MODELING OF VOLATILITY OF INTEREST AND TREASURY BILL RATES USING ARCH / GARCH FAMILY MODELS AND THEIR EFFECT ON PENSION FUND

ISAAC ONCHAGA ONDIEKI

I56/76584/2014

DEPARTMENT OF ACTUARIAL SCIENCE

SCHOOL OF MATHEMATICS

UNIVERSITY OF NAIROBI

A project submitted in partial fulfillment of the requirements for the degree of

Master of Science in Actuarial Science

December 2016.

Declaration

I declare that this research project is my original work and has never been submitted in any form as a credit academic qualification to my learning institution or any other university. It is submitted to post graduate department of actuarial science.

Signature…………………………………………………Date……………………………………

Isaac Onchaga Ondieki.

I56/76584/2014.

DECLARATION BY THE SUPERVISOR:

This research project has been submitted with our approval of as a university supervisor

Prof. Simwa
Professor, School of Mathematics,
University of Nairobi
Signature…………………………… Date…………………………………

ACKNOWLEDGEMENT

Foremost, I thank GOD for the far He has enabled me to have this project complete, it has not been easy.

My sincere gratitude to my supervisors' Prof. Simwa for his guidance availability throughout this process and your dedication to ensure I complete my project.

Thanks to Prof. Weke for encouraging me during this process.

I thank members of my family especially my wife Margaret, my daughter Sylvia, sons Benjamin, Shadrack and Nelson for their support during my project.

DEDICATION

This project is dedicated to my wife Margaret Kemunto, my sons Shadrack, Benjamin and daughter Sylvia for their support to the Completion of this project.

LIST OF ABBREVIATIONS

AR	:	Autoregressive ARCH
GARCH	:	Generalized Autoregressive Conditional Heteroscedasticity
EVT	:	Extreme Value Theory
PSTECM	:	Panel Smooth Transition Error Correction Model.
CBK	:	Central Bank of Kenya.
IRA	:	Insurance Regulatory Authority.
CAPM	:	Capital Asset Pricing Method.
MPC	:	Monetary Policy Committee.
AR	:	Autoregressive
AIC	:	Akaike Information Criterion
ACF	:	Auto relation Function.
PACF	:	Partial Autocorrelation Function
HQIC	:	Hannan-Quinn information criterion
SIC	:	Shwartz Information Criterion
PACF	:	Partial Autocorrelation Function
BIC	:	Bayesian Information Criterion

LIST OF SYMBOLS

Z_t	:	Identically and independent Standard Normal Random Variable.
ε_t	:	Error
y_t	:	Time series observations
y_t^2	:	Squared Observations
ψ_t	:	Set of Observations
r_t	:	Return
μ	:	Constant part of return or mean
$\alpha_o, \alpha_1, \beta$	:	Model parameters
σ_t	:	Standard deviation
σ_t^2	:	Conditional variance
σ	:	Volatility Estimate
b_i	:	Volatility Coefficient.

TABLE OF CONTENTS

LIST OF TABLES:

LIST OF FIGURES:

ABSTRACT

The interest rate and Treasury bill rates were converted into simple returns and modeled by use of ARCH and GARCH (1, 1) models. The GARCH (1,1) was preferred because it allows many parameters and considers conditional heteroscedasticity of data to assess volatility of interest rates and Treasury rates. Volatility measures the errors made in modeling returns. It was discovered that the average volatility is not constant but varies with time and can be forecast or predicted in both cases. Also a multifactor model was used to investigate how the two affect pension Fund, it was discovered that interest rates affected pension fund more than the treasury rates, and the model can be used to project growth of the fund.

CHAPTER ONE

1.0 INTRODUCTION

1.1 PENSION FUND

Pension Fund was established by the Government of Kenya through the act of Parliament and was last revised in 2009.It provided for regulation and granting, gratuities and other benefits to public service officers employed by the government of Kenya

The Government of Kenya also established Retirement Benefits Authority to regulate, supervise and promote retirement benefit schemes for private employers and their employees. They make contributions to this scheme. However, formal employers and workers are free to join this scheme. The benefits can be paid as long as the employee has worked for a period of ten years or above upon retirement or death whichever comes early.

The contributions are invested to earn interest so that the beneficially is paid principal plus interest earned. The money from this fund is invested in Government Treasury bills. The interest and Treasury bill rates changes are expected to affect the return on this fund.

1.2 Interest Rates:

Insurance firms, invest received premiums, largely in fixed income securities such as Treasury bills and Bonds. Changes in interest rates often affect returns from these securities over a period of time. The value of assets and liabilities of insurance firms change as interest rates change and thereby expose the company to risk.

This study focuses on interest rate environment and changes in Treasury bill prices and their effect on Pension Fund. The purpose is to establish whether there exists a relationship between interest rates changes, Treasury bill rates and the pension Fund. The suitable and appropriate Models, are required in analyzing the effects of interest rates changes charged by the banks and the changes on Treasury bill prices.

The use of stochastic processes and time series models are found to be appropriate in forecasting when applied in financial data in Economics. Many research papers carried out regarding this subject, focused on how interest rates affect the bond prices of non-life insurance investments on bonds.

Many countries in the world experience fluctuations in interest rates more often. For example in the USA in2012, the country experienced low interest rates that threatened the insurance industry according to Economic perspective, Kyal Berends *et al*(2013), In their research, noted that when interest rates fall, bond prices rise, also that some of the products like annuities may not be sold easily when interest rates are very low. The effect was more felt in large insurance firms than the small sized firms.

The changes in exchange rates fluctuations measures taken by central bank of Kenya too affect interest rates as in the case of 2015 when the bank lending interest rates shot up to between 18% to 26% from 14% the previous year.

The major problem with insurance companies is how to deal with the interest rate risk, In the USA interest rates swap derivatives were used to hedge changes in interest rates (Economic perspective, 2013)

In this study GARCH model (1, 1) is used to analyze the volatility in interest rates (bank lending spot rates) and Multifactor models are used to establish any relationship between interest rates, changes in rates of treasury bills and the pension fund. The two models are good in analyzing financial data that vary with time. These models are used mainly in time series applications.

Volatility is used to show periods of peak/crisis periods of interest rates changes and low periods or moderate.

1.3 Treasury Bills

A treasury bill is a paperless short-term borrowing instrument issued by the Government to raise funds from the public or institutional investors. Treasury bills are issued in maturities of 91, 182 and 364 days. Treasury bills are sold at a discounted price to reflect investor's return and redeemed at face (par) value. The difference between discounted value and par value represents the rate of return to the investor. Any investor is required to have an active Central Depository System (CDS) account with the CBK so that it is easy to trade in securities. The discounted price of the Treasury bill depends on the interest rate/yield quoted by the investor and is calculated using the following formula:

$$p = 100\left[\frac{1}{\left(1 + \left(\frac{r}{100} * \frac{d}{365}\right)\right)}\right]$$

Where,p-price per ksh.100

r -interest rate

d -Days to maturity

The interest rate in question is assumed to vary with bank lending rates. Already other investigations from other researchers have shown that there is a relation between interest rate and Bond rates, however I have not seen any one involving interest rates and Treasury bill rates. In this study, we only focus on Treasury bill rates whose maturity is 91 days. The other treasury bills have different rates, this is due to the fact that there are other factors affecting interest rates such as time to maturity.

1.4 Problem Statement*;*

Kenya has experienced a period of high interest rates from June 2015 to August 2016 before the Government intervened, the same period the Kenya shilling lost value to major foreign currencies including the US Dollar. The Central bank of Kenya took monetary and fiscal policy measures to stabilize the exchange rates, these resulted to the rise of bank lending rates from an average of 15% to 26% by December 2015.These interest rates shocks had impact on prices of commodities, purchasing power of salaried workers, affected stock returns of insurance companies. An intervention measures were necessary to lower the interest rate that is why the Government of Kenya through the act of parliament lowered the rates to 14% from 24%. Treasury bill prices equally were fluctuating. The value of returns from investments such treasury bills and Bonds changed significantly. There is need to know if these fluctuations would have been forecasted and measures taken in advance. How did fluctuations affected pension fund needs to be investigated.

