

**STOCHASTIC MODELLING FOR THE PENSION FUNDS COMPANIES; A  
CASE OLD MUTUAL PERSONAL PENSION PLAN**

**BY**

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## DECLARATION

This project is my original work and has not been presented for a degree in any other University

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This project has been submitted for examination with our approval as University Supervisor.

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**PROF. R. SIMWA**

## **DEDICATION**

I dedicate this work to my wonderful family for the priceless support.

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## **ACRONYMS AND ABBREVIATIONS**

**ALM:** Asset-Liability Model

**ARCH:** Autoregressive conditional heteroscedasticity

**ARIMA:** Autoregressive integrated moving average

**DB:** Defined Benefit

**DC:** Defined Contribution

**GDP:** Gross Domestic Product

**LTCM:** Long Term Capital Management

**NSSF:** National Social Security Fund

**OASDI:** Old Age, Survivor, and Disability Insurance

**RBA:** Retirement Benefits Authority

**SSA:** Social Security Administration

## ABSTRACT

The study aimed to determine stochastic modelling for the pension funds companies; a case of Old Mutual Personal Pension Plan. The study was guided by the following objectives; to identify economic factors affecting stochastic modelling in pension schemes, to determine a suitable stochastic model that can be used to model pensions for Old Mutual Personal Pension Plan and to establish the efficiency of the proposed pension schemes. The study adopted a case study research design. The target population was the management staff of Old Mutual Personal Pension Plan in the Nairobi branch. The data collection instruments that were used to collect data from the selected respondents was data collection sheet. Data was collected quantitatively from primary and sources. It was checked for accuracy and completeness. The data collected was analyzed using statistical package for social scientist (SPSS). The findings of the study were presented statistically in tables and figures. Inferential statistics of correlation, regression and Chi-square were employed. The study found that inflation, wage growth rate, long term interest rates and equity returns on assets were positively correlated to each other and significantly affected the efficiency of stochastic modelling. The study concluded that of the four variables results equity returns on assets affected the efficiency of the stochastic modelling the highest. This is attributed to the fact that pension schemes engage in investment of the funds of their pensioners and therefore the returns from this investment affect the effectiveness in performing their obligations. The study concluded that the proposed stochastic model was efficient and should be therefore employed in the pension funds companies. The study made the following recommendations; pension funds should invest wisely in order to ensure that their returns are good since their performance are crucial in meeting the needs of their pensioners and pension schemes should employ a suitable stochastic model that pay careful attention to inflation, wage growth rate, long term interest rates and equity returns on assets since these economic factors influence greatly the performance of pension schemes.



# **CHAPTER ONE**

## **INTRODUCTION**

### **1.1 Background of the Study**

A pension plan is a method for a prospective retiree to transfer part of his or her current income stream toward a retirement income (Davis and Hu, 2004). Pension plans are usually classified into two categories; defined-benefit plan-the pension fund, (e.g., employer) guarantees that the pensioner will receive a fixed, predefined, benefits upon retirement, regardless of the investment's performance and defined-contribution plan - the pension fund makes predefined contributions, usually tax exempt, toward a pool of funds, set aside for the pension fund's future benefit. The pool of funds is then invested on the retiree's behalf allowing her/him to receive benefits upon retirement. The final benefit received by the retiree depends on the investment's performance.

The benefits are paid upon the pensioner's retirement, usually in a lump sum (United Nations, 2007). However, some countries, such as the UK, members are legally required to purchase an annuity, which then provides a regular income. A public pension fund, is one that is regulated under public-sector law, while a private pension fund, is regulated under private-sector law, (United Nations, 2007). In a number of countries, the distinction between public or government pension funds and private pension funds may be difficult to assess. In others, the distinction is made sharply in the law, with very specific requirements for administration and investment.

Stochastic modeling is a form of financial modeling that includes one or more random variables. The main purpose of this kind of modeling is estimating that the possible outcomes tend to be within a forecast that it's work is to predict the conditions that are there for different situations. Pension schemes use them to plan their business in order to be successful in providing their services. Deterministic pension models, often rely on projections that are based on several assumptions concerning the "average" long-time behavior of the stock market (Russek, 2011). The use of a stochastic model displays handiest a pragmatic decision at the part of the modeler that any such model represents the best presently to be had description of the phenomenon under attention, given the statistics this is available and the universe of models known to the modeler.

Globally pension schemes have confronted crisis. for example, within the US, there was a \$1 trillion gap on the end of the monetary year 2008,, among the \$2.35 trillion that US states had to set aside to pay for their employees' retirement advantages and the \$3.35 trillion rate tag of these guarantees (Keegan, 2012). The existing price of unfunded obligations underneath Social safety as of August 2010 turned into approximately \$5.4 trillion (OASDI, 2010). Moreover, US state and local pension plans exhibit a structural shortfall that was likely pose a long-enduring problem, according to the US Congressional Budget Office (Russek, 2011). In the UK, many employees face retirement with an income well short of their expectations. Employees who pay into a Defined-contribution plan for 40 years, may get only half of the retirement income they could have expected (Coggan, 2011). According to the International Monetary Fund. Western economies would have to set aside an additional 50% of their 2010 GDP to support the retirees. Several reforms have been suggested to amend the pension crisis.

In South Africa, The South African pension machine consists of a non-contributory, method-tested public advantage software, diverse pension and provident fund arrangements and voluntary financial savings. Enterprise-based totally retirement plans have a long history in South Africa. The retirement funding machine has been in location in view that 1956 when the Pension funds Act turned into handed. But occupational retirement plans are limited to those employed in the formal quarter. Among those, the insurance rate is pretty high by way of global assessment and is predicted to be between 66% and 84%. For a few professions, employers are free to outline obligatory membership as an obligatory condition of employment. according to the national Treasury, the ratio of overall pension fund belongings to GDP in South Africa was approximately sixty three%, which compares favorably across the world (Stewart and Yermo (2009), in step with the 2007 annual report of the financial services Board, general pension fund property amounted to ZAR 1,284 billion (EUR 172 billion) in 2005, up from ZAR 1,098 billion (EUR 143 billion) in 2004. Therefore the control of this area may be very critical, considering that failure in the quarter results in very high losses and frustration in the economic system and for the pensioners.

### **1.1.1 Pension in Kenya**

The retirement blessings zone in Kenya consists of the civil provider scheme, the countrywide Social security Fund, occupational schemes and man or woman schemes, with a coverage fee of round 15% of the staff (10% of the contributors of the NSSF, 3% inside the civil provider scheme in 1.5% occupational schemes as well as 0.5% blanketed by way of person retirement advantage schemes).

