TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR RECTANGULAR CONCRETE BEAMS

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A thesis submitted in fulfilment of the requirements for the Degree of Master of Science in Civil Engineering to the Department of Civil Engineering

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other university.

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(Candidate)

This thesis has been submitted for examination with my approval as University Supervisor.

Prof. W.M. ONSONGO
(Supervisor)
To
My Wife SARASWATHY
and
Children JAI & KAVI
In order to provide shear resistance to reinforced concrete beams subjected to flexure and shear, a new type of shear reinforcement to be known as 'WAVE REINFORCEMENT' is examined experimentally and analytically. The experimental study is based on five beams incorporating wave reinforcement and/or vertical stirrups. It is aimed at resolving the acceptability and efficiency of wave reinforcement. The analytical study is aimed at verifying the adoptability of truss analogy for the design of wave reinforcement. Within the limitations of test programme, it is now confirmed that the wave reinforcement is acceptable as an alternate type of shear reinforcement and is also more efficient, functionally and economically than the vertical stirrups which are presently popular. It is also verified that the truss analogy can be safely used for the design of wave reinforcement. Again the compression field theory applied to a beam with wave reinforcement gives a good agreement between the predictions and test data. Detailing methods for the wave reinforcement and recommendations for computing the shear strength provided by them are proposed.
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NOTATION

The symbols listed below are those occurring frequently in this report. Other symbols used in one or two places only are defined where they occur and hence not included here.

\( A \) \hspace{1em} \text{cross-sectional area of member (=bh)}

\( A_s \) \hspace{1em} \text{area of bottom longitudinal steel (tensile)}

\( A_s' \) \hspace{1em} \text{area of top longitudinal steel (compressive)}

\( A_{sv} \) \hspace{1em} \text{area of two legs of vertical stirrups}

\( A_{sw} \) \hspace{1em} \text{cross-sectional area of wave reinforcement}

\( A_{we} \) \hspace{1em} \text{equivalent area of wave reinforcement in the typical section}

\( a_v \) \hspace{1em} \text{shear span from beam reaction to the first concentrated load point}

\( b \) \hspace{1em} \text{width of section}

\( d \) \hspace{1em} \text{distance from extreme compression fibre to centroid of (bottom) longitudinal tension reinforcement or effective depth of tension reinforcement}

\( d' \) \hspace{1em} \text{depth from extreme compression fibre to centroid of (top) longitudinal compression reinforcement}

\( E_c \) \hspace{1em} \text{static secant modulus of elasticity of concrete}
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<tr>
<td>$E_s$</td>
<td>modulus of elasticity of longitudinal tensile steel</td>
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<td>$E'_s$</td>
<td>modulus of elasticity of longitudinal compressive steel</td>
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<td>$F$</td>
<td>ultimate load</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>peak compressive stress for standard concrete cylinder test</td>
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<tr>
<td>$f_{cp}$</td>
<td>principal compressive stress in concrete</td>
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<td>$f_\ell$</td>
<td>stress in bottom longitudinal steel ($f_{\ell y} = \text{yield stress}$)</td>
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<td>$f'_\ell$</td>
<td>stress in top longitudinal steel ($f'_{\ell y} = \text{yield stress}$)</td>
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<tr>
<td>$f_{yv}$</td>
<td>characteristic strength of vertical stirrups</td>
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<td>$f_{yw}$</td>
<td>characteristic strength of wave reinforcement</td>
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<td>$h$</td>
<td>overall depth of beam</td>
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<tr>
<td>$jd$</td>
<td>lever arm for internal couple resisting bending moment</td>
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<tr>
<td>$M$</td>
<td>bending moment due to ultimate load for combined flexure and shear</td>
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<td>$M_p$</td>
<td>pure moment value</td>
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<td>$q$</td>
<td>shear flow</td>
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<td>$s_v$</td>
<td>spacing of vertical stirrups along the member</td>
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<td>v_s</td>
<td>Nominal shear stress resisted by vertical stirrups</td>
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<td>v_u</td>
<td>Ultimate shear resistance of reinforced concrete beam</td>
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<td>y</td>
<td>Neutral axis depth ( (y = y_p ) for pure flexure and ( y = y_n ) for combined flexure and shear)</td>
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<td>( \alpha )</td>
<td>Angle between the inclined leg of the wave reinforcement and longitudinal axis of beam</td>
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<td>( \varepsilon_{co} )</td>
<td>Concrete strain corresponding to peak concrete stress</td>
</tr>
<tr>
<td>( \varepsilon_{cp} )</td>
<td>Principal compressive strain in concrete</td>
</tr>
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<td>( \varepsilon_{ct} )</td>
<td>Extreme top fibre compression strain</td>
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<td>( \varepsilon_y )</td>
<td>Strain in bottom longitudinal steel ( (\varepsilon_y = yield ) strain)</td>
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<tr>
<td>( \varepsilon'_y )</td>
<td>Strain in top longitudinal steel ( (\varepsilon'_y = yield ) strain)</td>
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<td>( \varepsilon_t )</td>
<td>Principal tensile strain in concrete</td>
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<td>( \varepsilon_w )</td>
<td>Strain in wave reinforcement ( (\varepsilon_w = yield ) strain)</td>
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<td>( \theta )</td>
<td>Principal angle of compression measured with respect to the beam longitudinal axis</td>
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<td>( \Omega_c )</td>
<td>Concrete strain ratio defined as ( \frac{\varepsilon_c}{\varepsilon_{co}} )</td>
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Similarly, \( \Omega_{cp} = \frac{\varepsilon_{cp}}{\varepsilon_{co}} \) and \( \Omega_{ct} = \frac{\varepsilon_{ct}}{\varepsilon_{co}} \)
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TOWARDS AN EFFICIENT SHEAR REINFORCEMENT
FOR
RECTANGULAR CONCRETE BEAMS
CHAPTER 1  INTRODUCTION

The aim of shear design of reinforced concrete beams is to produce economically, beams whose ultimate strengths are governed by flexure rather than by shear and thus to forestall the possibility of a shear failure. Shear failures are non-ductile and the object of good design is to suppress such a failure. The lack of ductility produces a brittle or sudden failure and to provide an adequate safety against such a failure shear reinforcement (also known as web or transverse reinforcement) is used.