1.5 Objective;

To assess the volatility of interest rates and Treasury bill rates, and how each affect pension Fund

1.6 Specific Objectives

1. To assess volatility of bank lending interest rate from January 2012 and July 2016.

2. To assess the volatility of prices changes of treasury bills from January 2012 and July 2016.

3. To assess how interest rates and treasury bills affect pension fund.

1.7 Significance of the Study.

It will facilitate investigation of interest rate risk which can be indicator of the state of the economy. It will enable the Government to manage the volatility, in the long run and short run and provide Models for policy makers to help them predict future volatilities. This will be a guide to market of securities and other investments made by investors. The information can be used to make decisions in investments from the observed patterns of interest rate volatility. The information can be used to project on returns from pension fund investments.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Introduction

The Treasury bill prices and bank interest rates vary widely from period to the next. There are a number of factors that cause these variations in the rates. Theoretically interest rates also depend on a range of factors such as inflation, state of economy of the country, balance of payments and many others. Main focus is the numerical percentage of interest charged by Banks from time to time during the business period and how this variation affects the pension fund. Investments done by companies are likely to be affected by these variations making it difficult to accurately predict future profits. Returns from investments in Treasury bills can be predicted if the interest rates are fairly constant although this is not the case especially in Kenya. Treasury bills have a short period than Bonds that take long period to mature, and the interest rates do not remain constant over this long period. Therefore, the knowledge of how interest changes is very important in deciding the best investment portfolios for the investment companies in any given period.

2.2 REVIEW OF PREVIOUS STUDIES

A research done by Boubaker & Sghaier (2011), effect of interest rate and inflation rate on non-life insurance premiums, showed that both have an impact on non-life insurance premiums depending on the value of the inflation rate. They used Panel Smooth transition Error Correction Model (PSTECM) in analysis, the model took into account both short and long-run effects of changes in economic variables.

Research done by Kyal Berends et al (2010) on sensitivity of insurance firms to interest rates changes showed that fluctuations in interest rates affected life insurance business either positively or negatively. Stock price changes was analyzed with those of interest rates changes. Prices were taken at corporate level involving many firms other than the individual firms in USA.In this research a two-factor Model was used that was suggested by Brewer, Mondschean, and Strahan (1993) and James (1984).

Kenya operates on a free market economy. Interest rates in free economy depend on forces of demand and supply which vary with time. Volatility of interest rates can be studied by applying ARCH and GARCH models, which are appropriate in describing volatility in financial data with times series characteristics, Engle,(1992).The GARCH family models are good in capturing homoscedasticity and volatility clustering in financial data.GARCH (1, 1) model is more adequate in describing most financial time series Boleslaw et al, (1992)

The data where variances are not equal, suffers from what is called heteroscedasticity.

The GARCH models treat heteroscedasticity as a variance to be modeled, this mostly has been applied to non-time series models to find the standard errors to reduce heteroscedasticity.

For large sample size data concern of heteroscedasticity is minimal or less significance however for time series it can be noted that some periods are riskier than others however they are not scattered randomly. There's some degree of autocorrelation in the riskiness in the returns. The GARCH model which stand for Generalized Autoregressive Conditional Heteroscedasticity is applied in dealing with these situations mentioned above.

McNeil et *al* (2000) proposed a two stage model where GARCH model is fitted to return data and used to model the tail of residuals using EVT Model. EVT models may be used to model the risk of extreme values or rare events. The challenge is where extreme data is scarce. There is a challenge in determining whether a series is heavy-tailed or light –tailed, and also choosing the best method of estimating the parameters.

A model suggested by Engle (1982), allowed the use of best weights in forecasting the variance. The weights were taken to be parameters to be determined using the past observations made. It was calculated every day from the last 22 business days of the month.

Bollerslev (1986) gave a generalized model of GARCH of estimating the parameters. It proved to have yielded successful results in predicting conditional variances. Both interest rates and Treasury bill rates vary with time, the method described above can be used to determine the variance in each case as described in the next chapter.

In a study carried by Papadamou and Siriopoulos (2014) in UK, showed that fluctuations of interest rate affected significantly stock returns of companies. They investigated effect of Monetary Policy committee (MPC) on interest rate risk and insurance companies in UK. CAPM and Fama-French GARCH-models were used in modeling the interest rates and stock returns.

Investigation done by Mohammed Torkestani and ElhamBorujerdi (2014), found that there is a positive relationship between rates charged by life insurance firms and the bank rates in Iran,that's fluctuations occurring in bank rates resulted to changes in rates charged by insurance firms.

Maina,kamau and Kasungu(2013) noted in their research that the key component in foreign exchange market stability is political stability of a country otherwise high volatility in exchange rates will force firms to add risk premium to prices of their products. Usually measures taken by central Bank to maintain exchange rate stability have an effect on bank interest rates which directly or indirectly affect the prices of Treasury bills. Kenya operates on floating exchange rates which are highly volatile thus they do affect the interest rates.

CHAPTER THREE

3.0 METHODOLOGY

3.1 INTRODUCTION:

This chapter describes how data was obtained and the methodology used to conclude the study. The ARCH and GARCH models are discussed and the two factor Model used in regression analysis.

3.2 Research Design:

It is the collection and analysis of data in order to meet the set objectives through empirical evidence of past economic information, Schindler (2006).This research uses descriptive research design and looks at treasury/interest rates in the past and analyses it to come up with a model that can be used in forecasting future volatility of interest and Treasury bill rates. The data is obtained from the Central Bank of Kenya website.

3.3 Sample size of Treasury bills traded.

The sample size covers the period from January 2012 to July 2016, there are 4 weighted average rates of Treasury bills in a month, therefore in one year data considered is 4*12=48, The average rates for each month is determined so that there are 12 samples in a full year. .

3.4 Sample size of interest rates

The average spot rate of interest rates covering period from January 2012 to July 2016, calculated on weighted average for each month is considered. Each month has 12 weighted average rates times 4 for 4years 10 months in consideration, total is 55 Samples are considered

3.5 The ARCH MODEL

The ARCH models were introduced by Engle (1982). They are used to model financial data with time series characteristics. Suppose that $y_{1,}\ \ y_2 \ldots y_t$ time series observations and let ψ_t be the set of y_t up to time t, for $t \leq 0$. The process $\{y_t\}$ is an Autoregressive Conditional Heteroscedastic process of order p, ARCH (p), if:

$$r_t = \mu + y_t \tag{3.4.1}$$

$$y_t = \sigma_t \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2) \tag{3.4.2}$$

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j y_{t-j}^2 \tag{3.4.3}$$

With $\alpha_0 \geq 0$, $\alpha_j \geq 0$ and $\sum_{j=1}^{p} \alpha_j < 1$, as the ARCH model parameter limits.