The National Social Security Fund is a public provident fund, installed below an Act of Parliament. Those covered are employed folks, investors, the self-hired and, due to the fact 2004, a few employees within the informal region. It is obligatory for all those employing with at least five personnel to sign up their members, although open to all different people referred to above. Members of these schemes make contributions five 'percent' of month-to-month earnings as much as a most of 2 hundred shillings every month, which is the contribution fee for the ones making greater than KShs. 4,000. The employers have to pay 5% of the payroll, with challenge to a most of KShs. 400. Self-hired humans contribute a total of 5% in their month-to-month earnings, and not using a minimum or maximum income limits for contribution purposes specifically. Being effected from the start of June 2007, contributors of NSSF can increase the amount they save at any factor in time with an amount that could be less than or even equal to KShs. 1,000. The elderly-age pension advantages, are much at the disposal to the ones elderly 55 who've retired from employment but are insured. They're to be provided at the age 50 in the case where the individual is employed but not in insured. The new and also current retirees can acquire their blessings as a lump sum.

The Civil service Pension Scheme does cover all the participants of the Civil service; and is mounted beneath an Act of Parliament as the Pay As You Go system. It's far presently non-contributory, despite the fact that plans are underway to make it become a contributory machine.

Voluntary, and occupational pension plans may be administered via pension finances or provident price range, and via the DB or the DC arrangements. An organization or a group of those employing can also, on an optional basis, and could establish a

complementary professional pensioning plan for their employees. The plans are mostly set up via one unmarried organization. The membership to such occupational pensions or the provident plan is frequently mandatory for covered personnel. The minute the employee makes a decision to grow to be a member, but, withdrawal from club whilst being hired through the equal business enterprise is not allowed. And the employees who are inside five years of the plan retirement age, after they begin work with the organization or whenever there is a new plan is set up aren't qualified for membership within the case of the DB plans.

There aren't any legal regulations for employee or corporation contribution ranges. A standard plan requires employees to make contributions at a charge of 5% of earnings and employers contributing 10%. Personnel could be permitted to make any additional voluntary payment contributions to the plan excluding any kind of restrict (although the contributions are best tax-deductible until a certain limit). Then the full of employee and organisation contributions is tax-deductible as much as the restrict of the lower of KShs20,000 shillings or 30% of income. All funding profits earned with the aid of tax-registered retirement advantages schemes is tax-loose.

Plans may only be in a defined benefit, or in a defined Contribution in nature (around 80% are DC). The age for which these benefits end up available, isn't regulated and need to be laid down in plan policies. Upon attainment of the age of retirement, the provident plans will pay out a large payment. Pension plans, pay out benefits out as a month-to-month pension for the relaxation of the insured man or woman's lifestyles. Up to 1-third of total blessings can be changed into a lump-sum payment only if the specific plan is contributory (and it is 25% if non-contributory). The Board of Trustees provides the

scheme member the possibility to pick out their annuity provider and desired annuity. The plan rules do allocate exceptional advantage formulation for different categories of employees under certain occasions. Each lump sum payments, and annuities enjoy extra beneficent tax treatment, as long as the involved member retires on incapacity or age grounds, or the case that he has been a member of a retirement blessings scheme from the minimum of 10 years. These pensions are tax-unfastened for pensioners receiving as much as KShs26,000 and they have no other assets of income, or KShs15,000 if they produce other sources of income. As from June 2007, the pensions for the individuals above the age of sixty five are not taxable. And for the non-tax-registered retirement benefits schemes, contributions and investment earnings are taxable at regular tax costs. But, no in addition tax is exercised at the savings after the member retires.

Retirement benefits schemes are run with the aid of trustees. The half of these trustees are appointed by the participants, and half are nominated by using the company. All these trustees are required to engage the offerings of any belongings supervisor for the control in their assets. A refinement in 2005 abolished the opportunity for employees to withdraw all their belongings earlier than accomplishing the regular retirement age. Due to the fact then, employees can handiest withdraw their business enterprise's portion of contributions in the event that they have been participants of the scheme for much less than 12 months, and if the vesting rules allow them to make this kind of withdrawal. They are able to withdraw their very own contributions earlier than accomplishing the age of 55 if they're chickening out from a retirement blessings scheme. With effect from January 2008, they had been additionally be allowed to assign their savings as mortgage protection. In 2007, there had been around 1357 lively occupational pension schemes, of

which approximately 10% have been DB schemes. Most people of schemes are pension schemes as opposed to provident price range. As of 2006, pension fund assets amounted to around 250bn KES (c\$three.5bn). Investment regulations include as much as 70% in home and regional stocks, 15% offshore and 30% in real estate.

In terms of private pension preparations, 14 men or woman, DC-kind pension schemes exist, which cowl less than 1% of the population. They're in most cases, offered through coverage groups and are to be had to everybody. They're attractive to the ones people whose employers do now not offer a 401-k and to the self- employed. In might also 2007, the Zimele non-public pension plan, a voluntary retirement savings arrangement for all public- and personal-sector workers, changed into introduced. It will be managed by way of the personal company Zimele Asset management corporation. The plan operates on the basis of pooled funds. Contributions and funding earnings are exempt from tax. Amana private 401-k plan is the only different man or woman retirement blessings scheme this is similarly structured to the Zimele personal 401-k plan.

The pension machine in Kenya, has been supervised via the unbiased Retirement benefits Authority (RBA) because 2000, which oversees the 1997 Retirement benefits Act, which introduced regulation, protection and structure to the pension enterprise. The RBA maintains work to broaden the industry and teach trustees. However, challenges nevertheless stay for the RBA, along with bringing a few budget in line with the regulation, and to a completely funded stage. Publish-retirement poverty, low insurance, low contributions costs and the HIV/AIDS epidemic (which had reduced the life expectancy of Kenyans to underneath the ordinary retirement age) gift challenges for the Kenyan pension machine. The creation of a —0ll pillar has been proposed, imparting

established minimum pension to the population over sixty five. The Civil Servants Pension Scheme, non-contributory, can be reformed to introduce worker contributions and a revision of the retirement age from fifty five to 60. There have additionally been proposals to introduce a mandatory detail into the voluntary occupational and person schemes and to create a —fourth pillar inside the shape of tax incentives for own family aid and the purchase of a domestic

## **1.2 Problem Statement**

Pension finances are aimed toward presenting retirement profits for the pensioner. They are set up with the purpose of catering for individuals of their retirement days. However, the duty of a hard and fast, predefined quantity of advantages upon retirement exposes the insurer to a notable chance. The calculation of the benefits quantity is primarily based on monetary assumptions which can be difficult to measure or predict. These assumptions encompass lifespan of employees, returns earned by using pension investments, future taxes, and uncommon occasions, together with herbal screw ups. However, described-contribution plans transfer the hazard to the insured, who is depending on the pension fund overall performance upon his/her retirement date.