Various types of shear reinforcement have been successfully tested in the past (refer Sec. 2.5). However, despite tremendous research efforts on this subject a rational answer to when, where, what kind and how much shear reinforcement is required to satisfy its functions, is still elusive. But vertical stirrups are at present most popular.

In the search for an efficient type of shear reinforcement, a new type to be known as 'WAVE REINFORCEMENT' is proposed for the present study.
The wave reinforcement is experimentally and analytically studied, bearing in mind the restrictive factors such as time and resources available, for its adoptability to structural concrete beams.

By comparative experimental study on five beams using wave reinforcement and/or vertical stirrups, the thesis aims at resolving the acceptability of the wave reinforcement from the point-of-view of ultimate strength and deformations such as extent of cracking and deflection under service conditions. The study is also aimed at establishing the efficiency of wave reinforcement over vertical stirrups, both functionally and economically.

Many researchers have developed empirical equations for the design of shear strength of reinforced concrete members subjected to combined bending and shear. But the Codes 13 and 14 have accepted empirical approaches to compute the shear strengths provided by concrete and by the shear reinforcement. The latter utilize the 45 deg truss analogy. Although the above design approach is not based on a completely rational theory, the design equations have resulted in structures being designed which have performed satisfactorily during their intended life. This
thesis describes the theoretical framework for computing the shear strength provided by wave reinforcement based on truss analogy and accordingly the design recommendations have been proposed. The use of proposed design recommendations is illustrated by means of design examples.

The only relatively rational theoretical approach for design for shear available is the compression field theory. The compression field theory was originally developed by Collins [5] and was further studied by means of additional equations by Onsongo and Shitote [19 and 24] for a structural concrete beam with vertical stirrups subjected to flexure and shear. The above equations are suitably modified for the wave reinforcement and a computer program is prepared to predict the response characteristics of the beam. The study is also aimed at verifying the response characteristics predicted by the compression field theory and test data.

According to the knowledge of the author there is no similar work in this specific field at this date.
Regarding the role of web reinforcement, Kani [12] made the following comments:

"the main question with respect to the so-called shear strength of a reinforced concrete beam which confronts the designer is: where and what kind of shear reinforcement is to be used to prevent premature diagonal failure?"

In reply Swamy and others [12] felt that, "the main important question which need to be answered is 'when and how much' shear reinforcement is required to prevent diagonal failure and produce a ductile failure". It is relevant therefore, that in any study concerning shear reinforcement an answer should be found to when, where, what kind and how much shear reinforcement is required to satisfy its functions. In spite of tremendous research efforts a rational answer to the above is still elusive.

In this chapter the basic modes of shear failure, functions of shear reinforcement, current methods of analysis of shear strength of beams and the types of shear reinforcement presently in use are
reviewed. Though this review is not directly related to the present study of wave reinforcement it roughly establishes the current state-of-the-art.

2.1 MODES OF SHEAR FAILURE

The modes of shear failure in beams, other than deep beams, without web reinforcement (Fig. 2.1) as generally understood [1, 11, 13, 21 and 25] can be classified as follows:

Fig. 2.1 POSSIBLE MODES OF SHEAR FAILURE IN BEAMS WITHOUT WEB REINFORCEMENT
(a) Shear-compression failures:

These are typically observed for short beams for which $1.0 < a_v / d < 2.5$, where $a_v$ is the shear span and $d$ is the effective depth of tension reinforcement. In this failure mode, destruction of compression zone above the inclined crack takes place and ultimately the compression zone crushes.

(b) Shear-tension failures:

These are typically observed for normal and long beams for which $2.5 < a_v / d < 6.0$. In this failure mode, destruction of tension zone below the diagonal crack leads to splitting of the concrete at the level of tension steel.

(c) Diagonal-tension failures:

These are also typically observed for beams stated in (b) above. In this failure mode, destruction of tension zone below the diagonal crack results in the collapse.
Again, the diagonal-tension cracks also known as inclined cracks may be:

(a) Primary cracks - where tension cracks open first near the mid-depth of the beam,
   Either (i) independently in the vicinity of the neutral axis, when $a_y/d$ is small (known as web-shear cracks);

   or (ii) as a development of an existing flexural crack which is extended in an inclined direction, when $a_y/d$ is larger (known as flexure-shear cracks).

(b) Secondary cracks - where, upon further loading the inclined cracks may extend at either end leading to failure.

Taub, J., and Neville, A.M., [25] have observed that the modes of shear failure in reinforced concrete beams without web reinforcement are usually similar to that of reinforced concrete beams with web reinforcement, but in the latter case, the ultimate load is considerably higher and sudden collapse of
the beam does not take place.

2.2 FUNCTIONS OF SHEAR REINFORCEMENT

As Taub and Neville [25] have put it, the generally accepted function of shear reinforcement is to resist the opening or widening of the diagonal tension cracks and thus to prevent the failure of a reinforced concrete beam due to shear. It has also been noted [1] that the shear reinforcement in its secondary role to enhance the shear resistance of a beam:

(a) carries part of the shear
(b) transfers the shear across a potential inclined crack
(c) holds the longitudinal bars and increases their dowel capacity.