The conditions stated guarantees that the conditional variance be positive

3.5.1 Properties of ARCH (p) Model

(i)The Mean;

From equation 3.5.2, the conditional expectation and variance of x_t is, given the expectations of ε_t is zero, then the expectations of y_t is given as:

$$E(y_t) = 0$$

(ii) **The Second Moment or Variance**;

$$= E(y_t^2) = E(\sigma_t^2 \varepsilon_t^2)\ E(\sigma_t^2) \tag{3.4.4}$$

Since $\sigma^2 = 1$, following a standard normal distribution of ε_t.

$$E(\sigma_t^2) = \alpha_0 + \alpha_j \sum_{j=1}^{p} E(y_{t-1}^2) \tag{3.4.5}$$

Given $E(\sigma_t^2) = E(y_{t-1}^2)$ under stationarity assumption,

$$E(\sigma_t^2) = \frac{\alpha_0}{1 - \sum_{j=1}^{p} \alpha_j} \tag{3.5.6}$$

For ARCH (1), the variance is given by;

$$E(\sigma_t^2) = \frac{\alpha_0}{1-\alpha_1} \tag{3.4.7}$$

(iii)The Kurtosis;

First, the forth moment of the time series is obtained

,

$$E(y_t^4) = E\{(\sigma_t^2)^2 \varepsilon_t^4\} = E\{(\sigma_t^2)^2\}E(\varepsilon_t^4)$$

$$= 3E\{(\sigma_t^2)^2\} \tag{3.4.8}$$

$$E\{(\sigma_t^2)^2\} = E\{(\alpha_0 + \sum_{j=1}^{p} \alpha_j y_{t-1}^2)^2\}$$

$$= \alpha_0^2 + 2\alpha_0 \sum_{j=1}^{p} \alpha_j E(y_{t-1}^2) + \sum_{j=1}^{p} \alpha_j^2 E(y_{t-1}^4)$$

Substituting equation (3.5.8), we have;

$$E(y_t^4) = 3\{\alpha_0^2 + 2\alpha_0 \sum_{j=1}^{p} \alpha_j E(y_{t-1}^2) + \sum_{j=1}^{p} \alpha_j^2 E(y_{t-1}^4)\}$$

Under stationarity, $E(y_{t-1}^2) = E(\sigma_t^2) = \dfrac{\alpha_0}{1-\sum_{j=1}^{p}\alpha_j}$ and $E(y_t^4) = E(y_{t-1}^4)$

$$E(y_t^4) = 3\frac{\alpha_0^2(1+\sum_{j=1}^{p}\alpha_j)}{(1-\sum_{j=1}^{p}\alpha_j)(1-3\sum_{j=1}^{p}\alpha_j^2)} \tag{3.4.9}$$

The Kurtosis is given by;

$$= \frac{E(y_t^4)}{\{E(y_t^2)\}^2}$$

Substituting equations (3.5.7) and (3.5.9), we get;

$$3\frac{(1+\sum_{j=1}^{p}\alpha_j)(1-\sum_{j=1}^{p}\alpha_j)}{1-3\sum_{j=1}^{p}\alpha_j^2} \qquad (3.4.10)$$

Therefore, the kurtosis is

$$= 3\frac{1-\sum_{j=1}^{p}\alpha_j^2}{1-3\sum_{j=1}^{p}\alpha_j^2} \qquad (3.4.11)$$

When j=1, we get ARCH (1), then the Kurtosis of ARCH (1) is;

$$= 3\frac{1-\alpha_1^2}{1-3\alpha_1^2} \qquad (3.4.12)$$

Which is strictly greater than 3 unless $\alpha_1 = 0$. The kurtosis for a normally distributed random variable Z is 3. Thus, the kurtosis of y_t is greater than the kurtosis of a normal distribution, and the distribution of y_t has a heavier tail than the normal distribution, when $\alpha_1 > 1$.

3.4.2Fitting Procedure for ARCH model

There are two steps in fitting in the ARCH Model:

1st step: Plot the return of interest rates with time and Treasury rates with time, Log returns of both interest rate and treasury rates and analyze the autocorrelation function (ACF) and the partial autocorrelation function (PACF) between them.

An ARCH model assumes working with returns, therefore variables are to be converted into returns.

2nd step: Perform tests, such as ARCH effect test or the Q-test.

Autocorrelation detected has to be quantified. Quantification is done by the preceding the Ljung-Box-Pierce Q-test and Engle's ARCH test. Performing a Ljung-Box-Pierce Q-test, it can be verified approximately, the presence of any significant correlation in the returns when tested for up to 20 lags of the ACF at the 0.05 level of significance.

3.4.3 Weakness of ARCH model

Despite ARCH model able to capture the characteristics of financial time series data, it has some weaknesses which include; ARCH treats positive and negative returns in the same way (by past square returns) and restrictive in parameters. It often over-predicts the volatility, because it responds slowly to large shocks and volatility from it persists for relatively short amount of times unless p is large.

3.5 The GARCH Model

A new model was necessary to address the weaknesses of ARCH model; Bollerslev (1986) therefore proposed a useful extension of ARCH.The Generalized Autoregressive Centralized Heteroskedastic Model (GARCH) with only three parameters that allow for an infinite number of squared roots to influence the current conditional variance unlike the ARCH. This feature allows GARCH to be more persistence than ARCH model.

Although ARCH incorporates the feature of Autocorrelation observed in return volatility of most financial assets, GARCH improves ARCH by adding a more general feature of conditional heteroscedasticity. Parameters *p* and *q* in GARCH (*p, q*) are frequently used for modeling the volatility of financial returns; these models generate good estimates with few parameters.

The process y_t is a Generalized Autoregressive Conditional Heteroscedastic model of order p and q, GARCH (p, q) if:

$$r_t = \mu + y_t$$

$$y_t = \sigma_t \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \ldots\ldots + \alpha_p y_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots\ldots + \beta_q \sigma_{t-q}^2$$
$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (3.5.1)$$

Where q > 0, $p \geq o, \alpha_0 > 0, \alpha_i \geq 0$ for i =1, 2… p, $\beta_j \geq 0$ for j=1…q are the GARCH model parameter limits. Again these conditions are needed to guarantee that the conditional variance $\sigma_t^2 > 0$.

3.5.1 Properties of GARCH (p, q)

(i)The mean;

From equation (3.5.1), the conditional expectation and variance of x_t is:

$E(y_t) = 0$, since the expectation of ε_t is 0.

(ii)**The Second Moment or Variance;**

$E(y_t^2) = E(\sigma_t^2 \varepsilon_t^2) = E(\sigma_t^2)$ (3.5.2)

$$E(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i E(y_{t-i}^2) + \sum_{j=1}^{q} \beta_j E(\sigma_{t-j}^2) \quad (3.5.3)$$

Given $E(\sigma_t^2) = E(y_{t-1}^2) = E(\sigma_{t-j}^2)$ under stationary assumption,

$$E(\sigma_t^2) = \frac{\alpha_0}{1-(\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j)} \quad (3.5.4)$$

For GARCH (1, 1)

$$E(\sigma_t^2) = \frac{\alpha_0}{1-(\alpha_1 + \beta_1)} \quad (3.5.5)$$

(iii)**The Kurtosis;**

First the forth moment of the time series is obtained;

$$E(y_t^4) = E\{(\sigma_t^2)^2 \varepsilon_t^4\} = E\{(\sigma_t^2)^2\} E(\varepsilon_t^4) = 3E\{(\sigma_t^2)^2\}$$

But

$$E\{(\sigma_t^2)^2\} = E\{(\alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2)^2\}$$

$$= \alpha_0^2 + 2\alpha_0 \sum_{i=1}^{p} \alpha_i E(y_{t-i}^2) + 2\alpha_0 \sum_{j-1}^{q} \beta_j E(\sigma_{t-j}^2) + \sum_{i=1}^{p} \alpha_i^2 E(y_{t-i}^4) + \sum_{j=1}^{q} \beta_j^2 E[(\sigma_{t-j}^2)^2] + 2\sum_{i=1}^{p} \sum_{j=1}^{q} \alpha_i \beta_j E(y_{t-i}^2 \sigma_{t-j}^2)$$

