.7 EGARCH MODEL.

4.7.1 E-GARCH (1,1) with Error distribution QQ Plots.

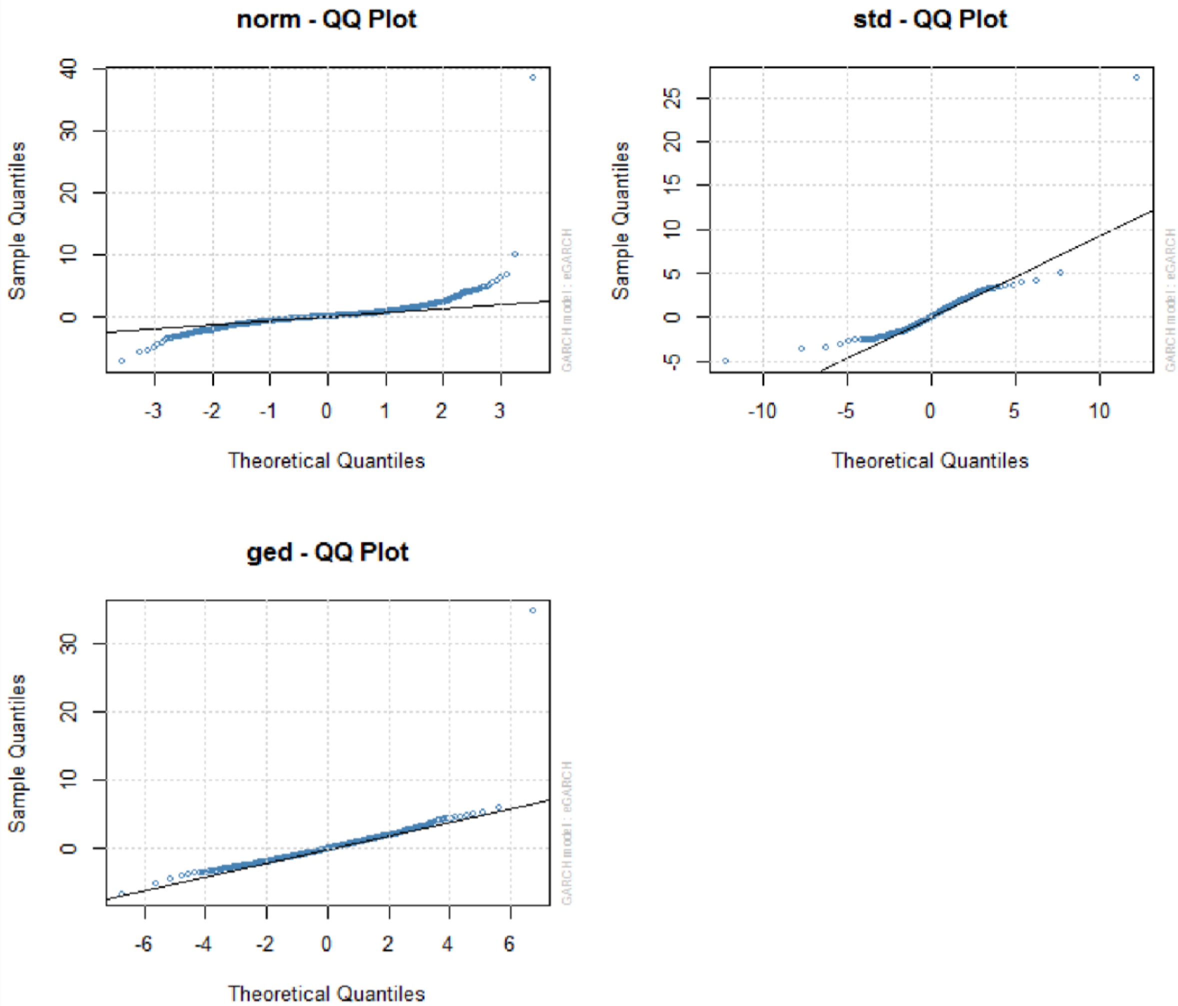

Figure 4.7 The QQ plot for E-GARCH (1,1) with residual distribution.

The norm - QQ plot suggests that the data is a poor fit when the residual distribution is Normal, thus the data is possibly non-normal. This is clearly indicated by the failure of the data to be linear at the tails suggesting a more heavily tailed distribution for the residuals. The std – QQ plot seems to have a relatively fair fit with student-t distribution being the residual distribution. GED best fits the data linearly.