Again, it is interesting to note the following statements regarding the functions of shear reinforcement:

(a) Evans and Kong [8] have observed that "an important function of stirrups is to prevent
the pressing down of the longitudinal reinforcement due to dowel-action force, and the consequent splitting of the concrete at the level of such reinforcement".

(b) Kani [12] has noted that "the purpose of web reinforcement is to provide reactions for the internal concrete arches which support the compressive zone of the beam".

(c) Park and Paulay [20] have noted that "suitably detailed web reinforcement will preserve the integrity and therefore the strength of the beam mechanism, allowing additional shear forces to be resisted by the truss mechanism".

(d) Collins and Mitchell [6] have stated that "the primary function of the stirrups is to hold the beam together in the lateral direction".

Whatever be the observations regarding the function, the shear reinforcement substantially augments the shear resistance of beams by carrying a part of the shear and in turn increases the ductility of the beam and eliminates the danger of a premature
non-ductile failure. This warrants the provision of a suitable type of shear reinforcement.

2.3 SHEAR TRANSFER IN BEAMS WITH TRANSVERSE REINFORCEMENT

Though the mechanism of shear transfer in beams is a subject of controversy, the mechanism proposed in the ASCE-ACI Committee 426 [1] explains basically the internal forces at a typical inclined crack (Fig. 2.2).

Shear Carried by:

- $V_{cz} =$ concrete compression zone
- $V_s =$ shear reinforcement
- $V_{sy} =$ interface shear transfer component
- $V_d =$ longitudinal steel (dowel shear)

Fig. 2.2 INTERNAL FORCES AT INCLINED CRACK DUE TO APPLIED SHEAR (Based on [1])
These mechanisms are such, that the applied shear force $V$, is resisted by:

(a) the shear force carried across the uncracked concrete compression zone, $V_{cz}$;

(b) the transverse component of the force due to interlocking of aggregate particles (interface shear transfer) across a crack, $V_{ay}$;

(c) the transverse shear force induced in the main flexural reinforcement by dowel action, $V_d$; and

(d) the transverse shear force carried by the shear reinforcement crossed by the diagonal crack, $V_s$.

Hence the applied shear force, $V = (V_{cz} + V_{ay} + V_d) + V_s$

$$= V_c + V_s$$  \hspace{1cm} (2.1)

where $V_c$ is equal to the bracketed quantities in the above expression. This is defined as the 'concrete contribution' or the shear carried by the concrete.
Again, typical contributions [1] of the various internal forces with properly detailed stirrups are shown in Fig. 2.3.

![Diagram of shear transfer in beams with well-anchored reinforcement]

**Fig. 2.3** SHEAR TRANSFER IN BEAMS WITH WELL-ANCHORED REINFORCEMENT

(Based on [1])

It is however noted that there is still disagreement as to the presence and roles of these
components of shear transfer in beams and the mechanisms of shear transfer is not yet well understood [1].

2.4 METHODS OF ANALYSIS OF SHEAR STRENGTH OF BEAMS

In designing for shear, the codes [3 and 4] adopted the use of nominal or design shear stress, \( v \), at any cross-section calculated as,

\[
v = \frac{V}{bd}
\]  

(2.2)

Again \( v = v_c + v_s \) where \( v_c \), is the shear stress resisted by concrete and \( v_s \), is the shear stress resisted by transverse reinforcement. The value of \( v_c \) is assumed to be equal to the shear strength recorded in a similar beam without transverse reinforcement when a diagonal crack was either first noted or was estimated to have traversed the neutral axis and empirical equations are proposed to compute its value. The value of \( v_s \) is customarily based on the 'truss analogy'.

2.4.1 The Truss Analogy

Ritter and Mörsch's theory commonly referred to as the 45 deg truss analogy makes use of a truss model for the design of shear reinforcement in reinforced concrete beams. According to this analogy, after the formation of the diagonal cracks the beam tends to behave like a pin-jointed plane truss: the bottom longitudinal bars and the stirrups constitute the tension members, while the top flange and the inclined concrete struts in the web are the compression members. The forces in the truss were then determined from the consideration of equilibrium.

The ASCE-ACI Committee 426 [1] which is one of the important references concerning shear transfer mechanisms noted that the truss analogy does not take into consideration the presence of several significant factors associated with shear transfer namely \( V_{cz} \), \( V_{ay} \) and \( V_d \) (refer section 2.3). Thus in the Codes [3] and [4] this added shear capacity was taken as the 'concrete contribution'.

The ASCE-ACI Committee 426 [1] has suggested that regardless of the shortcomings the truss analogy is "an excellent conceptual tool in the study of beams with shear reinforcement". Again, Rensaa in his contribution to the paper by Taub and Neville [25] has stated that,

"the truss analogy is an excellent method of indicating the function of shear reinforcement and it gives quite accurate results provided realistic slopes of the 'compression diagonals' are being used".

It is verified by many researchers that the slope of compression diagonals vary along the length of beam. But the problem lies in its accurate determination. Incorporating the compatibility of strains which was otherwise neglected in the truss analogy, Collins [5 and 6] has proposed a rational theory to compute the inclination of the diagonal compression based on the compression field theory.

It should be noted that the Codes [3] and [4] utilize the truss analogy for the design of shear reinforcement.
2.4.2 The Compression Field Theory

The compression field theory [5 and 6] assumes that after cracking, the reinforced concrete beam can resist no tension and that the shear will be carried by a field of diagonal compression which resulted in the following expression for the angle of inclination of the diagonal compression ($\theta$).