When $i = j = 1$, we get GARCH (1, 1)

$$E\{(\sigma_t^2)^2\} = \alpha_0^2 + \alpha_1^2 E(y_{t-1}^4) + \beta_1^2 E\{(\sigma_{t-1}^2)^2\} + 2\alpha_1\beta_1 E(y_{t-1}^2\sigma_{t-1}^2) + 2\alpha_0\alpha_1 E(y_{t-1}^2) + 2\alpha_0\beta_1 E(\sigma_{t-1}^2)$$

$$= \alpha_0^2 + (3\alpha_1^2 + 2\alpha_1\beta_1 + \beta_1^2)E\{(\sigma_{t-1}^2)^2\} + 2\alpha_0(\alpha_1 + \beta_1)E(\sigma_{t-1}^2) \quad (3.5.6)$$

Assuming the process is stationary, $E\{(\sigma_t^2)^2\} = E\{(\sigma_{t-1}^2)^2\}$

Hence

$$E\{(\sigma_t^2)^2\} = \frac{\alpha_0^2 + 2\alpha_0(\alpha_1 + \beta_1)E(\sigma_{t-1}^2)}{1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2}$$

$$= \frac{\alpha_0^2 + 2\alpha_0^2(\alpha_1 + \beta_1)}{(1 - \alpha_1 - \beta_1)(1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2)}$$

$$E(y_t^4) = 3E\{(\sigma_t^2)^2\}$$

$$= 3\frac{\alpha_0^2 + 2\alpha_0^2(\alpha_1 + \beta_1)}{(1 - \alpha_1 - \beta_1)(1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2)} \quad (3.5.7)$$

The Kurtosis is given by;

$$= \frac{E(y_t^4)}{\{E(y_t^2)\}^2}$$

Substituting equation (3.7.5) and equation (3.7.7), we get;

$$= 3\frac{1 - (\alpha_1 + \beta_1)^2}{1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2} \quad (3.6.8)$$

Which is strictly greater than 3 unless $\alpha_1 = 0$

The same fitting procedure is applicable for a general GARCH (p, q).

3.5.2 Forecast of Conditional Variance in GARCH model

The formula used to calculate the multi-step forecasts of the conditional variance for the GARCH (1, 1) model is illustrated below, the variance equation is

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.5.9}$$

Denote the forecast origin by n and the forecasted value by h and let F_n be the information set available at time n. For $h = 1$, the 1-step ahead forecast of the conditional variance is simply

$$E(\sigma_{n+1}^2 | F_n) = E(\alpha_0 + \alpha_1 y_n^2 + \beta_1 \sigma_n^2 | F_n)$$

$$= \alpha_0 + \alpha_1 y_n^2 + \beta_1 \sigma_n^2 \tag{3.5.10}$$

For $h = 2$, 2-step then, it becomes

$$E(\sigma_{n+2}^2 | F_n) = E(\alpha_0 + \alpha_1 y_{n+1}^2 + \beta_1 \sigma_{n+1}^2 | F_n)$$

$$= \alpha_0 [1 + (\alpha_1 + \beta_1)] + \alpha_1 (\alpha_1 + \beta_1) y_n^2 + \beta_1 (\alpha_1 + \beta_1) \sigma_n^2 \tag{3.5.11}$$

Recursively, it is easily seen that for $h = j$, the j -step ahead forecast of the conditional variance of the GARCH (1, 1) model is

$$E(\sigma_{n+j}^2 | F_n) = \alpha_0 \sum_{k=o}^{j-1} (\alpha_1 + \beta_1)^k + \alpha_1 (\alpha_1 + \beta_1)^{j-1} y_n^2 + \beta_1 (\alpha_1 + \beta_1)^{j-1} \sigma_n^2$$

$$= \alpha_0 + (\alpha_1 + \beta_1) * E(\sigma_{n+j-1}^2 | F_t) \tag{3.5.12}$$

Therefore, the forecasts of the conditional variances of GARCH (1, 1) model can be computed recursively.

The GARCH (1,1) model is good in predicting volatility changes .The model describe the time evolution of the average of squared errors i.e. magnitude of uncertainty however they fail in to explain why uncertainty tends to cluster. It does well in some periods and worse in others. The interest rates are modeled to determine the moderate and peak periods of changes using volatility. The same is applied to changes in rates of Treasury bills.

3.5.3 Conditional Error Distributions

The following error distributions are analyzed to select the best distribution to use in forecasting in GARCH (1,1)**.**

3.5.4 Normal Distribution

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad , -\infty < x < \infty \quad (3.5.13)$$

3.5.5 Student- t Distribution

When $\nu \to \infty$ the distribution converges to a standard Normal

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)\left(1+\frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}} \quad (3.5.16)$$

3.5.6 Generalized error distribution

$$f(x) = \frac{\lambda.s}{2.\Gamma\left(\frac{1}{s}\right)} . \exp(-\lambda^s . [x-\mu]^s \quad (3.5.15)$$

Where,

λ –scale parameter

μ-location parameter

$\mu(z)$-Euler Function

s- Shape Parameter.

3.5.7 Model selection criteria

Selection criteria is used to find out whether the fitted model gives an optimum balance between persistence and goodness-of-fit. The common models used for criteria selection are Akaike Information Criterion (AIC), Hannan- Quinn criterion (HQ), Bayesian Information criterion (BIC), Schwarz Information criterion (SIC), and Log likelihood criterion.

Equation of each model is as:

AIC = -2log (maximum likelihood) + 2k where k= p + q +1 if the model contains an intercept or a constant term and k = p + q.

BIC = -2log (L) +2(m)

HQ = -2log (L) + 2mlog (log (n)).

SIC = -2log (L) + (m + mlog (n))

Log (L) is the log likelihood.

The model that gives the minimum or lowest value is the most desirable to use.

3.6 MUILTFACTOR MODEL

This is a financial model that employs multiple factors in its computations to explain market phenomena or equilibrium asset prices. The multifactor model can be used to explain either an individual security or a portfolio of securities. It compares two or more factors to analyse relationships between variables and the security's resulting performance.

The multifactor model was considered for this study as the variables are mainly macro-economic and financial data. These variables have unique characteristics that can only be handled by the P_i is the performance on the pension Fund

a_i, c_i are the constant and random parts respectively of the component of pension fund performance

$I_1 \ldots I_4$ are the systematic economic factors that influence the performance of pension fund obtained from the GARCH forecasts. The volatility of the individual variables which were modeled using the GARCH model were consolidated to give the overall pension plan performance.

A multifactor model for the pension plan performance is given by an equation of the form:

$$P_i = a_i + b_{i,1}I_1 + b_{i,2}I_2 + b_{i,3}I_3 + b_{i,4}I_4 + c_i \qquad (3.5.16)$$

Where:

$b_{i,4}$ is the sensitivity estimates of the economic variables, that is they represent the standard deviations of variables which were obtained after the factors have been modeled.

The following are the assumptions of Multifactor model:

The factor realizations, I_t are stationary with unconditional moments

$$E(I_t) = \mu_I$$
$$\text{cov}(I_t) = E\{(I_t - \mu_I)(I_t - \mu_I)'\} = \Omega_I \qquad (3.5.17)$$

The specific error term c_{it} , are uncorrelated with each of the common factors, I_{ki} ,

$$\text{cov}(I_{kt}, c_{it}) = 0 \text{ for all } k, i \text{ and t}$$

Error terms c_{it} are serially uncorrelated and contemporaneously uncorrelated across assets

$$\text{cov}(c_{it}, c_{js}) = \sigma_t^2 \text{ For all } i = j \text{ and } t = s$$

3.6.1 Fitting Multi-factor model

The following are the steps required to fit the multifactor model;

Step1: Obtain the values of $b_i's$ of the variables using GARCH model as the standard deviations of these variables.

Step2: Forecast the economic variables $I_i's$ using GARCH model

Step3: Linearly combine these values to obtain a multifactor model with a_i being the base value or constant term and c_i the random variable error term with a zero error mean.