4.7.2 E-GARCH (1,2), E-GARCH (2,1), E-GARCH (2,2) with Normalized, Student - t and GED Error distribution QQ plots.

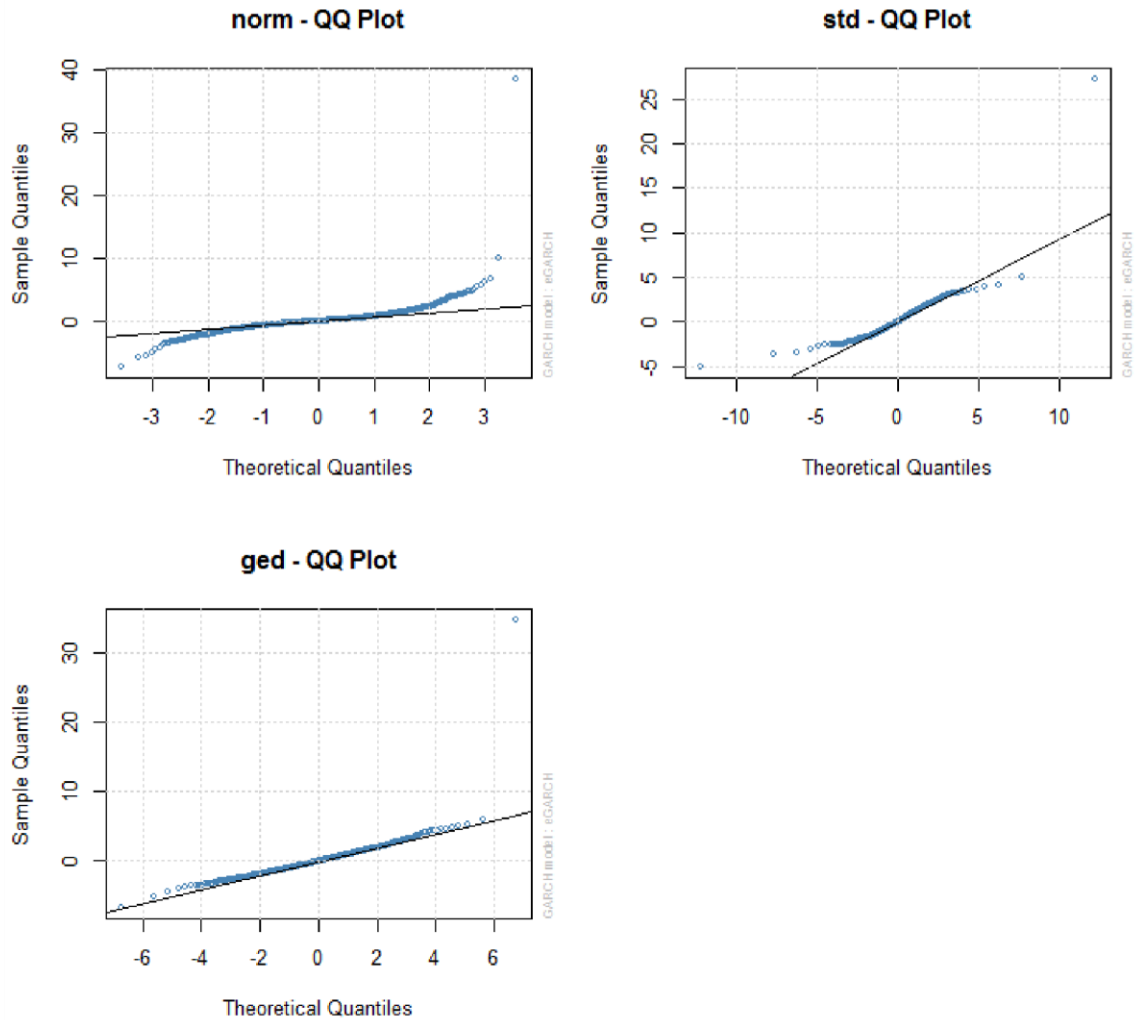

Figure 4.8 The QQ plot for E-GARCH (1,2), E -GARCH (2,1) & E -GARCH (2,2)with residual distribution.

The QQ plot for E-GARCH have similar properties to those of E-GARCH (1,1) for the residual distribution. Therefore, just like the GARCH, and GJR GARCH models, GED residual distribution best fits the data implying that the data is heavily tailed.

Table 4.5: E-GARCH with residual distribution.

MODEL	ERROR DISTRIBUTION	AIC	LOG LIKELIHOOD
E-GARCH (1,1)	Normal Distribution	-0.6499	104.3484
	Student-t Distribution	-0.9047	107.7257
	Generalized Error Distribution	**-0.9128**	**107.8332**
E-GARCH (1,2)	Normal Distribution	-0.6528	104.3966
	Student-t Distribution	-0.9041	107.7273
	Generalized Error Distribution	-0.9126	107.8401
E-GARCH (2,1)	Normal Distribution	-0.5572	103.1434
	Student-t Distribution	-0.9036	107.731
	Generalized Error Distribution	-0.9122	107.8453
E-GARCH (2,2)	Normal Distribution	-0.6517	104.4025
	Student-t Distribution	-0.9029	107.7324
	Generalized Error Distribution	-0.9111	107.8399

Based on AIC and LLvalue on the results on table 4.6, E-GARCH (1,1) with residual distribution being generalized error distribution is the model that best fits the data.