Using the compatibility condition of strains, it was proved that (refer equation 3.13):

$$\tan^2 \theta = \frac{\varepsilon_L + \varepsilon_{cp}}{\varepsilon_v + \varepsilon_{cp}} \quad (2.3)$$

where $\varepsilon_L$ = longitudinal tensile strain

$\varepsilon_v$ = transverse tensile strain

$\varepsilon_{cp}$ = diagonal compressive strain

Collins [6] has also applied the equilibrium equations of truss analogy, the compatibility conditions of strains and the stress-strain relationships of concrete to the design of shear reinforcement rationally and accurately.

The compression field theory was used to
predict the full behavioural response of prestressed and non-prestressed concrete beams in shear and torsion [6]. The rationale behind this theory has attracted many researchers [18, 19, 24, and 29]. Onsongo [18] has extensively studied the compression field theory and used it to predict the response of reinforced concrete beams subjected to combined torsion, flexure and axial load. It was further extended [19 and 24] to study the behavioural response of structural concrete subjected to flexure and shear.

In this present study, the truss analogy and the compression field theory are suitably modified to form the basis of the theoretical framework for the wave reinforcement (Chapter 3).

2.5 TYPES OF SHEAR REINFORCEMENT

Various types of shear reinforcement which have been adopted successfully in practice are:

(a) vertical stirrups
(b) inclined stirrups
(c) bent-up bars
In this section, a brief attempt is made to identify and compare their effectiveness in enhancing the shear strength of beams.

2.5.1 Vertical Stirrups

Vertical stirrups are most commonly used to attain the various functions stated in Section 2.1. These are also known as lateral reinforcement or web reinforcement or links which are bent into recommended shapes and located perpendicular to axis of member. A typical arrangement of vertical stirrups is shown in Fig. 2.4.
Various codes specify empirical spacing limits for vertical stirrups. Table 2.1 gives the extract of spacing limits specified in ACI 318M-83 [4] and in BS 8110: Part 1: 1985 [3].

Table 2.1

<table>
<thead>
<tr>
<th>SPACING LIMITS FOR VERTICAL STIRRUPS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACI 318 M-83 [4]:</strong></td>
</tr>
<tr>
<td>Spacing of shear reinforcement placed</td>
</tr>
<tr>
<td>perpendicular to axis of member shall not</td>
</tr>
<tr>
<td>exceed ( \frac{d}{2} ) in non-prestressed members</td>
</tr>
<tr>
<td>(and ( \frac{3}{4}h ) in prestressed members, nor</td>
</tr>
<tr>
<td>600 mm).</td>
</tr>
<tr>
<td><strong>BS 8110: Part 1: 1985 [3]:</strong></td>
</tr>
<tr>
<td>Spacing of links in the direction of the</td>
</tr>
<tr>
<td>span should not exceed 0.75d, the</td>
</tr>
<tr>
<td>horizontal spacing should be such that</td>
</tr>
<tr>
<td>no longitudinal tension bar is more than</td>
</tr>
<tr>
<td>150 mm from a vertical leg; this spacing</td>
</tr>
<tr>
<td>should in any case not exceed d.</td>
</tr>
</tbody>
</table>

The vertical stirrups should tightly surround
the longitudinal tensile steel and anchored at both ends to develop the design yield strength at a critical section and to attain the following beneficial effects [31]:

(a) extra concrete force contribution due to the prevention of diagonal crack propagation; and

(b) extra dowel action force from longitudinal tensile reinforcement due to the support.

Also, Park and Paulay [20] have noted that the vertical stirrups will prevent the opening of the bond-generated splitting of concrete along the longitudinal tensile steel in the shear span. Taub and Neville [25] observed that "vertical stirrups alone cannot always economically guarantee a full protection from shear failure of beams because they do not lie in a direction normal to the diagonal tension cracks".

2.5.2 Inclined Stirrups

Inclined stirrups (Fig. 2.5) making an angle of 45° or more with longitudinal tension reinforcement
is used as a shear reinforcement [4, 6]. From the test results [25] and study of free-body diagrams, it was inferred that stirrups inclined at $45^\circ$ to the axis of the beam was most effective in resisting the widening of diagonal tension cracks provided slippage at the contact between the inclined stirrups and the longitudinal tensile reinforcement is eliminated.

![Inclined Stirrups Diagram](image)

**Fig. 2.5 INCLINED STIRRUPS**

ACI Building Code [4] specifies that the inclined stirrups should be so spaced that every 45 deg line extending towards the reaction from the middepth of member $d/2$ to longitudinal tension reinforcement should be crossed by at least one line of shear reinforcement.

The following observations regarding the use of
inclined stirrups are worth-noting:

(a) That maximum shear resistance is attainable with a smaller amount of inclined stirrups and it is therefore economical to provide such a reinforcement [8 and 25].

(b) That the difficulty of elimination of slippage at the contact between inclined stirrups and the longitudinal steel makes this type of shear reinforcement impracticable [16].

(c) That the inclined stirrups are effective only in one direction and hence should not be used when there is a possibility of load reversal unless it is provided in both directions [20].

2.5.3 Bent-up Bars

Main steel is utilised for shear reinforcement where it is no longer required in the tension face of the beam. Thus the longitudinal tensile
reinforcement is bent upwards (Fig. 2.6) from the tensile flange into the compression flange, normally near the ends of the beam where the shear force is large and the bending moment is small. ACI Building Code [4] specifies that the longitudinal reinforcement with bent portion making an angle of 30 deg or more with the longitudinal tension reinforcement and spaced as explained in Sec. 2.5.2 may be used as a shear reinforcement.