CHAPTER FOUR:

DATA, RESULTS AND DISCUSSION

4. 0 INTRODUCTION

This section deals with the analysis and interpretation of the results of monthly interest rates and Treasury bill rates volatilities from January 2012 to July 2016.

The raw data used for interest rates was obtained from central Bank of Kenya website: www.centralbank.go.ke/commercial- banks-weighted-average-rates/ and raw data for Treasury bill rates was obtained from: www.centralbank.go.ke/treasury-bonds/ and IRA Yearly Reports available in www.ira.go.ke/annual/reports.

3.6 Data

Descriptive Statistics

The raw data of interest rates considered in this study is the weighted average for each month from January 2012 to July 2016.

As already mentioned earlier the interest rate is converted to returns before using the ARCH/GARCH models. Similarly the Treasury bill rates will analyzed using the same formula for returns.

$$l_t = \left(\frac{p_t}{p_{t-1}} - 1\right) * 100\%$$

Where

p_t -The current interest rate

p_{t-1} –The previous interest rate

The interest rate of return is;

Formula for returns is given as:

$$r_t = log\frac{l_t}{l_{t-1}}$$

Where r_t is returns up to period t

3.7 Raw data analysis

The time plot of the monthly interest rates is as shown in figure 1;

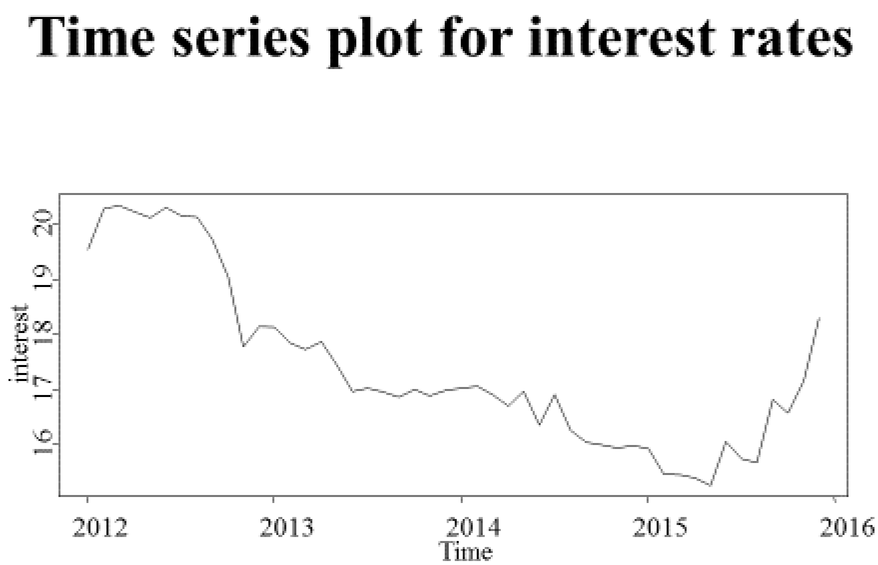

FIGURE 0-1: RAW DATA PLOT OF INTEREST RATES VS. TIME IN MONTHS.

The interest rate trend can be seen clearly gradually decreasing from peak at December2012 to February 2015 and starts rising again to July 2016. Clustering is also observed. The volatility characteristics of financial time series data can be clearly seen from the fall –rise of interest rates.

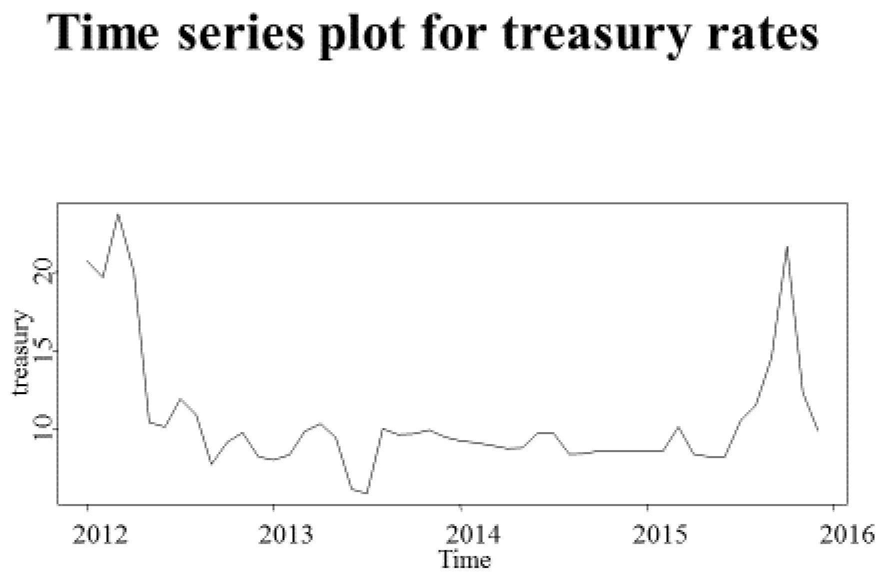

FIGURE 0-2: PLOT FOR TREASURY BILL RATES VS. TIME (MONTHS)

The trend of Treasury bill rates is clearly falling from peak December 2012 to the middle of the year and fairly stabilizes up to April 2015, it rises again sharply towards July 2016. Volatility characteristic of financial time series data is clearly seen. In both cases it observed highest rates occur at the same period .It shows that is a correlation which need further investigation.

3.8 Time-series characteristics for both plotted.

The plot clearly shows variation of interest rates with time, peak period and low periods occurring at the same showing correlation features. Both rates are time dependent or display characteristics of time-series financial data.

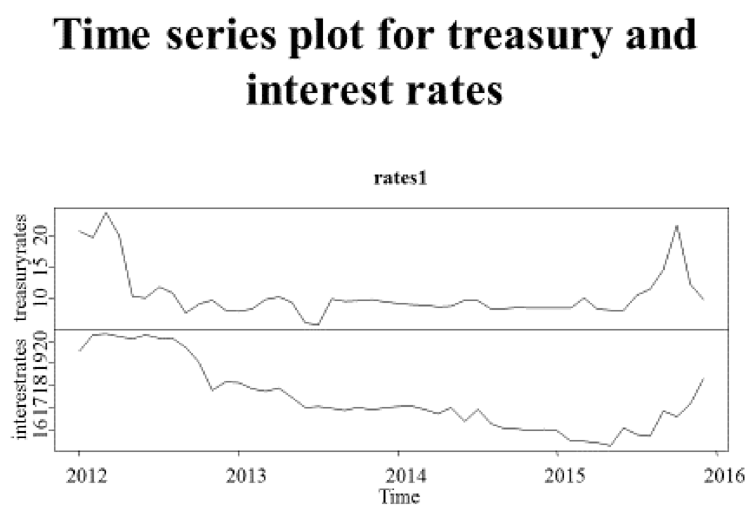

FIGURE 0-3: COMBINED PLOT FOR INTEREST RATES AND TREASURY BILL RATES.

Peak periods are 2012 and 2016 when rates were very high. We can proceed to the autocorrelation and partial correlation.