4.8 Model Comparison and Selection

Table 4.6: The comparison of GARCH (1,1), GJRGARCH (1,1) & E-GARCH (1,1).

MODEL	ERROR DISTRIBUTION	AIC	LOG LIKELIHOOD
GARCH (1,1)	Generalized	-0.901778	107.677
GJR-GARCH (1,1)	Generalized	-0.90123	107.815
E-GARCH (1,1)	**Generalized**	**-0.9047**	**107.7257**

Based on the results in table 4.7, the model that appears to best fit the dataset is E-GARCH (1,1) with generalized error distribution. The E-GARCH model is a better fit since it can capture the leverage effects. The E-GARCH model also has an advantage over the GJR-GARCH model in that, even if the parameters are negative, variance remains positive because the variable modelled is ln ($\delta_t{}^2$).

Additionally, the QQ-plots, AIC and Log Likelihood suggested that E-GARCH model with generalized error distribution is a better fit for the dataset than E-GARCH (1,1) with student-t residual distribution. The chosen model is E-GARCH (1,1) with GED.

4.9 THE SIMULATION.

4.9.1 Conditional SD Simulation Density

A simulation with the model E-GARCH (1, 1) appears to fit the structure of the original return series as depicted in Figure 4.11 and Figure 4.12 below:

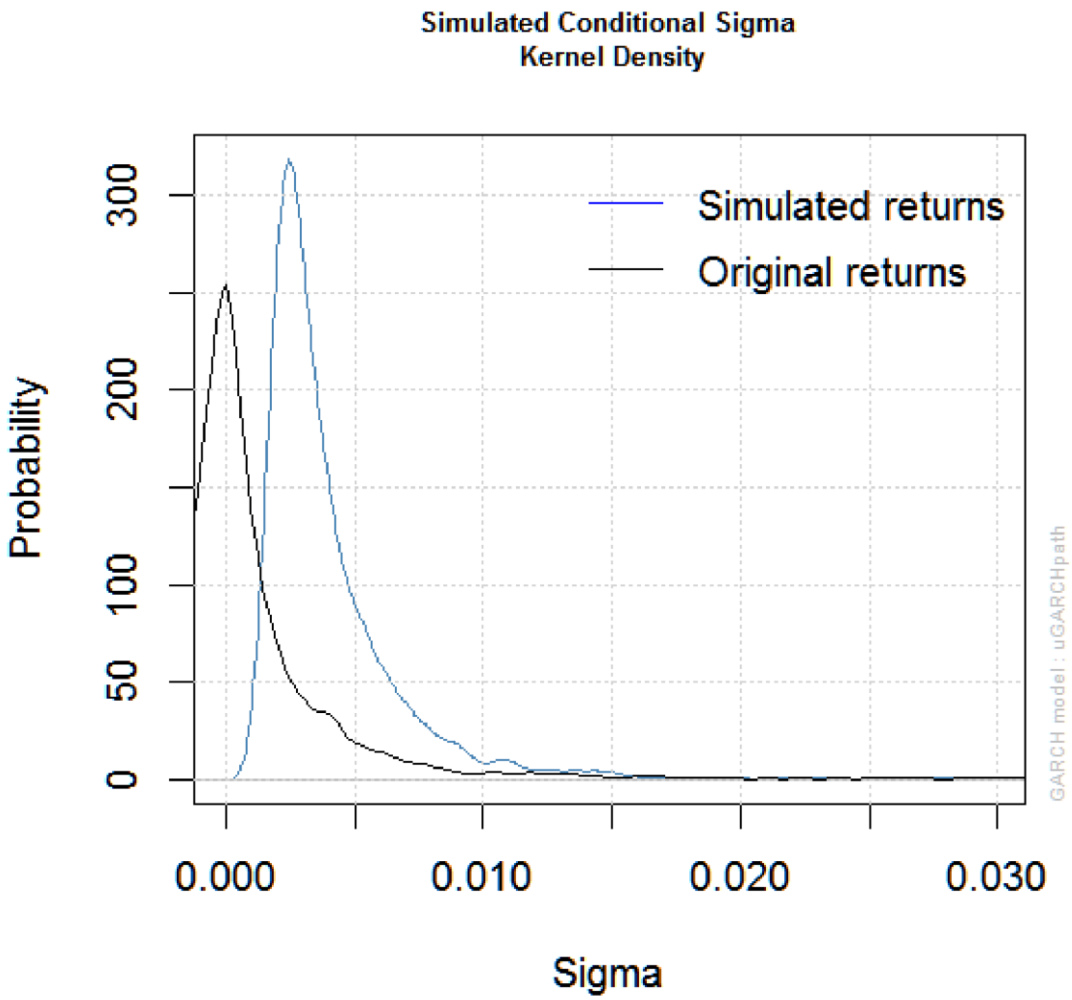

Figure 4.9: Conditional standard deviation simulations

4.9.2 Return Series Simulation Path Density

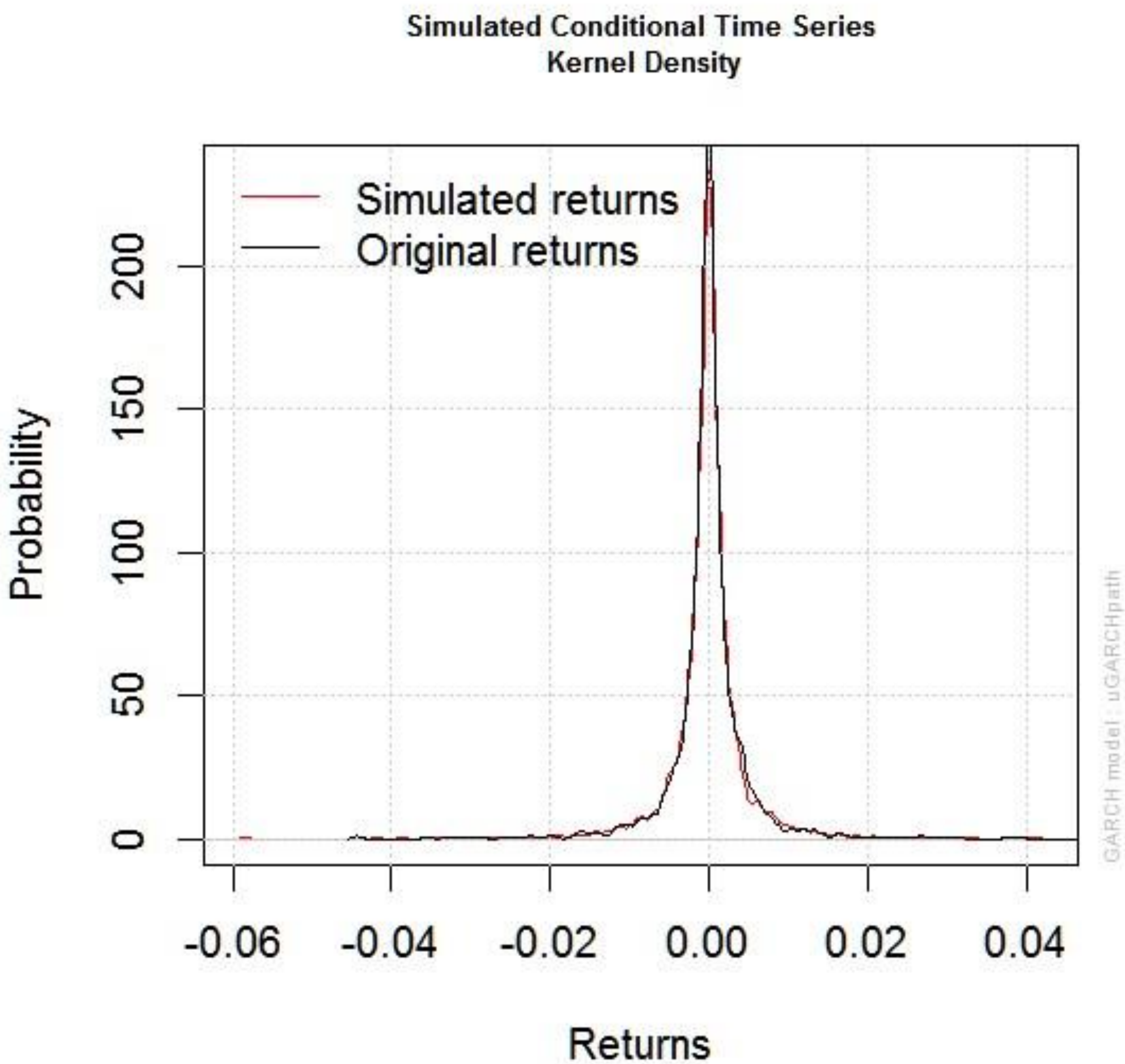

Figure 4.10: Return series simulation path density.

4.10 Residual Analysis of the Models

Table 4.7: E-GARCH (1,1) with GED parameter estimates.

Conditional Variance Dynamics				
GARCH Model	E-GARCH (1,1)			
Mean Model	ARFIMA(0,0,0)			
Distribution Model	GED			
Optimal Parameters				
Parameter	**Parameter Estimate**	**Standard Error**	**t-value**	**P-Value**
Omega	-0.558348	0.349971	-1.5954	0.110621
Alpha1	0.047247	0.026697	2.1443	0.032010
Beta1	0.950922	0.030666	31.0090	0.000000
Gamma1	0.427086	0.120499	3.5443	0.000394
Shape	0.781789	0.025126	31.1151	0.000000

The results gives positive value of Gamma1 and significant at 1% level of significance. The magnitude is 0.427086 and the sign is positive. Past events have effects on the future volatility. The values of α_1 and β_1 are significant at 1% significance level.