The bent-up bars alone as shear reinforcement create stress complications in the concrete resulting in a lower shear strength and much wider cracks (than similar beams with vertical stirrups) and produce excessive compression and hence bursting of the concrete near the bends [25]. Kani [12] has also noted that the inclined bent-up bars in preventing a diagonal failure over vertical stirrups is undisputed but from practical considerations a combination of vertical stirrups and bent-up reinforcement should be used. A typical combination of vertical stirrups and bent-up bars is shown in Fig. 2.6.
B.S. Code of Practice [3] specifies that when bent-up bars are used, at least 50% of shear resistance provided by the steel should be in the forms of links. In beams with a combination of bent-up bars and vertical stirrups, it was noted [25] that the vertical stirrups prevent the pressing down of the tensile bar at the lower end of the diagonal crack and the resulting splitting of the concrete. Also the vertical stirrups carry a part of the diagonal tension thus relieving some of the force in the bent-up bars and in the tension reinforcement.
2.5.4 Orthogonal Reinforcement

An orthogonal shear reinforcement consists of vertical stirrups and one or more layers of horizontal bars (other than longitudinal tensile steel) which are placed at right angles (to vertical stirrups). A typical orthogonal shear reinforcement with one layer of horizontal reinforcement is shown in Fig. 2.7.

Fig. 2.7 ORTHOGONAL REINFORCEMENT

It was noted [20] that the horizontal bars strengthen the contribution of the concrete but will have no effect on the shear strength of a beam (since truss mechanism is unaffected). It was noted [25] that the orthogonal shear reinforcement is efficient especially for deep beams.
2.5.5 Welded Wire Fabric

Welded wire fabric (either smooth or deformed) as shear reinforcement consists of a series of longitudinal and transverse cold-drawn steel wires placed at right angles to each other and welded together at all points of intersection. Welded wire fabric is placed perpendicular to axis of member, the size and spacing depending on the requirements of the design.

Welded wire fabric has been found to increase the number of, and to decrease the width of, cracks. It may be due to the use of a large number of smaller bars in preference to a few large bars. It is effective for crack control and was observed that [26] the welded wire fabric offers promise in slender webs of prestressed deep girders and for resisting torsion.

The ASCE-ACI Committee 426 [1] has also recorded that the welded wire mesh stirrups with mats of 50 to 100 mm spacing of stirrup bar were the best with respect to crack widths and compressive stresses in the web.
2.6 CHOICE OF SHEAR REINFORCEMENT

For beams with identical dimensions and steel content the maximum width of shear cracks for average types of shear reinforcement was found by Leonhardt [15] to be the smallest in beams with closely spaced inclined stirrups followed by a beam with vertical stirrups. The widest cracks were observed in a beam with bent-up bars (Fig. 2.8).

![Graph showing typical load vs maximum crack width curves](Image)

Fig. 2.8 TYPICAL LOAD Vs MAXIMUM CRACK WIDTH CURVES (Based on [15])

Taub and Neville have concluded after extensive experimental study [25] that "beams differing in web
reinforcement only exhibit the first diagonal crack under the same load. It is only after the cracking has started that the behaviour of a beam depends on the type of web reinforcement used and the further development of cracks and the ultimate load are a function of this reinforcement).

A typical combination of vertical stirrups and inclined stirrups shown in Fig. 2.9 is modified to derive the advantages of both. Hence the evolution of wave reinforcement (refer Sec. 3.1).
In the next chapter, the theoretical framework is established to reinforced concrete beams with wave reinforcement subjected to flexure and shear.
CHAPTER 3  THEORETICAL FRAMEWORK

This chapter aims in introducing the wave reinforcement as used in the present experimental study (refer Sec. 4.4.1), along with the detailing methods. In addition, the equations pertaining to the computation of nominal shear strength provided by wave reinforcement based on truss analogy are derived. Also explained is compression field theory for predicting the behaviour of beams with wave reinforcement.

3.1 WAVE REINFORCEMENT

The wave reinforcement is a steel bar consisting of vertical and inclined legs forming acute angles with two or more bends in the opposite directions (in the same plane). A typical wave reinforcement with bending dimensions is shown in Fig. 3.1.

The wave reinforcement can be identified by degrees of arc $\alpha$, between the horizontal and inclined leg, namely $\alpha$ deg wave reinforcement. But in preparing the bar bending schedule, it can preferably be defined in terms of lengths A and B (refer also Sec. 3.2). Minimum hook allowance should also be
considered during bar bending.

The bending of wave reinforcement can be achieved by making use of the same bar bending arrangement used for vertical stirrups (Fig. 3.2). By marking a control line (at $\alpha$ deg as shown) the bending of $\alpha$ deg wave reinforcement can be carried out. It can be seen that an angle $\alpha$ equals to 45 deg is easy to conceive by the bar bender at all construction sites.

**Fig. 3.1 TYPICAL BENDING DIMENSIONS FOR WAVE REINFORCEMENT**
3.2 DETAILING OF WAVE REINFORCEMENT

Refer to the part details of beam shown in Fig. 3.3 (also refer Fig. 4.4 for more details). The wave reinforcement (bar mark 3) can be identified either as:

(a) $45^\circ$ deg wave reinforcement ($\alpha = 45^\circ$)

or (b) $2Y803 \, -370 \, -525$, where

- $2Y803$ denotes two units of wave reinforcement using Y8 bars of bar mark 3
- $370$ denotes the dimension $A$, in mm
- (which is kept always equal to $(d-d')$)
- $525$ denotes the dimension $B$, in mm
Fig. 3.3 PART DETAILS OF BEAM  
(Showing bar marks)

A typical bar bending schedule for the above beam is shown in Fig. 3.4.