The demanding situations faced with the aid of pension finances corporations in meeting their obligations to their pensioners have affected the self-assurance of people in those funds for the reason that experience of pensioners have painted a picture of trouble in getting access to their advantages and some have accessed less that they'd expected. These demanding situations have been felt at an international scale. a number of the current occurrences consist of the Russian economic crisis (1998), right here numerous elements, consisting of artificially high constant exchange charge and continual monetary



deficit led the Russian authorities to devalue the Ruble, default on home debt, and claim a moratorium on charge to overseas creditors (Feridun, 2004).”As a result, inflation reached 84% that 12 months. Banks closed down. Tens of millions of human beings misplaced their life savings. As an instantaneous consequence, US Hedge funds collapsed, inclusive of long time Capital management (LTCM), which received a \$3.6B bailout (Greenspan, 2007) under the supervision of the Federal Reserve. The Dot-com bubble (2000) where NASDAQ Composite misplaced 78% of its value. \$5 trillion loss inside the marketplace price of corporations. The maximum latest was the Subprime loan disaster (2008) where people misplaced extra than a quarter of their net worth. Housing prices dropped, GDP commenced contracting, and unemployment charge rose from 5% to 10%. S&P500 fell fifty seven% from its October 2007 height. US total country wide debt rose shape sixty six% GDP pre-crisis to over 103% publish-crisis. Those challenged have posed the want to conduct a study to on pension budget as a way to shed mild on what can be accomplished to ensure that the sector performs. This study therefore aimed to decide stochastic modelling for the pension funds businesses; a case of vintage Mutual non-public 401-k.

### **1.3 Objective**

#### **1.3.1 Main Objective**

The main objective of the study was to determine stochastic modelling for the pension funds companies; a case of Old Mutual Personal Pension Plan

#### **1.3.2 Specific Objective**

The study was guided by the following objectives;

- i) To identify economic factors affecting stochastic modelling in pension schemes
- ii) To determine a suitable stochastic model that can be used to model pensions for Old Mutual Personal Pension Plan
- iii) To establish the efficiency of the proposed pension schemes

#### **1.4 Research Questions**

The study sought to answer the following research questions;

- i) What are the economic factors affecting stochastic modelling in pension schemes?
- ii) Which is the suitable stochastic model that can be used to model pensions for Old Mutual Personal Pension Plan?
- iii) What is the efficiency of the proposed pension schemes?

#### **1.5 Significance of the Study**

The study will be significance to the following parties;

##### **Management of pension schemes**

The findings will be of significance to the management of pension schemes. This is because they will provide information on the suitable stochastic model that the firms can use in order to ensure that they are able to meet their obligations and be profitable.

### **Policy Makers**

The study will be of significance to policy makers since it will provide information on the stochastic models employed and their suitability. It will give the policy makers insight in formulating policies that will ensure that the industry is profitable

### **Future Scholars**

The study will be of relevance to future scholars since it will provide a body of literature that scholars interested in looking at the same or other related areas can refer to. The study will also contribute to knowledge in the field of stochastic modelling and pensions system operations

### **1.6 Scope of the Study**

The study aimed to look at stochastic modelling for the pension funds companies; a case of Old Mutual Personal Pension Plan. The study specifically looked at; economic factors affecting stochastic modelling in pension schemes, suitable stochastic model that can be used to model pensions for NSSF and to establish the efficiency of the proposed pension schemes. The study was done through a case study research design of Old Mutual Personal Pension Plan Nairobi offices. The study was conducted between the months of September and October 2016.

## **CHAPTER TWO**

### **LITERATURE REVIEW**

#### **2.0 Introduction**

This chapter covers the review of literature related to the study it will highlight the contribution of other scholars in the field. The literature will involve economic factors affecting stochastic modelling in pension schemes, suitable stochastic models and efficiency of proposed models.

#### **2.1 Economic Factors affecting Stochastic Modelling in Pension Schemes**

Projections are made for indicators of a pension scheme in order to be able to assess some of the future financial capability of the pension plan in general. These projections include demographic and macroeconomic factors as well as pension scheme indicators.

DB pension plan modeling has been investigated in discrete time by Cairns and Parker (1997), Dufresne (1989), Haberman (1994) as well as in continuous time by Cairns (1996) with the rates of return modeled as a single stochastic variable. Zhang and Hou (2011) calculated the optimal investment strategy for a DC pension plan where the wage increase and inflation are stochastic. Chang and Cheng (2002) used stochastic interest rates and inflation rates to model a DB pension plan. A stochastic wage index was also calculated as a function of the inflation rates.

To perform a DB pension plan valuation in Canada, actuaries are required to determine the values of the economic and demographic variables. For economic variables, the future wage increases, and the future return on the assets are needed. Usually, a building block approach is used to determine the wage increases.

That is, future wage increase is determined as the sum of three components: future expected inflation, future general productivity increases, and future merit increases. The expected return on the assets is a weighted average of the returns of each asset class in the target asset mix. For DC pension plans, an assumption is that employee's pension account should include a similar asset classes same as in the assets for the DB pension. Because the projections of a Pension plan is a series of valuations at different future time points, economic variables which are necessary for projecting a pension plan are similar to those required for valuation of a pension.

## **2.2 Suitable stochastic Models**

Inside the complex environment wherein pension finances have to function, ALM requires an instrument to discover and manage risks. Such a tool is a stochastic simulation model, which mimics the conduct of a pension fund by using incorporating randomness to gain a statistical pattern of possible effects (Campbell and Viceira, 2005). ALM modeling is accordingly a key method in strategic threat control. It includes growing mathematical eventualities of the destiny evolution of property and liabilities, given sure assumptions approximately the statistical residences of the variables that have an effect on the evolution of assets and liabilities. Even though dynamic models have established a higher match for real world scenarios, they do have their drawbacks, as due to their complexity they may be tougher to recognize and interpret (Campbell and Viceira, 2005).

There are many approaches to generate these situations. The traditional technique become to create a critical state of affairs, and to carry out a few strain testing around it. With time the fashions have end up more sophisticated, concerning stochastic simulation of

property and liabilities (Gerstner, et al., 2008). Modern-day ALM studies depend upon stochastic models that generate heaps of situations, with distinctive possibilities attached to each. While the conventional ALM research targeted on asset optimization with a deterministic view of liabilities, these days ALM is increasingly used to simulate the consequences of regulations for distinct stakeholders whilst complying with the requirements of the regulatory government. On this feel, ALM systems are used as integrated making plans systems to simultaneously determine investment, funding, and—if applicable—indexation regulations, thereby balancing the desires of the unique stakeholders.