Key:

Blue line for Interest rates

Dark line for Treasury rates

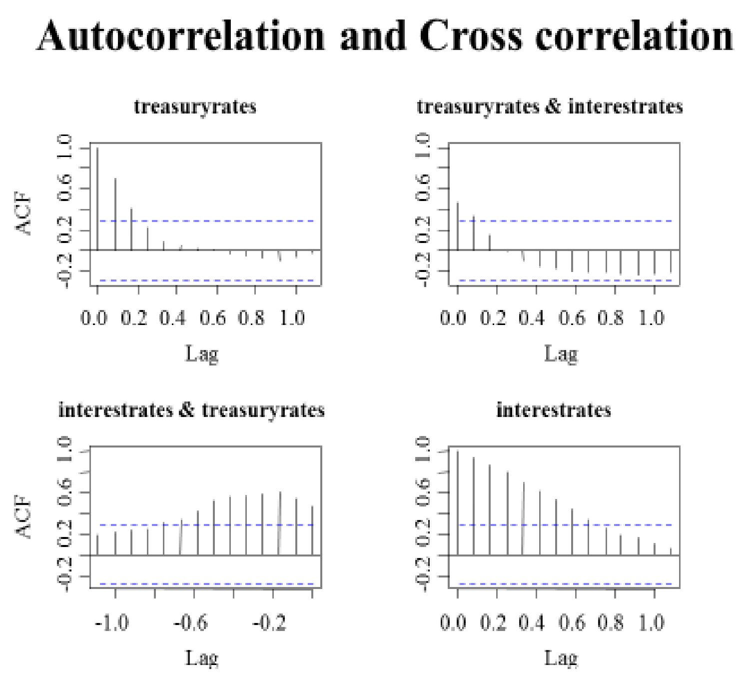

FIGURE 0-4: THE AUTOCORRELATION AND CROSS-CORRELATION OF INTEREST AND TREASURY RATES.

3.9 ARCH EFFECT TESTS

We can show the ARCH effects by using simple and log returns, log interest rates and Treasury rates.

3.9.1 Simple and Log returns Descriptive Statistics.

	Interest	Treasury
nobs	54.000000	54.000000
NAs	0.000000	0.000000
Minimum	-0.066176	-0.478443
Maximum	0.072704	0.695058
1. Quartile	-0.011587	-0.078151
3. Quartile	0.005225	0.047109
Mean	-0.001140	-0.000985
Median	-0.003751	-0.006121
Sum	-0.061540	-0.053171
SE Mean	0.003249	0.025909
LCL Mean	-0.007656	-0.052952
UCL Mean	0.005377	0.050982
Variance	0.000570	0.036249
Stdev	0.023874	0.190392
Skewness	0.747587	0.658052
Kurtosis	3.074789	3.906281

TABLE 0-1: SIMPLE AND LOG RETURN INTEREST AND TREASURY RATES:

Based on the results of basic statistics of data, mean of simple and log return for interest and Treasury rates are -0.001140 and -0.000985 respectively are very close to zero. The values of kurtosis are 3.074789 and 3.906281 which is greater than 3 hence the data exhibits excess kurtosis showing heavy tailed distribution. The values of skewness are 0.747587 and 0.658052 for interest rate and Treasury rates respectively which are greater than zero.

3.9.2 Standardized Residuals Tests:

Jarque-Bera Test R Chi^2 = 25.11513 statistic-p-value = 3.518182e-06

	Interest	**Treasury**
Test statistics	10.52586	13.91997
Parameter	12	12
p-value	0.5699285	0.305848

TABLE 0-2: LJUNG BOX TEST FOR LOG RETURN FOR INTEREST AND TREASURY RATES.

From Ljung box test for log returns, p-value and test statistics suggest that ARCH effects are significant since p-value are less than one

3.9.3 Plot for log return of Treasury rates.

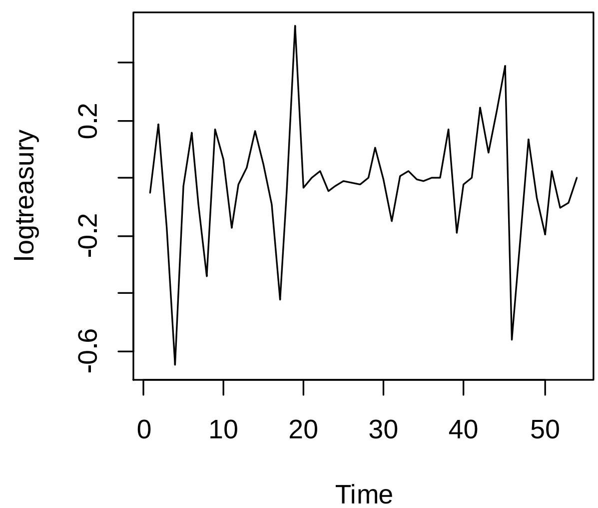

FIGURE 0-5: LOG RETURNS OF TREASURY BILLS VS. TIME IN MONTHS.

From the log returns plots of returns, volatility clustering can be clearly seen where there's a fall-rise or rise-fall of rates.. The mean reverting property can also be seen clearly where the returns revolve around a certain value. ARCH characteristics is shown from the figure, the fact it shows both negative and positive values of log returns.

3.10 GARCH MODEL

The square log plots for interest rates and Treasury rates are shown the figures below,

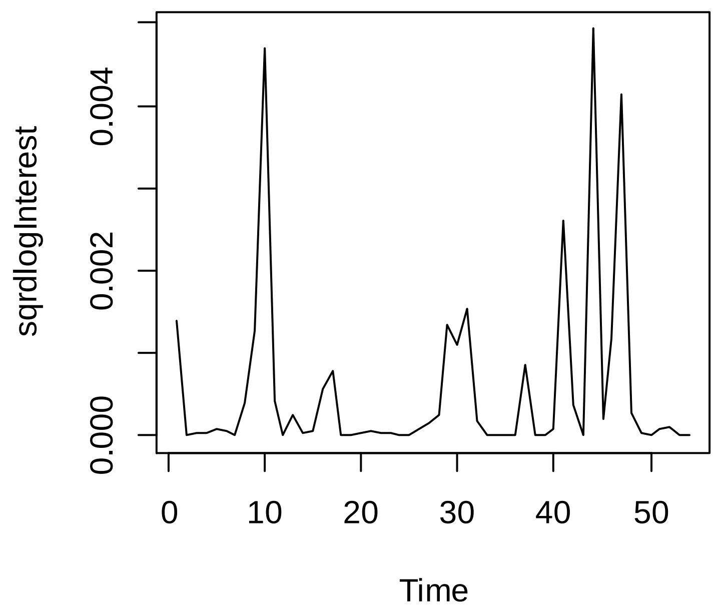

FIGURE 0-6: PLOT OF LOG RETURNS OF INTEREST RATES.

It is clearly seen from the plots GARCH effects differ from Arch due to fact that it treats returns as positive only. Volatility is again clearly shown, for interest rates are very high in the 10^{th} and between 40^{th} and 50^{th} months. The advantage of GARCH model is that it treats all returns as positive**.**

DISTRIBUTIONS MODELS SELECTION CRITERIA

3.10.1 NORM, Sd-t, and GED

There are three distributions suggested, thus Normal distribution, Student T test and GED ,we use the following formulas to test their suitability before it is used in the GARCH (1,1) modeling of the interest rates and Treasury bill rates changes. We can use the QQ plots to see which one tries to fit the data in linear model.

The results of QQ plots is as shown below;

QQ plot for GARCH(1,1),Normal Distribution

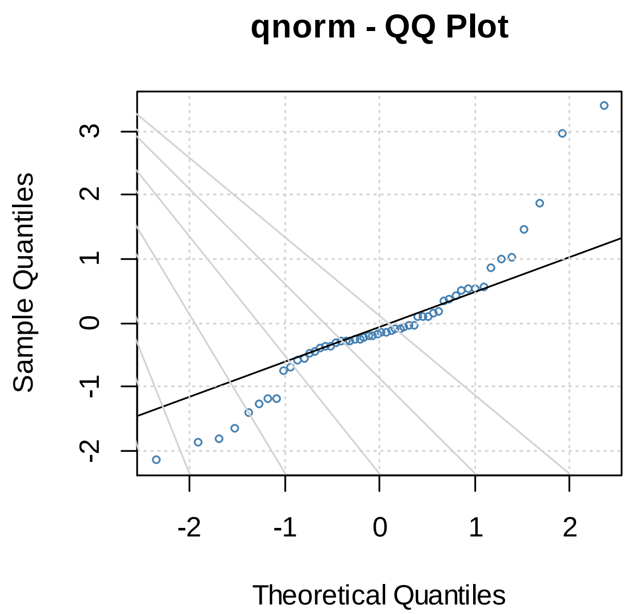

FIGURE 0-7: QQ PLOT OF GARCH (1,1) WITH NORMAL DISTRIBUTION.