<table>
<thead>
<tr>
<th>Member</th>
<th>Bar mark</th>
<th>Type and size</th>
<th>No. of mbrs</th>
<th>No. of bars in each</th>
<th>Total No.</th>
<th>Length of each bar (mm)</th>
<th>Shape</th>
<th>A (mm)</th>
<th>B (mm)</th>
<th>C (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Y25</td>
<td></td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3150</td>
<td></td>
<td>2600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Y8</td>
<td></td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2775</td>
<td></td>
<td>2600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Y8</td>
<td></td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>3125</td>
<td></td>
<td>370</td>
<td>525</td>
<td>370</td>
</tr>
<tr>
<td>4</td>
<td>R6</td>
<td></td>
<td>1</td>
<td>8</td>
<td>8</td>
<td>1075</td>
<td></td>
<td>365</td>
<td>115</td>
<td></td>
</tr>
</tbody>
</table>

* Specified in multiples of 25 mm  
* Specified in multiples of 5 mm  
BS 4466 (1981)

Fig. 3.4 TYPICAL BAR BENDING SCHEDULE
The wave reinforcement should be properly placed so that the tension in the shear reinforcement is distributed along the length of the beam balancing the outward thrusts of the diagonal concrete compressions. Again the inclined legs of the wave reinforcement should be so placed as to cross the anticipated inclined cracks effectively (Fig. 3.5a).

Fig. 3.5 PLACEMENT OF WAVE REINFORCEMENT

Improper placing such as wave reinforcement not extending beyond the section where the shear
reinforcement is not required (Fig. 3.5b(i)) and the inclined legs placed in the direction nearly parallel to the anticipated crack direction (Fig. 3.5b(ii)) should be avoided for they defeat the very purpose of providing shear reinforcement to beams. Again, the wave reinforcement is placed to the inner side of the outer longitudinal tensile bar as shown in Fig. 3.6 such that it starts at the bottom of the beam (above the support) and eventually ends at the top, but beyond the section where the shear reinforcement is not required. The wave reinforcement is tied to the bottom and top longitudinal bars at their points of contact using 2 or 3 strands of steel wires. A minimum of two wave reinforcement should be placed at all sections. The vertical stirrups are provided at all sections where the vertical legs of wave reinforcement exist (refer Sec. 4.4.1 and 5.1) and this arrangement provides a steel cage of required shape and rigidity.

Fig. 3.6 CAGE WITH WAVE REINFORCEMENT
(Without vertical stirrups)
As Park and Paulay [20] have stated, the "detailing based on an understanding of and feeling for the structural behaviour of reinforced concrete is likely to require as much creative power as the derivation of structural actions by mathematical analysis". It is therefore suffice to say that the detailing of wave reinforcement should be properly understood in order to derive the best results.

3.3 TRUSS ANALOGY FOR WAVE REINFORCEMENT

Consider a truss model for a cracked beam (Fig. 3.7) where $\theta$, is the arbitrary angle (in degrees) of inclination of concrete diagonals in compression measured with respect to the longitudinal axis of the beam and $\phi$, is the angle between the inclined leg of the wave reinforcement and longitudinal axis of beam.

The wave reinforcement may be assumed to constitute:

(a) a system of vertical stirrups, and
(b) a system of inclined stirrups formed by the vertical and inclined legs of the wave
reinforcement respectively. When the bars crossed by the crack reach the yield strength \( f_{yw} \), the nominal shear strength provided by the vertical legs \( V_{wv} \), and the inclined legs \( V_{wi} \), can be computed using truss analogy [3, 8, and 20] but by considering one inclined leg for each wave reinforcement over a typical region, as follows:

\[
V_{wv} = A_{sw} f_{yw} \cot \theta \left( \frac{d-d'}{s_w} \right) \quad (3.1)
\]

and,

\[
V_{wi} = A_{sw} f_{yw} \left( \cos \alpha + \sin \alpha \cot \theta \right) \left( \frac{d-d'}{s_w} \right) \quad (3.2)
\]

Fig. 3.7 TRUSS MODEL FOR A BEAM WITH WAVE REINFORCEMENT

Thus the nominal shear strength provided by the wave reinforcement \( V_{sw} \) can be computed as:

\[
V_{sw} = V_{wv} + V_{wi} \quad (3.3)
\]
It can be noted in Sec. 4.4.1 that additional vertical stirrups are also provided at all sections where vertical legs of wave reinforcement exist. The necessity of such a provision is well-established based on test results as outlined in Sec. 5.1. Therefore, the additional shear strength $V_{sv}$, provided by the vertical stirrups would be,

$$V_{sv} = A_{sv} f_{yv} \cot \theta (\frac{d-d'}{s_w})$$

(3.4)

Hence the nominal shear strength $V_s$, provided by shear reinforcement having a combination of wave reinforcement and vertical stirrups is computed as

$$V_s = V_{sw} + V_{sv}$$

(3.5)

To avoid crushing of concrete, Collins and Mitchell [6] have proposed that the angle of inclination, $\theta$, must satisfy the limits given by the following:

$$10 + \frac{35(\tau_n/f'_c)}{0.42-50e_z} < \theta < 80 - \frac{35(\tau_n/f'_c)}{0.42-65e_t}$$

(3.6)

where $\tau_n$ is the nominal shear stress given by $V_n/b(d-d')$. For highly stressed members, it was also recommended [6] that $\theta$ will be restricted to a narrow range of values close to 45 deg while for
lightly loaded members the range of allowable angles will be very wide (in the limit 10 deg ≤ θ ≤ 80 deg); and for convenience a value of θ somewhat larger than the smallest allowable angle but constant over the length of the beam may be chosen. For the present study the 45 deg truss analogy accepted in Codes [3] and [4] is chosen in order to compute the shear strength provided by wave reinforcement.