Each ALM device has its personal particular characteristics, principally ruled by way of the underlying stochastic strategies and distribution features (Campbell and Viceira, 2005). The principle financial uncertainty comes from converting asset fees and hobby fees, and from transition probabilities. But how do they behave? Questions which includes “Are there differences among brief-time period and long-time period conduct?” (Gerstner, et al., 2008) must be asked, and if the answer is yes, they have to be modeled. Moreover, modeling generally concentrates only on stocks and bonds, so the question arises whether funding products need to additionally be covered (Gibbs, and McNamara, 2007).

Asset liability management for pension finances is a threat management method, which takes under consideration the property, the liabilities, and additionally the interactions among the special rules which the board of a pension fund can practice (Berardinelli et al. 2007). The board of a pension fund must locate proper regulations that assure with big probability that the solvency of the fund is enough for the duration of

the making plans horizon and, at the identical time, all promised gain bills might be made. The solvency is the capacity of the pension fund to satisfy all promised payments within the long-run. Generally, the solvency at a sure time second is measured because the investment ratio.

Consider that this is the ratio of belongings and liabilities. Underfunding happens whilst the funding ratio is less than one. Another manner of characterizing underfunding is by using announcing that the excess is poor, where the surplus is the difference between the fee of the property and the price of the liabilities. The excess is the a part of the reserves of the pension fund that isn't wanted for paying advantage payments. The funding ratio modifications through the years, specifically due to fluctuations in the liabilities and inside the belongings. Consequently, a pension fund rebalances its asset portfolio and adjusts as an instance its contribution charge regularly, with the intention to control adjustments of the investment ratio over time. In case of distress, the sponsor of the fund may also should assist out with a remedial contribution. In the ALM selection system, conflicting interests of different events exist. Inside the subsequent segment, we will appearance in greater detail on the pastimes of different parties.

The popularity of ALM in pension funds seems to have risen in recent years. Society of Actuaries (2003) affords a useful definition: ALM is the practice of coping with an enterprise selections and actions interested by appreciate to assets and liabilities are coordinated. ALM can be described as an ongoing system of formulating, enforcing, monitoring and revising strategies related to belongings and liabilities to acquire agencies economic targets, given the corporations risk tolerances and other constraints. ALM is applicable to, and crucial for, sound management of the finances of

any organization that invests to satisfy its destiny cash waft desires and capital requirements.

Most manifestly, ALM ought to be the cornerstone of any pension organization's investment coverage. Feinberg [2002] reports that many pension funds at the moment are carrying out extra asset/liability studies especially due to the deterioration of their funded reputation. She has interviewed many pension fund managers to study the motive for the recent boom in the recognition of the asset/liability studies. The demand for those asset/legal responsibility research has occurred due to diverse reasons, including: market situations, switching from defined benefit finances to described contribution finances, extra contributions, expanded liabilities due to the infant boomers retirement and adjustments inside the destiny benefits shape.

according to Chernoff [2003], a pension fund cannot just maximize its go back via the use of traditional efficient frontier approach the precise manner is to healthy pension property towards pension liabilities, and he simplifies: healthy the belongings and the liabilities and go to mattress Ito [1995] argues that the goal of pension fund asset control is to provide investment for the pension liabilities, but a pension fund sponsor has also a secondary purpose that is the fulfillment of an "profits spread" (i.e. the advantageous hole between property and liabilities), as this income unfold can reduce the requirement for future contributions.



### **2.3 Efficiency of Proposed models**

In preference to using traditional deterministic modeling, researchers were utilizing stochastic modeling for the assessment of the lengthy-time period financial ability of social security schemes, specifically for the OECD countries. Ronald Lee, Shirad Tuljapurkar and others have produced a series of research to discuss the effects of demographic adjustments at the long-term budget of the U.S. social safety system. A populace forecast for the length 1994-2070 became made with the aid of the introduction of stochastic forecasts for mortality and fertility, and the use of a deterministic forecast for net immigration. Assumptions from the Social security management (SSA) deterministic forecasts were acquired for the economic variables, however stochastic forecasts are made for the productivity increase and interest charges, following an impartial AR(1) tactics. in addition they employ stochastic forecasts in some other paper for those variables, as a substitute the productiveness increase charge tends to observe an ARMA limited mean (ARMA-CM) model, and the interest price follows an AR(1). The two papers have the authors inspecting the volume to which these variables make contributions to the uncertainty of economic forecasts via as an alternative permitting these variables to be deterministic or stochastic.

For the influences of demographic modifications handiest on the U.S. old Age, Survivor, and incapacity coverage (OASDI) accept as true with budget, Tuljapurkar and Lee (2000) makes stochastic populace forecasts with stochastic mortality and fertility, and deterministic internet immigration, even as taking values of financial variables, together with the inflation, actual hobby and productivity increase fees, and exertions force participation price from the Trustees' 1995 record.

Full-size differences in feasible future trajectories of the fund are located, after which propose using stochastic forecasts with supplementary strategies to examine uncertainty. In a similar paintings for stochastic projections of the Social safety believe Fund, Lee et al. (2003) also make stochastic forecasts for real wage increase, the interest price, and fairness returns the use of a vector auto regression. They then study numerous plans for reaching lengthy-term solvency by using raising the normal retirement age, increasing payroll taxes, and making an investment some part of the fund inside the inventory marketplace. Further examining the influences of internet immigration in the social protection fund, Lee et al. (2004) make stochastic forecasts for this variable, and the consequences show that stochastic net immigration makes simplest little difference to the opportunity distribution of the old-age dependency ratio, and thus a small distinction in trust fund projections.

In growing their very own lengthy-term actuarial version to evaluate the Social safety administration (SSA)'s seventy five-year projections of the believe funds, the Congressional budget workplace (2001) makes stochastic forecasts for demographic variables (inclusive of mortality, fertility, and immigration) and economic variables (inclusive of actual salary increase, unemployment, inflation, actual hobby fee, disability occurrence fee, and incapacity termination charge). As an instance, they estimate immigration and fertility with an ARMA (4,1) model, and real salary boom and mortality upgrades with AR(1) methods. To observe the simulated effects, they use each Monte Carlo strategies and bootstrap simulations so that it will generate an opportunity distribution for destiny results.

Fehr and Habermann (2004) talk the sustainability of the German pension device below demographic uncertainty during 2001-2050. The paper makes stochastic forecasts for fertility and mortality, however uses deterministic net immigration, to acquire stochastic population forecasts, even as economic variables are predicted by way of using an overlapping generation version with three sectors, i.e. household, manufacturing, and government. They discover that demographic changes have sizeable effects on the advantage stages inside the lengthy-time period. The paper then appears for the effects of changes in contribution fees and replacements costs at the financial balance of the gadget, in which generational fairness is likewise in focus. Børlum (2004) uses the same method, however takes net immigration to be stochastic, and unearths additionally that the demographic uncertainty may have extensive influences on the lengthy-time period advantage degrees of various participating cohorts in the Danish PAYG DB pension machine during the 2000-2050 simulation period.