This is clearly indicated by the failure of the data to be linear at the trails, it suggests a heavily tailed distribution for the residuals since norm QQ plots poorly fits.

QQ plots for GARCH (1, 1), student T distribution.

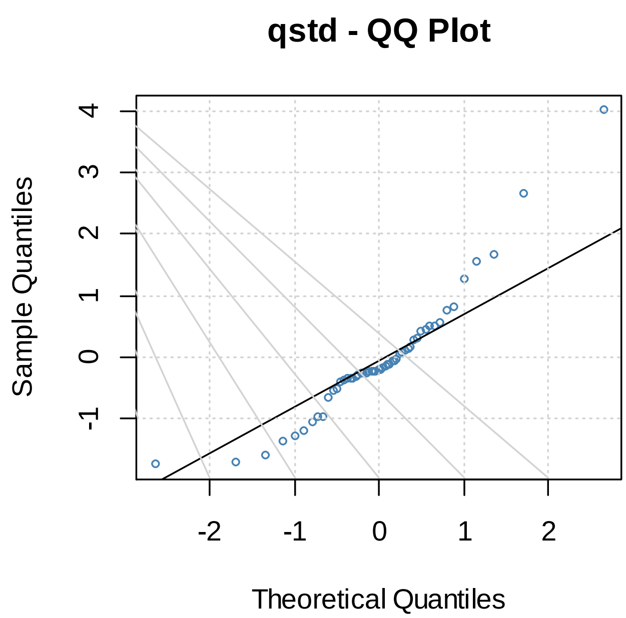

FIGURE 0-8: QQ PLOTS FOR GARCH (1, 1), STUDENT T DISTRIBUTION.

The std-QQ plot seems to have relatively fair fits with student-t distribution being the residual distribution

QQ plot for GARCH (1, 1), GED distribution.

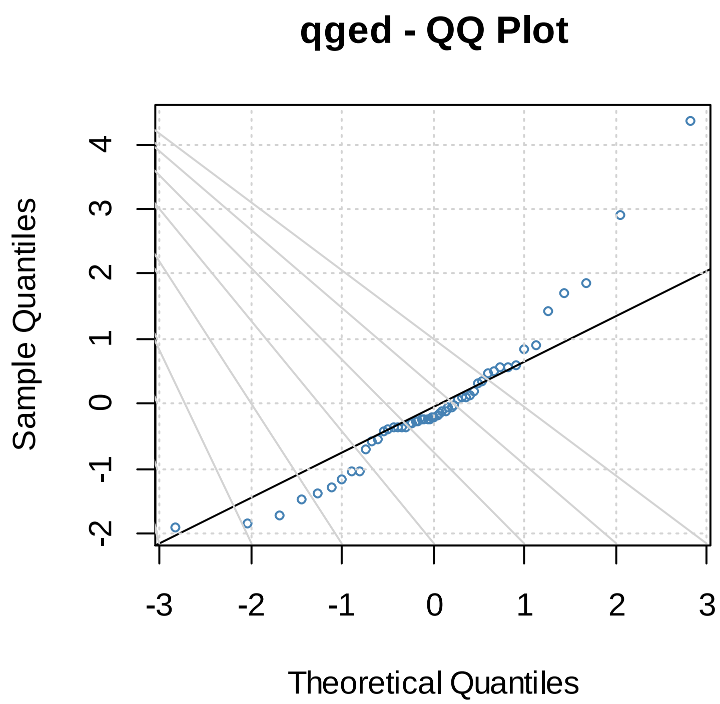

FIGURE 0-9: QQ PLOT FOR GARCH (1, 1), GED DISTRIBUTION

It is clearly that the GED-QQ plot fairly fit than the student-t distribution. The data fits along the line.it seems that three distributions fits the data into linear model, however GED stands out to be the best where the gap between the line and fitted data is very small as compared to the other two. Based on this information, now the GED distribution with GARCH (1, 1) is used to predict the future volatility of interest and Treasury rates.

3.10.2 Testing using AIC, BIC, SIC, HQIC and LL

The tests showed the following results, Criterion Statistics:

AIC	BIC	SIC	HQIC	LL
-4.646305	-4.529806	-4.64O056	-4.597690	128.2882

TABLE 0-3: CRITERION STATISTICS

The criteria used in selecting the model is the one with minimum or smallest value. It is evident from the figures that the lowest is -4.646305 which is the AIC.

3.10.3 COMPARISON OF AIC OF NORMAL DIST., STUDENT T, AND GED;

We perform yet another test along with the condition $\alpha + \beta < 1$ for GARCH for each distribution to choose the best distribution to be used in GARCH (1, 1) model. The results are displayed in the following tables shown below;

		Standardized	Residuals Tests	Statistic p-Value
Jarque-Bera Test	R	Chi^2	25.10385	3.538088e-06
Shapiro-Wilk Test	R	W	0.9045178	0.000407079
Ljung-Box Test		Q(10)	12.90447	0.2290624
Ljung-Box Test	R	Q(15)	19.03649	0.2120849
Ljung-Box Test	R	Q(20)	19.9813	0.4591
Ljung-Box Test	R^2	Q(10)	13.91629	0.1768442
Ljung-Box Test	R^2	Q(15)	14.97993	0.4528642
Ljung-Box Test	R^2	Q(20)	15.20514	0.7645518
LM Arch Test	R	TR^2	15.30931	0.2249553

Error Analysis:	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
Omega	1.671e-04	9.585e-05	1.743	0.0813.
Alpha1	5.649e-01	3.745e-01	1.508	0.1315
Beta1	2.858e-01	1.772e-01	1.613	0.1067

TABLE 0-4: GARCH (1, 1) NORMALIZED DISTRIBUTION

The condition of the coefficients is obeyed as shown below.

The sum of $\alpha + \beta < 1$ as shown by the values of alpha and beta,

$0.5649+0.2858 = 0.8507 < 1$, this is an indicator that volatility is persistence. The values of α and β are at 1% significance level.

	Estimate	Std. Error	t value	Pr(>|t|)
Omega	2.865e-04	2.377e-04	1.205	0.228078
alpha1	1.000e+00	7.147e-01	1.399	0.161750
Beta1	1.000e-08	1.342e-01	0.000	1.000000
Shape	2.817e+00	8.255e-01	3.413	0.000642 ***

Table 0-5: GARCH (1, 1), Student T test.

The sum of $\alpha + \beta < 1$ with student-t distribution in GARCH(1,1),values are shown as:

$0.00000001+ 0.002817 < 1$

	Estimate	Std. Error	t value	Pr (>|t|)
Omega	2.446e-04	1.283e-04	1.907	0.0565.
Alpha1	8.098e-01	6.061e-01	1.336	0.1816
Beta1	1.000e-08	1.525e-01	0.000	1.0000
Shape	1.000e+00	2.307e-01	4.334	1.47e-05

Table 0-6: GARCH (11) GED

Sum of α + β < 1 as indicated by the values below,

0.80978050+0.00000001<1 Volatility is persistence in all the three distributions. The more accurate method of choosing the distribution is summarized in next section.

3.10.4 SELECTION CRITERION FOR NORM, STD t AND GED

We can use the AIC value to the one with the lowest value from the summarized table below.