Assuming θ = 45 deg, the shear strength \( V_s \) provided by α deg wave reinforcement combined with vertical stirrups as explained earlier can be computed as

\[
V_s = \left[ A_{sv} f_{v} + A_{sw} f_{yw} \right. \\
\left. + A_{sw} f_{yw} (\cos \alpha + \sin \alpha) \right] \left( \frac{d-d'}{s_w} \right)
\]

(3.7)

If same diameter bars are used, \( A_{ss} = A_{sv}/2 = A_{sw} \), and \( f_{ys} = f_{v} = f_{yw} \) then

\[
V_s = A_{ss} f_{ys} \left( \frac{d-d'}{s_w} \right) (3+\cos \alpha+\sin \alpha)
\]

(3.7a)

where \( A_{ss} \) = the cross-sectional area of each bar of shear reinforcement

\( f_{ys} \) = characteristic strength of shear reinforcement
For a typical case of 45 deg wave reinforcement \((\alpha = 45^\circ)\) combined with vertical stirrups as stated earlier, noting that \(d - d' = s_w\) and \(\theta = 45\) deg, the above equations can be used to obtain,

\[
V_s = A_{sv} f_{yv} + A_{sw} f_{yw} + 1.414 A_{sw} f_{yw} \\
= A_{sv} f_{yv} + 2.414 A_{sw} f_{yw} \tag{3.8}
\]

Again, if equal diameter bars are chosen

\[A_{ss} = A_{sw} = \frac{1}{2} A_{sv} \quad \text{and} \quad f_{ys} = f_{yw} = f_{yv}, \quad \text{then} \]

\[
V_s = 4.414 A_{ss} f_{ys} \tag{3.8a}
\]

In a general form, for a 45 deg wave reinforcement in combination with vertical stirrups of same diameter,

\[
V_s = A_{we} f_{ys} \tag{3.9}
\]

where \(A_{we}\) is defined as the equivalent area of shear reinforcement over a typical region of width, \(s_w\) (Fig. 3.11). Again it can be proved that,

\[
A_{we} = A_{ss} (2N + 2.414 n) \tag{3.10}
\]

where \(N\) is the number of two-legged vertical stirrups and \(n\) the number of wave reinforcement, at the section
considered.

For one vertical stirrup and two wave reinforcement provided in the region considered, Eq. 3.10 becomes

\[ A_{we} = 6.828 \, A_{SS} \] (3.10a)

It is to be noted that due to the concrete 'surround' at the bends (Fig. 3.8), the slippage of inclined legs of wave reinforcement does not exist, which was otherwise a problem observed with inclined stirrups (refer Sec. 2.5.2). Thus due to the 'anchorage' provided by the concrete, the wave reinforcement can develop the design yield strength.

Fig. 3.8 'ANCHORAGE' PROVIDED BY THE CONCRETE
The adequacy of the equations has been verified and elaborated in Sec. 5.1.2. The simplicity of truss analogy for wave reinforcement as outlined is illustrated in Appendix A using a design example. In addition, Chapter 7 provides the proposed design recommendations for shear with wave reinforcement based on truss analogy.

3.4 COMPRESSION FIELD THEORY

The compression field theory which is well-established [5, 6, 18, 19 and 24] can be used to rationally explain the behaviour of a uniformly cracked reinforced concrete beam. The compression field theory incorporates:

(a) compatibility conditions in terms of average strains; and
(b) equilibrium conditions in terms of concrete stresses.

Shitote [24] has successfully applied the compression field theory for the analysis of reinforced concrete beams under combined flexure and shear, using vertical stirrups as shear reinforcement. In this section the above theory as applicable to wave reinforcement is explained.
3.4.1 Compatibility Conditions in Terms of Average Strains

In formulating the compatibility conditions, Collins [5] assumed that the beam is uniformly cracked over the region and the direction of average principal compressive stress in the concrete coincided with the direction of the average principal compressive strain, \( \theta \). It is also assumed that all strains at a point in a plane are compatible.

The compatibility requires that the strains in the reinforcement and the concrete equal those in the member, which can be represented by Mohr's circle of strain (Fig. 3.9). The strain circle is based on:

(a) average principal compressive strain, \( \varepsilon_{cp} \) (assumed positive when compressive);
(b) angle \( \theta \), between the principal compressive strain direction and the longitudinal direction;
(c) average strain in the longitudinal direction, \( \varepsilon_l \) (assumed positive when tensile);
(d) average strain in the transverse direction, \( \varepsilon_v \) (assumed positive when tensile).
The average strains are obtained by averaging the strains measured over a region involving uncracked and cracked portions.

![Strain Circle Diagram](image)

**Fig. 3.9** COMPATABILITY CONDITIONS FOR AVERAGE STRAINS FOR CONCRETE
(Based on COLLINS [5])

From the geometry of the strain circle, it
can be seen that:

\[ \gamma_{lt}/2 = (\varepsilon_v + \varepsilon_{cp})/\tan \theta \]  \hspace{1cm} (3.11)

and, \[ \varepsilon_v = \frac{\gamma_{lt}/2}{\tan \theta} - \varepsilon_{cp} \]  \hspace{1cm} (3.12)

By eliminating \( \gamma_{lt} \) from the above two equations, the following equation can be obtained:

\[ \tan^2 \theta = \frac{\varepsilon_t + \varepsilon_{cp}}{\varepsilon_v + \varepsilon_{cp}} \]  \hspace{1cm} (3.13)

This represents the compatibility condition in terms of average strains \( \varepsilon_t, \varepsilon_v \) and \( \varepsilon_{cp} \) which provides the necessary condition to determine the value of \( \theta \).