The paper by using Kitamura et al. (2005) uses the Asset-liability version (ALM) with stochastic simulation to analyze the Japanese Public personnel' Pension Scheme. Amongst various variables within the version, they make stochastic forecasts for the increase fee of charges, increase rate of wages, and fee of investment return of the pension reserve fund with the intention to examine the appropriateness of an indexation rule, which impacts destiny gain tiers and financial situations of the scheme

Box and Jenkins (1976) proposed the use of the autoregressive incorporated transferring average (ARIMA) model to version time collection. Since then, a lot of variations of the model have been created. Box and Jenkins' model assumed constant innovation variance while Engle (1982) proposed a model with non-constant variances conditional on the

past. Engle's model is called autoregressive conditional heteroscedasticity (ARCH) model.

Hamilton (1989) considered a nonlinear stationary process. He proposed a model in which the trend of a series changes in response to discrete unobserved events. In other words, the time series follows an ARIMA model but the trend parameter changes when the underlying regime changes. The transition of the regime from one state to another is assumed to follow a Markov chain. This model is generally called a regime switching model. Since the publication of Hamilton's paper, regime switching models have been generalized to model the effect of the underlying regime on different parameters in ARIMA and ARCH models (for example Hamilton and Susmel (1994)).

In the meantime, Wilkie (1986, 1995) has proposed to model different economic series together through a cascade model. He first modeled inflation. Then he took advantage of the correlation between economic series to model them in layers, using one layer to explain another. For example, wages are modeled using inflation with the error terms following an autoregressive model of order one. In this report, only the following models are considered: white noise model; autoregressive model of order one (AR(1)); autoregressive conditional heteroscedasticity model of order one (ARCH(1)); regime switching model with two regimes; and transfer function (Wilkie model).

For the ARCH (1) model, the series is modeled as an AR (1) model with non-constant variances based on the previous observation. Also, three versions of the ARCH (1) model are considered with different parameters in the variance components set to zero. For the regime switching model, the class of models is also restricted to the AR (1) and ARCH (1) models only.

For the transfer function model, both one lag and two lags are considered. Only the five models mentioned above are considered because these models are simple enough to implement the pension plan projections, and flexible enough to do an adequate job to model the dynamics of the economic variables. Simple models can help actuaries, employers, and employees understand the dynamics of economic series more easily. This is an important aspect because stochastic modeling is not common outside the academic sector, and therefore, simplicity will encourage the usage of time series models

## CHAPTER THREE

### RESEARCH DESIGN AND METHODOLOGY

#### 3.1 Introduction

In this chapter we discuss the various methods of modeling that have been used on pension funding. Some of the approaches to modeling pension funds are; Deterministic Modeling, Sensitivity Analysis, and Stochastic Modeling

#### 3.11 Independent and Dependent Variables

Mathematical models may additionally include a few variables. For example, a mathematical version of a bus experience, ought to consist of three variables, bus speed, the time and, distance traveled. A mathematical expression relates the variables: the distance is calculated by multiplying the time factor by the speed. Any values could be set from the speed in use and the total time covered as long as they are around the vehicle's physicality; the maximum speed and the fuel tank capacity, and these are called independent variables. The distance traveled is a dependent variable and depends on the speed and time.

The model is quite less complex due to the fact that it's just a bus covering a certain distance at some constant speed. Additional acceleration period as well as deceleration period enhances this simple model. New nondependent variables and some mathematical relationships are created from every case there being an acceleration and a deceleration. In addition, new dependent variables might be added, just like the distance covered all through various elements of the ride. The effects are intermediate, and the very last end result is the total distance protected.

### **3.2 Deterministic Modeling**

In the case of deterministic modeling, specific values for each time frame covered between the start to the cease of the expected duration are for the non-established variables. The values are called the “assumptions”. In addition, the association between the nondependent variables, and any values obtained when the model is run, which are the dependent variables, should be already determined first. The term deterministic distinguishes this kind of modeling from the other models used such as stochastic modeling, in the modeling some or even the entire independent variables adopt the given values which is based on the likelihood that the given values will be realized. In this modeling the link between the no dependent and dependent variables could not be already fixed.

To be able to project trust fund balances, assumptions are made on these three kinds of nondependent variables: they are demographic, economic and program specific variables.

### **3.3 Sensitivity Analysis**

Deterministic modeling may not consider input variables considered the most crucial to the end result. It is hard to determine if, for example, a very small change in mortality improvement rate tends to have a much greater or sometimes even little effects on the trust fund equilibrium, compared to the quite small changes in inflation rate of the prices addition and it’s difficult to determine the scope for the change in a particular inclination for a given variable is due to a change in some different inclination for some other different variable. The conclusive results after specific adjustments are made to model inputs shows the need to ascertain the sensitivity.

Sensitivity analysis could be useful for better understanding of the long-range prospects to a trust fund. For the analysis, Intermediate-cost projection is commonly used, and if an assumption is changed from an original value it will show how the actuarial equilibrium has changed. Some of the assumptions studied are; total fertility rate, disability incidence rates, consumer price index, death rates, real interest rates, real wage differentials, disability termination rates, net immigration. The three projections make it possible for varying results to be seen when all the assumptions are changed; how sensitive the projection results is to the individual eight assumptions is evaluated through sensitivity analysis.

### **3.4 Stochastic Modeling**

Stochastic modelling makes use of the probabilities of independent variables to ascertain dependent variables. To develop a stochastic model, a range of all possible values is determined for all nondependent variables and a probability is allocated to each. This result is known as the “opportunity distribution of the independent variable”. A number of strategies may be used.

First method bases chance distributions of unbiased variables, on empirical studies of real-international phenomena they constitute. Values selected for every 12 months over the projection length, is independent of the values selected for other years.

A second approach, one this is utilized by the Social protection actuaries in stochastic projections, which bases the value of an unbiased variable every year at the values in previous years, in addition to with some random every year fluctuation. Within the analysis, the relationships are described in a manner that, inside the loss of the random



fluctuation, the values for a few given year would be same to those below the intermediate set of assumptions.

The techniques are important for correlation and covariance but the impartial variables selected couldn't be absolutely independent. as an example; productivity increase compared to GDP boom are correlated positively, every time the productivity boom is excessive, the GDP growth tends to be high, and while productiveness boom turns into low, the GDP increase becomes low as properly. Any other case is evaluating GDP growth and unemployment which are extremely negatively correlated; an excessive fee of unemployment may slow down the GDP boom and vice versa

A well designed stochastic analysis involves; determination of probability distributions, their corresponding correlations, and the covariances for non dependent variables. The precision involved in carrying out the process determines the value of a stochastic model. Once the process is complete, the trials results are tabulated and ordered, and statistical inferences could be made.