The sum of $\alpha_1 + \beta_1 = 0.998169 < 1$. Hence persistence in volatility

Mean equation : $X_t = Z_t + 0.047247Z_{t-1}$

Our variance equation becomes:

$$\ln\left(\delta^2{}_t\right) = -0.558348 + 0.950922\ln\left(\delta^2_{t-j}\right) + 0.427086 \left|\frac{u_{t=i}}{\sqrt{\delta_{t=i}}}\right| + \sum_{k=1}^{r}\gamma_k \frac{u_{t-k}}{\sqrt{\delta_{t-k}}}$$

Table 4.8: E-GARCH (1,1) with GED conditional distribution Weighted Ljung-Box Test on Standardized Residuals

Lag	Test Statistic	P-value
Lag[1]	3.476	0.06226
Lag[2*(p+q)+(p+q)-1][2]	4.933	0.04238
Lag[4*(p+q)+(p+q)-1][5]	7.775	0.03351
H_o: No serial correlation		

Table 4.9: E-GARCH (1,1) with GED conditional distribution Weighted Ljung-Box Test on Standardized Squared Residuals

Lag	Test Statistic	P-value
Lag[1]	0.006725	0.9346
Lag[2*(p+q)+(p+q)-1][5]	0.013184	1.0000
Lag[4*(p+q)+(p+q)-1][9]	0.016688	1.0000

The high p-values lead us to accept the null hypothesis which is further strengthened by the observation of the ACF.

Table 4.10: E-GARCH (1,1) with GED conditional distribution Weighted ARCH LM Tests results.

ARCH Lag	Test Statistic	Shape	Scale	P-value
ARCH Lag[3]	0.0007012	0.500	2.000	0.9789
ARCH Lag[5]	0.0038767	1.440	1.667	0.9999
ARCH Lag[7]	0.0057890	2.315	1.543	1.0000

The ARCH LM tests the null hypothesis that there are no more ARCH effects in the residuals. From the p-values as listed above, at 1%,5% and even 10% level of significance, we can conclude that there are no more ARCH effects in the residuals which indicates that the volatility model is correctly specified.

4.11 FORECASTING

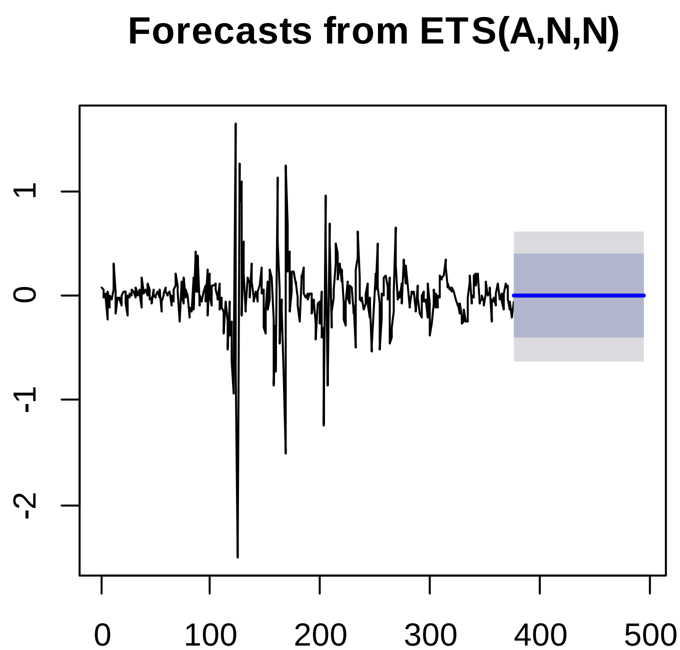

Figure 4.11 Ten Years Volatility Forecast

From Figure 4.11 it is clear that the next 10 years predictions for the inflation rate are constant. Inflation rate stability at 5% in the previous three months. Here, for the next 10 years predictions are plotted as blue line, 80% prediction interval as a blue shaded area and the 95% prediction interval as grey shaded area.