MODEL	DISTRIBUTION	SELECTION CRITERION	VALUES
GARCH11	NORM	AIC	-4.646305
GARCH11	STD t	AIC	-4.851885
GARCH11	GED	AIC	-4.852741

TABLE 0-7: AIC TABLE FOR THE THREE DISTRIBUTIONS:

We choose on the model distribution which has the least AIC which Garch (1,1) is GED, in this case is GED Distributions has the lowest value.

Finally the distribution has been chosen which can be used in the GARCH (1,1) to forecast variance of the interest rates and Treasury bill rates discussed in the next page.

3.11 FORECASTING PLOTS

Essentially any model chosen should be able to predict the possible future occurrences under given conditions such as Market, Economic conditions, etc. In the model, forecast for the next 12 months is projected as shown below. Figure below shows forecast interest rates from 55^{th} month to 67^{th} month.It shows the rates are farely constant. This is because of political stability ,agricultural produceis sufficient and there prices are stable.The prices of petroleum prices are farely constant.The interest rates are now controled the by the act of parliament.

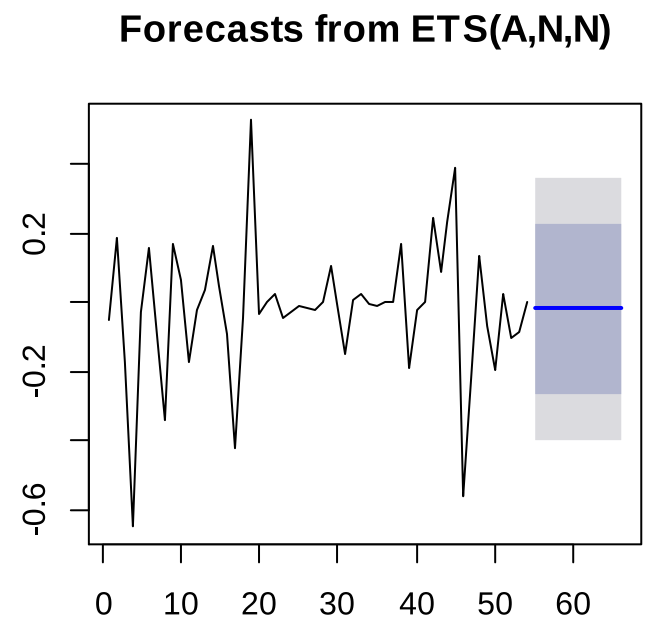

FIGURE 0-10: FORECAST PLOT FOR NEXT 12 MONTHS

From figure 10, it is clear that the interest rates will remain constant in the next 10 months shown at 5% prediction blue/dark shaded area, reason being measures taken by the government to reduce interest rates, availability of agricultural products, prices of petroleum products remaining constant .For the next 12 months predictions at 80% interval are shown in the shaded blue area, and at 95% prediction interval as grey shaded area.

3.12 MULTIFACTOR MODEL

Using the GARCH (1, 1) model, the standard deviation estimates for the variables are obtained. The standard deviation of interest rate was 0.603457. The treasury rate has a standard deviation of 0.58028. These values are represented by $b_i's$ in the multifactor model. The base value a_i was set to be 300 billion shillings. Combining these values we get the multifactor model as:

$$P_i = 300{,}000{,}000{,}000 + 0.603457 I_1 + 0.580287 I_2 \quad (4.10.1)$$

Where P_i is the pension plan value at time i, I_1 is the interest rates, I_2 is the treasury rates.Using the equation of the multifactor model and substituting the standard deviations of the variables together with their forecasted values, we obtain the projected pension plan values for the next 10 years.

The projected 10 year pension plan values.

Year	Pension plan Value
1	2.567327×10^{15}
2	2.709412×10^{15}
3	2.851079×10^{15}
4	2.992966×10^{15}
5	3.134863×10^{15}
6	3.276769×10^{15}
7	3.418687×10^{15}
8	3.560617×10^{15}
9	3.702563×10^{15}
10	3.844382×10^{15}

TABLE 0-8: FORECAST FOR THE NEXT 12 MONTHS.

The model above was used to forecast the growth of pension fund as the interest and Treasury bill rates change with time. Other factors affecting the fund such corruption that is becoming a virtue to some people was assumed constant, The projection is if 300billion is invested in the beginning of year 1 then after10 years fund would grow up to 3.844382×10^{15}

From the model, it can be seen that when all other values remain constant at time zero, the pension plan performance would be 300 billion. This value was taken to be the base value in which the pension plan was assumed to have started with.

From the model, it can be seen that when all other values remain constant at time zero, the pension plan performance would be 300 billion. This value was taken to be the base value in which the pension plan was assumed to have started with. According to these values of the variables, there is evidence that the interest rate affected the pension plan performance more than Treasury bill rates.

Interpretation of Findings

From the above Multifactor Model, the study found out that macroeconomic variables and interest rates influence the pension plan performance more than the treasury rates. The study established that the coefficient for the interest rate is high meaning that the interest rate significantly influence the pension plan performance in Kenya. These finding contradicts the findings of Najarzadeh et al (2009) who found out that that the interest rates have a negative impact on the pension performance in long run and have a positive impact in the short term.

CHAPTER FOUR

4.0 CONCLUSION

The volatility of interest rates and Treasury rates was modeled using ARCH and GARCH(1, 1) and tested QQ-plots,it was seen clearly from results that volatility of interest rate varies with time. There were large variations followed by large variations of variance and also small changes followed by other small changes. The Treasury rates displayed same results. The GARCH(1, 1) was used with GED distribution to forecast the volatility for the next 12 months. Result showed volatility changes are small due to the fact that interest rates and Treasury rates are expected to remain fairly constant.

The Multifactor Model was used to assess the effects of volatility of interest and Treasury rates on pension fund. The study established that the coefficient for interest rate is high than that of Treasury rates meaning that interest rates significantly influence the pension fund performance in Kenya.

4.1 Limitations of the study.

The Treasury bill rates are given on weekly basis, the data has high frequency if analyzed on daily basis than monthly averages. I used monthly averages so that same model is applied to interest rates which were only available as monthly weighted averages.

There are many other factors affecting pension fund apart from interest rates and Treasury rates and are not quantified in the model therefore required more time to investigate.

5.0 REFERENCES

Akaike, H (1974). A New look at the statistical Model in identification. I.E.E.E. *Transaction on Automatic Control*, 19(6), 719-723.

Bollerslev, T. (1986).*Generalized Autoregressive Conditional Heteroscedasticity*. Journal of Econometrics, 64, 93-110.

Bollerslav T, Chou R.Y, Kroner K.F, (1992). ARCH Modeling in Finance, *"A Review of the Theory and Empirical Evidence*". J. Economic.52:5-59

Central Bank of Kenya (2015).*Monetary Policy Committee Report*, October 2015

Engle, R.F. (1982).*Autoregressive Conditional Heteroscedasticity with Estimate of variance of United Kingdom inflation*. Econometrica, 40, 987-1007.

McNeil A, Frey R (2000). *Estimation of tail-related risk measures for heteroscedastic financial time series*: an extreme value approach "J.Empir.financ.7:271-300.

Papadamou, S. and Siriopoulos, C, (2014).Interest rate risk and the creation of the monetary policy committee: *Evidence from banks' and life insurance companies, stocks in the UK*,*Journal of Economics and business* ,71,pp45-67.

Sharku, G, Leka, B. and Bajrami, E. (2011).Considerations on Albanian life insurance Market, *Romanian Economic Journal*, 14(39), pp.133-150.

McNeil A, Frey R (2000). *Estimation of tail-related risk measures for heteroscedastic financial time series*: an extreme value approach "J.Empir.financ.7:271-300.

www.centralbank.go.ke/commercial- banks-weighted-average-rates/:
www.centralbank.go.ke/treasury-bonds/