Not all the cases have the independent and dependent variables relationship known prior; in such a case the generalized format of this model is better when specified. In such cases, regression analysis is much preferred. The regression analysis techniques lets in the derivation of the fine possible definition of the relationships via using the determined past values from both the independent and based variables. From there a version for which destiny values of the dependent variables may be forecast may be subtle from the formed relationships.

## CHAPTER FOUR

### DATA ANALYSIS, PRESENTATION, INTERPRETATION AND DISCUSSION

#### 4.0 Introduction

This chapter presents the findings of the study, interpretation and discussion. The information gathered from the field was subjected to sifting and then it was presented in terms of tables. The findings are as follows.

#### 4.1 Economic Factors affecting Stochastic Modelling

The study sought to determine the economic factors affecting stochastic modelling. This was done by conducting a correlation analysis between the economic factor and stochastic modelling, the findings are presented in table 4.1

**Table 4.1 Correlations between Economic Factors and Stochastic Modelling**

		Stochastic modelling	Inflation	Wage Growth rate	Long-term Interest rate	Equity returns on assets
Stochastic modelling	Pearson Correlation	1	.399**	.346**	.412**	.356**
	Sig. (2-tailed)		.000	.002	.000	.001
	N	62	62	62	62	62
Inflation	Pearson Correlation	.399**	1	.593**	.552**	.591**
	Sig. (2-tailed)	.000		.000	.000	.000
	N	62	62	62	62	62
Wage Growth rate	Pearson Correlation	.346**	.593**	1	.387**	.532**
	Sig. (2-tailed)	.002	.000		.001	.030
	N	62	62	62	62	62
Long term interest rates	Pearson Correlation	.412**	.552**	.387**	1	.387**
	Sig. (2-tailed)	.000	.000	.001		.001
	N	62	62	62	62	62
Equity returns on assets	Pearson Correlation	.423**	.521**	.327**	.327**	1
	Sig. (2-tailed)	.000	.000	.001	.001	
	N	62	62	62	62	62

The findings indicate that there was a significant relationship between inflation and wage growth rate  $P=0.000$ , inflation and Long term interest rate  $P=0.000$ , inflation and equity returns on assets  $P=0.001$ , wage growth rate and long term interest rate  $P=0.001$ , wage growth rate and equity returns on assets  $P=0.030$  and long term interest rates and equity returns  $P=0.001$ .

The correlation results indicate that there was a significant relationship between stochastic modelling and inflation  $P=0.000$ , stochastic modelling and wage growth rate  $P=0.002$ , stochastic modeling and long term interest rates  $P=0.000$ , stochastic modelling and equity returns on assets.

These results indicate that the economic factors that affect pension funds are positively correlated to each other indicating that each has an effect on the other. These economic factors also have a significant correlation to the efficiency of stochastic modelling.

#### **4.2 Suitable Stochastic Model**

The study sought to determine the most suitable stochastic model, this was done by running a regression analysis of the variables under study. In order to conduct a regression analysis, there are a number of assumptions that were made; this include;

Multiple linear regression needs at least 3 variables of metric (ratio or interval) scale. A rule of thumb for the sample size is that regression analysis requires at least 20 cases per independent variable in the analysis, in the simplest case of having just two independent variables that requires  $n > 40$ . G\*Power can also be used to calculate a more exact, appropriate sample size.

A regression model was used to test the stochastic model, the following equation was employed to the study;

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e$$

Where Y= Suitable Stochastic Model

$X_1$ = Inflation

$X_2$ = Wage rate growth

$X_3$ = Long term interest rate

$X_4$ = Equity return on assets

$\beta$  = Beta Value

E = Error Term

A regression with stochastic regressors could be applied to obtain the proposed model, and then considering one of the assumptions for the ordinary least square(OLS), with random regressors,  $E = (e|x)$ , in the case where the assumption is violated, the following are the ways to deal with it;

- **If x is correlated with e**

If  $(e) = 0$  , and  $cov(x, e) = 0$  ends up replacing the assumption, then OLS is biased in a small samples if the replacement is true but the assumption is not

Therefore, OLS coefficient estimators are consistent and they are asymptotically normal.

In case the new assumption is violated, the result is OLS is biased and inconsistent.

Coefficient on x will picks up the effects of the parts of **e** that are correlated with it in

addition to the direct effects of  $x$ . And the direction of bias depends on the sign of correlation between  $x$  and  $e$ . Therefore, this is exactly analogous to omitted-variables bias

**Measurement error;** We suppose that the dependent variable is measured accurately, but that we measure  $x$  with error:  $\tilde{x}_i = x_i + \eta_i$

Then the estimated model is  $y_i = \beta_1 + \beta_2 \tilde{x}_i + (e_i - \beta_2 \eta_i)$ . And because  $\eta$  is part of  $\tilde{x}$  and as a result correlated with it, the composite error term is now correlated with the actual regressor, this means that  $b_2$  is biased and inconsistent. But if  $e$  and  $\eta$  are independent and normal, then

$\text{plim } b_2 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\eta^2} \beta_2$ . We conclude that the estimator is biased toward zero. And if most

of the variation in  $\tilde{x}$  comes from  $x$ , then the bias will be quite small. Well, as the variance of the measurement the error growth is in connection to the variation in the true variable, the magnitude of the bias also increases. For a worst-case limit, if the true  $x$  doesn't vary across our sample of observations, and all of the variation in our measure  $\tilde{x}$  is random noise, then the expected value of our coefficient is zero.

The best solution is getting a much better measure. The alternatives are instrumental variables, or the direct measurement of degree of the measurement error.

**Omitted-variables bias;** The omitted variable is included in error. If the omitted variable is somehow correlated with the included variable, then the OLS estimator of coefficient on the included variable is definitely biased and inconsistent.

**The Simultaneous-equations bias (simultaneity bias);** We suppose that  $y$  and  $x$  are part of a much larger theoretical system of equations:  $y = \beta_1 + \beta_2x + e$

$$x = \gamma_1 + \gamma_2y + u$$

Then the two variables are ‘jointly determined’ and both of them are endogenous. Then there is ‘feedback’ from  $y$  to  $x$ , or ‘reverse causality’, that is actually bidirectional.  $e \Rightarrow y \Rightarrow x$ , and so  $e$  and  $x$  are correlated

- **Instrumental variables**

Referring from the method of moments analysis by the OLS estimators, then the following population moment conditions were assumed;

We have  $E(e) = 0$ ,  $\text{cov}(x, e) = 0$  to derive the OLS normal equations as sample moment conditions:

$$\frac{1}{N} \sum_{i=1}^N \hat{e}_i^2 = 0, \quad \frac{1}{N} \sum_{i=1}^N x_i \hat{e}_i = 0$$

If the  $\text{cov}(x, e) \neq 0$ , subsequently the population moment conditions are invalid, and it will result to getting biased and inconsistent estimators from the OLS sample moment conditions.