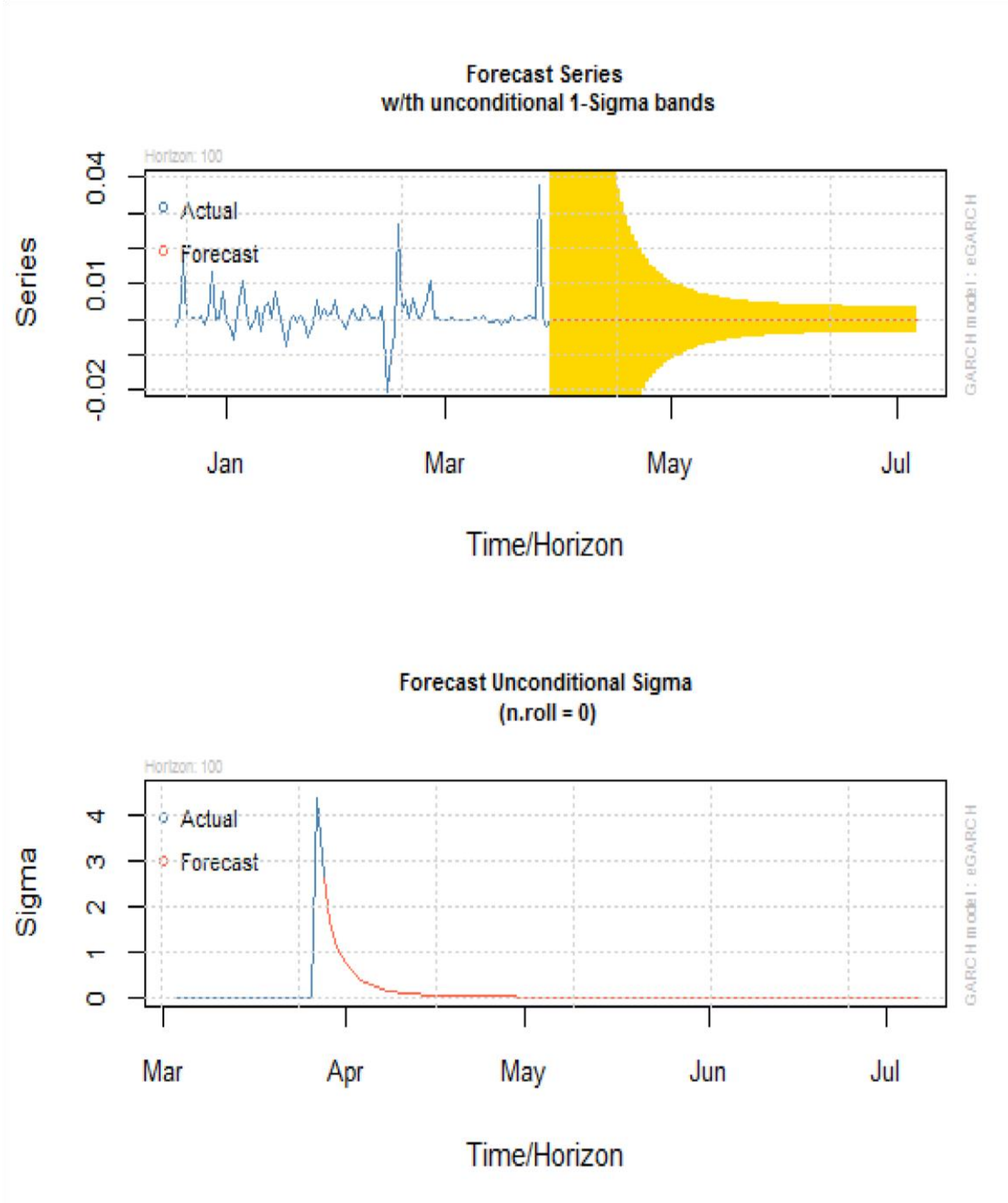

Figure 4.12: Monthly Forecast Series and Forecast Volatility.

CHAPTER FIVE

SUMMARY CONCLUSION AND RECOMMENDATION

5.1 SUMMARY

Modelling and forecasting the volatility of returns in stock markets has become a fertile field of empirical research in financial markets. It is also crucial to various groups such as importers, exporters, investors, policy makers, governments etc.

This study opted for all specifications of the GARCH models although empirical evidence shows that the lower specifications are able to sufficiently capture the characteristics of inflation rates while at the same time upholding the principle of parsimony. Various GARCH, GJR-GARCH and E-GARCH models were fitted with variations being made to the conditional distribution used i.e. normal, student-t and generalized error distribution.. The E-GARCH model provides a better fit than the GARCH model and its advantages over the GARCH model are that first, it can capture leverage effects and secondly, that there is no restriction that the parameters α_1 and β_1 must be positive.

5.2 CONCLUSION

Based on the empirical results presented, the following can be concluded; the study finds strong evidence that inflation monthly returns could be characterized by the above mentioned models. For the period specified, the empirical analysis was supportive to the symmetric volatility hypothesis, which means returns are volatile and that positive and negative shocks (good and bad news) of the same magnitude have the same impact and effect on the future volatility level. The parameter estimates of the GARCH (1,1) models (alpha and beta) indicates a high degree of persistent in the conditional volatility of returns on the Kenyan inflation rates which means an explosive volatility. To summarize, the results from all GARCH specifications applied in this study for the periods explain that explosive volatility process is present in inflation returns over the sample period.