**The instrumental-variables estimator;** it can be derived from the method of moments.

We suppose that  $y = \beta_1 + \beta_2x + e$ , but also suppose that  $\text{cov}(x, e) \neq 0$ . We let  $\mathbf{z}$  to be a variable with the following properties:

- $\mathbf{z}$  has no direct effect on  $y$ . So, it does not belong in the equation alongside  $x$ .  
Meaning that  $\mathbf{z}$  affects  $y$  only through  $x$ , but not independently.

- $\mathbf{z}$  is exogenous. So it is not correlated with  $\mathbf{e}$ .
- $\mathbf{z}$  is much strongly correlated with  $\mathbf{x}$ , and  $\mathbf{x}$  is the endogenous regressor.

Now this makes  $\mathbf{z}$  a valid instrumental variable. And we can exploit  $\text{cov}(\mathbf{z}, \mathbf{e}) = 0$  as the second moment condition in the place of  $\text{cov}(\mathbf{x}, \mathbf{e}) = 0$ , which is clearly not true for this model. Therefore, the sample moment conditions are;

$$\sum_{i=1}^N \hat{e}_i = \sum_{i=1}^N (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) = 0$$

$$\sum_{i=1}^N \hat{e}_i z_i = \sum_{i=1}^N z_i (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) = 0$$

Solving the normal equations, yields  $\hat{\beta}_2 = \frac{\sum_{i=1}^N (z_i - \bar{z})(y_i - \bar{y})}{\sum_{i=1}^N (z_i - \bar{z})(x_i - \bar{x})}$

We compare this to the standard OLS slope estimator  $b_2 = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^N (x_i - \bar{x})(x_i - \bar{x})}$

In the matrix terms,  $\hat{\beta} = (\hat{\mathbf{Z}}\mathbf{X})^{-1}\hat{\mathbf{Z}}\mathbf{y}$  vs  $\mathbf{b} = (\hat{\mathbf{X}}\mathbf{X})^{-1}\hat{\mathbf{X}}\mathbf{y}$

The properties of IV estimator; It is consistent as long as  $\mathbf{z}$  is exogenous. It is asymptotically normal, that is,

$$\hat{\beta}_2 \sim N \left[ \beta_2, \frac{\sigma^2}{r_{xz}^2 \sum_{i=1}^N (x_i - \bar{x})^2} \right], r_{xz} \equiv \widehat{\text{corr}}(x, z)$$

We then estimate  $\sigma^2$  by  $\sigma_{IV}^2 = \frac{\sum_{i=1}^N (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2}{N-2}$

Weak instruments in this case are: If  $r_{xz}$  is near zero, then the variance of  $\hat{\beta}_2$  is large and then the IV estimator is quite unreliable.

### Two-stage least squares

Having more instruments than the endogenous regressors, we then have an ‘overidentified’ system with different choices of instruments. And suppose that  $x_k$  is endogenous, but the first  $K - 1$  regressors are exogenous. And suppose that  $z_1$  through  $z_L$  are  $L$  valid instruments. Any linear combination of the instruments ends up being admissible. We choose the one that has more correlation with  $x_K$

To get that, we regress  $x_K = \gamma_1 + \gamma_2 x_2 + \dots + \gamma_{K-1} x_{K-1} + \theta_1 z_1 + \dots + \theta_L z_L + v_K$  and use the fitted values  $\hat{x}_K$  as the instrument for  $x_K$

This constitutes to doing two separate regressions, a first-stage regression of  $x_K$  on the exogenous  $x$  variables and the instruments  $z$ , and then a second-stage regression of

$$y = \beta_1 + \beta_2 x_2 + \dots + \beta_{K-1} x_{K-1} + \beta_K \hat{x}_K + e^*$$

So the estimators of  $\beta$  from the second-stage regression are what we call 2-stage least squares estimators. It’s not exactly like doing two separate regressions because, our estimator of the error variance uses the actual values of  $x_K$  rather than the fitted values:

$$\hat{\sigma}_{IV}^2 = \frac{\sum_{i=1}^N (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{i,2} - \dots - \hat{\beta}_K x_{i,K})^2}{N - K}$$

A 2-stage least squares can easily extend to multiple endogenous regressors, as long as we have more of the independent instruments than the endogenous regressors. Suppose there are  $G$  ‘good’ exogenous regressors,  $B = K - G$  ‘bad’ endogenous regressors, and



also L ‘lucky’ instrumental variables.  $L > B$  means overidentified,  $L = B$  is just identified,  $L < B$  means underidentified (and this cannot be estimated by IV)

$$y = \beta_1 + \beta_2 x_2 + \dots + \beta_G x_G + \beta_{G+1} x_{G+1} + \dots + \beta_K x_K + e$$

The first-stage regressions:  $x_{G+j} = \gamma_{1j} + \gamma_{2j} x_2 + \dots + \gamma_{Gj} x_G + \theta_{1j} z_1 + \dots + \theta_{Lj} z_L + v_j, j = 1, \dots, B$

Getting fitted values:  $\hat{x}_{G+j} = \hat{\gamma}_{1j} + \hat{\gamma}_{2j} x_2 + \dots + \hat{\gamma}_{Gj} x_G + \hat{\theta}_{1j} z_1 + \dots + \hat{\theta}_{Lj} z_L, j = 1, \dots, B$

Regressing original equation and then replacing endogenous regressors with fitted values we have:

$$y = \beta_1 + \beta_2 x_2 + \dots + \beta_G x_G + \beta_{G+1} \hat{x}_{G+1} + \dots + \beta_K \hat{x}_K + e^*$$

The test for the model is presented in table 4.2

**Table 4.2 Regression Analysis**

Model Summary						
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate		
1	.610a	0.57	0.53	0.5756		
ANOVA						
Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	16.311	4	4.078	12.307	.000 <sup>a</sup>
	Residual	27.502	83	0.331		
	Total	43.813	87			
a. Predictors: (Constant)						
b. Dependent Variable: d						
Coefficients						
Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.
		B	Std. Error	Beta		
1	(Constant)	0.889	0.777		1.142	0.257
	Inflation	0.392	0.105	0.331	3.738	0.000
	Wage rate growth	-0.315	0.113	-0.245	-2.783	0.007
	Long term interest rate	0.172	0.086	0.174	1.998	0.049
	Equity return on assets	0.517	0.123	0.374	4.118	0.000
a. Dependent Variable: Stochastic modelling						

The model summary from the regression model indicated that about 57% of the data could be accounted for in the regression while it indicated a significant relation ( $p=0.000$ ) to imply that the data that had been employed in the regression model had not been computed by chance.