5.3 RECOMMENDATION

This study recommends that univariate time series models, where other economic variables that could influence the volatilities in the monthly rate of inflation such as exchange rates, amount of money supply, interest rates and others will be modelled along the rates of inflation. The inclusion of these other variables could help identify which of them contribute more to the variability in the monthly rates of inflation in Kenya. Another important area of research in modelling inflation rate would be use of Bayesian statistics

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APPENDIX: R CODES

```
library(fGarch)
#Inflation Rates
data=read.csv("inflationrates.csv");data
mydata=data$inflation;mydata
#Simple Returns
p=c()
for(i in 2:length(mydata)){
s.r=(mydata[i]/mydata[i-1])-1
p=c(p,s.r)
}
Inflation=p;Inflation
basicStats(Inflation)
#log Returns
logInflation<-diff(log(mydata)); logInflation
basicStats(logInflation)
acf(logInflation)
pacf(logInflation)
plot.ts(logInflation)
sqrdlogInflation= logInflation^2; sqrdlogInflation
plot.ts(sqrdlogInflation)
acf(sqrdlogInflation)
pacf(sqrdlogInflation)
plot.ts(sqrdlogInflation)
Box.test(logInflation,lag=1,type='Ljung')
ai=logInflation-mean(logInflation)
Box.test(ai^2,lag=1,type='Ljung')
m2=garchFit(~garch(1,1),data=logInflation,trace=F)
summary(m2)
plot(m2)

symoredata=read.csv("symore data.csv",header=T)
attach(symoredata)
dat1=symoredata$inflation
ts.plot(dat1,main="RAW DATA PLOT Inflation",ylab="prices(Inflation)")
dat2=symoredata$CPI
ts.plot(dat2,main="RAW DATA PLOT CPI",ylab="prices(CPI)")
library(rugarch)
library(parallel)

#Log return
l.rinflation=diff(log(dat1))
ts.plot(l.rinflation,main="log returns plot inflation",ylab="log
returns(inflation)")


# testing ARCH effects
cbind(Box.test(l.rinflation,lag=12),Box.test(l.rCPI,lag=12))

library(rugarch)
library(parallel)

#GARCH(1,1)-Normalized Error Distribution
```

```
garch11.norm=garchFit(formula = ~ garch(1, 1), data =l.rinflation,cond.dist
= c("norm"),include.mean =F)
summary(garch11.norm)
plot(garch11.norm)
#GARCH(1,1)-Student-t Error Distribution
garch11.std=garchFit(formula = ~ garch(1, 1), data =l.rinflation,cond.dist =
c("std"),include.mean =F)
summary(garch11.std)
plot(garch11.std)

#GARCH(1,1)-GED Error Distribution
garch11.ged=garchFit(formula = ~ garch(1,1), data =l.rinflation,cond.dist =
c("ged"),include.mean =F)
summary(garch11.ged)
plot(garch11.ged)

#GARCH(1,2)-Normalised Error Distribution
garch12.norm=garchFit(formula = ~ garch(1, 2), data =l.rinflation,cond.dist
= c("norm"),include.mean =F)
summary(garch12.norm)
plot(garch12.norm)

#GARCH(1,2)-Student-t Error Distribution
garch12.std=garchFit(formula = ~ garch(1, 2), data =l.rinflation,cond.dist =
c("std"),include.mean =F)
summary(garch12.std)
plot(garch12.std)
#GARCH(2,1)-Normalised Error Distribution
garch21.norm=garchFit(formula = ~ garch(2,1), data =l.rinflation,cond.dist =
c("norm"),include.mean =F)
summary(garch21.norm)
plot(garch21.norm)

#GARCH(2,1)-Student-t Error Distribution
garch21.std=garchFit(formula = ~ garch(2,1), data =l.rinflation,cond.dist =
c("std"),include.mean =F)
summary(garch21.std)
plot(garch21.std)

#GARCH(2,2)-Normalised Error Distribution
garch22.norm=garchFit(formula = ~ garch(2,2), data =l.rinflation,cond.dist =
c("norm"),include.mean =F)
summary(garch22.norm)
plot(garch22.norm)

#GARCH(2,2)-Student-t Error Distribution
garch22.std=garchFit(formula = ~ garch(2,2), data =l.rinlation,cond.dist =
c("std"),include.mean =F)
summary(garch22.std)
plot(garch22.std)
#gjr garch with student t disribution,,,,
```

```
spec.gjrmodel1.std=ugarchspec(variance.model=list(model="gjrGARCH",garchOrde
r=c(1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.mode
l="std")
gjrmodel1.std=ugarchfit(l.rinflation,spec=spec.gjrmodel1.std)
gjrmodel1.std
plot(gjrmodel1.std,which=9)

#gjr garch with ged disribution,,,,

spec.gjrmodel1.ged=ugarchspec(variance.model=list(model="gjrGARCH",garchOrde
r=c(1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.mode
l="ged")
gjrmodel1.ged=ugarchfit(l.rinflation,spec=spec.gjrmodel1.ged)
gjrmodel1.ged
plot(gjrmodel1.ged,which=9)


#gjr garch with student t disribution,,,,CPI

spec.gjrmodel1.std=ugarchspec(variance.model=list(model="gjrGARCH",garchOrde
r=c(1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.mode
l="std")
gjrmodel1.std=ugarchfit(l.rCPI,spec=spec.gjrmodel1.std)
gjrmodel1.std
plot(gjrmodel1.std,which=9)

#gjr garch with ged disribution,,,,CPI

spec.gjrmodel1.ged=ugarchspec(variance.model=list(model="gjrGARCH",garchOrde
r=c(1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.mode
l="ged")
gjrmodel1.ged=ugarchfit(l.rCPI,spec=spec.gjrmodel1.ged)
gjrmodel1.ged
plot(gjrmodel1.ged,which=9)


# E-garch models with normal plus ged one

spec.emodel1.norm=ugarchspec(variance.model=list(model="eGARCH",garchOrder=c
(1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.model="
norm")
emodel1.norm=ugarchfit(l.rinflation,spec=spec.emodel1.norm)
emodel1.norm
plot(emodel1.norm,which=9)



spec.emodel1.ged=ugarchspec(variance.model=list(model="eGARCH",garchOrder=c(
1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.model="g
ed")
emodel1.ged=ugarchfit(l.rinflation,spec=spec.emodel1.ged)
emodel1.ged
plot(emodel1.ged,which=9)
```

```
# E-garch models with normal

spec.emodel1.norm=ugarchspec(variance.model=list(model="eGARCH",garchOrder=c
(1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.model="
norm")
emodel1.norm=ugarchfit(l.rCPI,spec=spec.emodel1.norm)
emodel1.norm
plot(emodel1.norm,which=9)

# E-garch models ged

spec.emodel1.ged=ugarchspec(variance.model=list(model="eGARCH",garchOrder=c(
1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.model="g
ed")
emodel1.ged=ugarchfit(l.rinflation,spec=spec.emodel1.ged)
emodel1.ged
plot(emodel1.ged,which=9)


spec.emodel1.ged=ugarchspec(variance.model=list(model="eGARCH",garchOrder=c(
1,1)),mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.model="g
ed")
emodel1.ged=ugarchfit(l.rCPI,spec=spec.emodel1.ged)
emodel1.ged
plot(emodel1.ged,which=9)

spec.eGARCH11.ged=ugarchspec(variance.model=list(model="eGARCH",garchOrder=c
(1,1)),
mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.model="ged"
,fixed.pars=list(mu = 0,omega=-1.496285,alpha1=0.076862,beta1=0.829872,
gamma1=0.539965,shape=1.201080))
set.seed(123)
egarch11.ged.sim=ugarchpath(spec.eGARCH11.ged, n.sim=2700)
egarch11.ged.sim

#Conditional SD Simulation Density
#Use the plot method to plot simulated series and conditional volatilities
plot(egarch11.ged.sim, which=3, col="red")
lines(density(l.rinflation),col="black")
legend('topright', c('Simulated returns', 'Original returns'),
col =c("blue","black"), lty = c(1, 1), bty = 'n')

#Conditional SD Simulation Density
#Use the plot method to plot simulated series and conditional volatilities
plot(egarch11.ged.sim, which=3, col="red")
lines(density(l.rinflation),col="black")
legend('topright', c('Simulated returns', 'Original returns'),
col =c("red","black"), lty = c(1, 1), bty = 'n')

library(parallel)
library(rugarch)
```

```
spec.eGARCH11.ged=ugarchspec(variance.model=list(model="eGARCH",garchOrder=c
(1,1)),
mean.model=list(armaOrder=c(0,0),include.mean=F),distribution.model="ged"
,fixed.pars=list(mu = 0,omega=-0.558348,alpha1=0.057247,beta1=0.950922,
gamma1=0.427086,shape=0.781789))
set.seed(123)
egarch11.ged.sim=ugarchpath(spec.eGARCH11.ged, n.sim=376)
egarch11.ged.sim

#Conditional SD Simulation Density
#Use the plot method to plot simulated series and conditional volatilities
plot(egarch11.ged.sim, which=3, col="red")
lines(density(l.rinflation),col="black")
legend('topright', c('Simulated returns', 'Original returns'),
col =c("blue","black"), lty = c(1, 1), bty = 'n')

plot(egarch11.ged.sim, which=4, col="red")
lines(density(l.rinflation),col="black")
legend('topright', c('Simulated returns', 'Original returns'),
col =c("red","black"), lty = c(1, 1), bty = 'n')
# Conditional SD Simulation Path
plot(egarch11.ged.sim, which=1)

#FORECASTING

# predictive accuracy
library(forecast)
accuracy(logInflation)
# predict next ten future values
library(forecast)
forecast(logInflation, 10)
plot(forecast(logInflation, 10))

forc=ugarchforecast(emodel1.ged,n.ahead=100,mse=c("cond"),nx=round(0.25*leng
th(l.rinflation)),plot=T)
plot(forc)
plot(forc,which=3)
#Volatility analysis.
plot(egarch11.ged.sim, which=1)
```