The ANOVA (table 4.12) for the regression indicated that the results computed using the regression model were significant ( $F= 12.307$ ,  $p<0.000$ ) implying that the regression model was significant. It can be used to predict the dependent variable.

The results of the study indicate that inflation had a significant relationship with stochastic modelling performance with a  $P=0.000$ . These findings indicate that the inflation level that the pension funds experienced affected the efficiency of their stochastic modelling.

The results indicate that there was a significant relationship between wage growth rate and stochastic modelling performance with  $P=0.007$ . These findings indicate that the rate at which the wages of the pensioners grew had an effect on the application of stochastic modelling of the firm and how efficient they would be.

The results also indicate that there was a significant relationship between long term interest rate and stochastic modelling performance with  $P=0.049$ . These findings indicate that the long term interest rates of an economy has effect on the ability of the firm to employ stochastic modelling effectively.

The results indicate that there was a significant relationship between equity return on assets indicate and stochastic modelling performance with a  $P=0.000$ . The return on assets of the pension funds plays a significant role in determine the performance of the stochastic modelling. This can be attributed to the fact pension funds are general involved in investment of the pensioners funds and therefore when they are able to get good returns for their equity investment, then their models is significant.

From the coefficients, the study was able to illustrate the completion of projects as indicated in the equation below:

Stochastic modelling= 0.889 +0.392 (Inflation) - 0.315(wage rate growth) +0.172(Long term interest rate) + 0.517 (Equity returns on asset)

From the regression equation, it is evident equity returns on assets contributed significantly to the efficiency of the stochastic modelling 51.7% to the pension funds companies followed by Inflation with 39.2%, then wage rate growth with 31.5% and lastly long term interest rate with only 17.2%.

The result of the equity returns on assets affecting the stochastic model more could be attributed to the fact that pension firms engage in investment and rely on the returns to provide for their pensioners need and therefore failure in investment plans or its success could greatly influence the efficiency of the model.

The suitable stochastic model for the study is therefore as follows;

$$Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + e$$

Where: x= is the independent variables indicated by X1 = inflation, X2 = wage growth rate, X3 = long term interest rates, X4 = Equity returns on assets, Y = The dependent variable (stochastic modelling),  $\beta$  = The unknown parameters; this may be a scalar or a vector and e= Error

#### **4.3 Efficiency of the proposed stochastic Model**

The study sought to determine the efficiency of the proposed stochastic Model. This was done by running a Chi-Square Tests. The findings are presented in table 4.3

**Table 4.3 Chi-Square Tests for the Efficiency of the proposed stochastic Model**

<b>\Chi-Square Tests</b>			
	Value	Df	Asymp. Sig. (2-sided)
<b>Pearson Chi-Square</b>	17.295 <sup>a</sup>	5	0.004
<b>Likelihood Ratio</b>	16.971	5	0.005
<b>Linear-by-Linear Association</b>	14.192	1	0
<b>N of Valid Cases</b>	62		

**a. 9 cells (75.0%) have expected count less than 5. The minimum expected count is .43.**

The tests indicated that the proposed stochastic model was efficiency with a P= 0.004 and a likelihood ratio of 0.005. This findings indicate that the proposed stochastic model was efficient and should be therefore employed in the pension funds companies.

## CHAPTER FIVE

### SUMMARY CONCLUSION AND RECOMMENDATIONS

#### 5.0 Introduction

This chapter presents the summary, conclusion and recommendations of the study.

#### 5.1 Summary of the Findings

The findings indicate that there was a significant relationship between inflation and wage growth rate  $P=0.000$ , inflation and Long term interest rate  $P=0.000$ , inflation and equity returns on assets  $P=0.001$ , wage growth rate and long term interest rate  $P=0.001$ , wage growth rate and equity returns on assets  $P=0.030$  and long term interest rates and equity returns  $P=0.001$ .

The correlation results indicate that there was a significant relationship between stochastic modelling and inflation  $P=0.000$ , stochastic modelling and wage growth rate  $P=0.002$ , stochastic modeling and long term interest rates  $P=0.000$ , stochastic modelling and equity returns on assets.

These results indicate that the economic factors that affect pension funds are positively correlated to each other indicating that each has an effect on the other. These economic factors also have a significant correlation to the efficiency of stochastic modelling.

The model summary from the regression model indicated that about 57% of the data could be accounted for in the regression while it indicated a significant relation ( $p=0.000$ ) to imply that the data that had been employed in the regression model had not been computed by chance.

The ANOVA (table 4.12) for the regression indicated that the results computed using the regression model were significant ( $F= 12.307$ ,  $p<0.000$ ) meaning that the regression model was significant. It can be used to predict the dependent variable.

From the coefficients, the study was able to illustrate the completion of projects as indicated in the equation below:

Stochastic modelling=  $0.889 + 0.392$  (Inflation) -  $0.315$ (wage rate growth) + $0.172$ (Long term interest rate) +  $0.517$  (Equity returns on asset)

From the regression equation, it is evident equity returns on assets contributed significantly to the efficiency of the stochastic modelling 51.7% to the pension funds companies followed by Inflation with 39.2%, then wage rate growth with 31.5% and lastly long term interest rate with only 17.2%.

The suitable stochastic model for the study is therefore as follows;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e$$

Where:  $x$ = is the independent variables indicated by  $X_1$  = inflation,  $X_2$  = wage growth rate,  $X_3$  = long term interest rates,  $X_4$  = Equity returns on assets,  $Y$  = The dependent variable (stochastic modelling),  $\beta$  = The unknown parameters; this may be a scalar or a vector and  $e$ = Error

The tests indicated that the proposed stochastic model was efficiency with a  $P= 0.004$  and a likelihood ratio of 0.005. This findings indicate that the proposed stochastic model was efficient and should be therefore employed in the pension funds companies.

## **5.2 Conclusion**

On the economic factors that affect pension funds the study concluded that all these factors are positively correlated to each other and significantly affect the efficiency of stochastic modelling. These factors include inflation, wage growth rate, long term interest rates and equity returns on assets.

The study also concluded that of the four variables results equity returns on assets affected the efficiency of the stochastic modelling the highest. This is attributed to the fact that pension schemes engage in investment of the funds of their pensioners and therefore the returns from these investment affect the effectiveness in performing their obligations. The study concluded that the proposed stochastic model was efficient and should be therefore employed in the pension funds companies.

## **5.3 Recommendations**

The study made the following recommendations;

- i) Pension funds should invest wisely in order to ensure that their returns are good since their performance are crucial in meeting the needs of their pensioners.
- ii) Pension schemes should employ a suitable stochastic model that pay careful attention to inflation, wage growth rate, long term interest rates and equity returns on assets since these economic factors influence greatly the performance of pension schemes.



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