In addition, the following relationships can be obtained:

(i) the principal tensile strain, \( \varepsilon_t \) (tension positive):

\[ \varepsilon_t = \varepsilon_{cp} + \varepsilon_t + \varepsilon_v \]  \hspace{1cm} (3.14)
(ii) the shear strain relative to the longitudinal and transverse directions, $\gamma_{lt}$:

$$\gamma_{lt} = 2\sqrt{(\varepsilon_L + \varepsilon_{cp}) (\varepsilon_V + \varepsilon_{cp})} \quad (3.15)$$

(iii) the maximum shear strain, $\gamma_m$:

The diameter of the circle is a measure of the maximum shear strain. Thus,

$$\gamma_m = \varepsilon_V + \varepsilon_V + 2\varepsilon_{cp} \quad (3.16)$$

3.4.2 Equilibrium Conditions in Terms of Concrete Stresses

It was established [5 and 19] that for a cracked beam model subjected to a shear force $V$ and a flexural moment $M$, an element on the plane of shear flow will be subjected to shear stress $\tau$, longitudinal stress $\sigma_{cl}$, and a transverse strain, $\sigma_{ct}$ (Fig. 3.10).

Onsongo [19] has shown from the Mohr's circle of strain that:

(a) $\tau = \frac{1}{2} f_{cp} \sin 2\theta \quad (3.17)$
The equations obtained [19 and 24] based on the equilibrium in the longitudinal direction and the shear flow variation are given in Appendix B.
The transverse equilibrium conditions can be obtained by assuming that the section is fully cracked in the region below the neutral axis under the loading and the concrete can carry no tension. It implies that in the post-cracking loading range, the applied loads are resisted by a field of diagonal compression in the concrete, while the reinforcing steel resists the induced tension.

Assuming that within the spacing, $s_w$ (Fig. 3.11), the applied shear and the internal compression stresses in concrete are uniform, the equilibrium of transverse forces can be identified by the following equations.
Thus \[
\frac{A_{we}f_w}{s_w} = q \tan \theta
\]

Substituting \( t_w = A_{we}/s_w \), equation (3.21) becomes

\[
\tan \theta = \frac{t_w f_w}{q}
\]  

(3.22)

3.4.3 Stress-Strain Relationships of Steel and Concrete

The local stress- local strain relationships determined from standard material tests may vary in a structural concrete beam under loading depending on the bond between the reinforcement and the concrete, the distribution of the reinforcing bars within the concrete and the discontinuities occurring at cracks. Due to the fact that the 'actual' stresses and strains along the member is difficult to obtain, [5] for simplicity the idealised stress-strain relationships as explained
below are considered in the prediction of the behavioural response of the beam.

The idealised stress-strain curve for the reinforcement (Fig. 3.12a) is based on the simple bi-linear function defined by Young's Modulus $E_s$, namely:

\[
stress ~ f_s = E_s \varepsilon s f_y \quad (3.23)
\]

(a) For reinforcement

For the normal-weight concrete, the compressive stress-strain curve (Fig. 3.12b) is based on the parabolic expression, namely:

\[
stress ~ f_{cp} = f_c (2\varepsilon_{cp} - \varepsilon_{cp}^2) \quad (3.24)
\]
where \( \eta_{cp} = \frac{\varepsilon_{cp}}{\varepsilon_{co}} \). This expression is much simpler and reasonably accurate than the more complex relationships (of Kabaila, Hognestad and Desayi and Krishnan). Again, it was noted that for low strains (roughly, \( \varepsilon_{cp} < 0.3 \varepsilon_{co} \)) the straight-line expression may be a reasonable approximation and for high strains (roughly, \( \varepsilon_{cp} > 1.2 \varepsilon_{co} \)) the parabolic expression underestimates the stresses.

Collins and Mitchell [6] have suggested the following stress-strain relationship:

\[
f_{cp} = f'_c (2\eta_{cp} - \eta_{cp}^2)/(0.80+0.34\varepsilon_t)
\]

(3.25)

based on the verification that the principal compressive stress in a cracked concrete is a function not only of the principal compressive strain \( \varepsilon_{cp} \) but also of the co-existing tensile strain, \( \varepsilon_t \). The above equation is used in the solution technique.

3.4.4 Solution Technique

A solution technique for the prediction of the behaviour of a beam using the compression field theory
is given in Appendix B along with the steps and equations used. A sample calculation is also provided to illustrate the procedure. Again, a computer program based on the above solution technique is provided in Appendix C.
CHAPTER 4  EXPERIMENTAL STUDY

Bearing in mind the restrictive factors such as the type of equipment available, machine capacity, laboratory space and resources, the experimental study was planned as explained in detail in the following sections.

4.1  OBJECTIVES OF THE EXPERIMENTAL STUDY

The primary purpose of the experimental study was to verify whether the wave reinforcement can be a better alternative to the vertical stirrups as shear reinforcement, to rectangular reinforced concrete beams. Accordingly the following objectives were aimed at:

(a) To test whether the wave reinforcement can be an acceptable type of shear reinforcement and also a better alternative to the vertical stirrups from the point-of-view of ultimate strength and cracking.

(b) To verify the response characteristics predicted...
by the compression field theory model, in particular the strains and angles of principal compression (refer Chapter 3).

The experience of experimental study was also to be used in comparing the efficiency of wave reinforcement with vertical stirrups and in formulating design recommendations involving wave reinforcement.

4.2 PARAMETERS OF THE TEST PROGRAMME

The type of shear reinforcement, namely vertical stirrups and/or wave reinforcement, the bending details associated with wave reinforcement and the amount of steel content were the basic parameters varied in the tests. Details regarding the choice of test specimens are given in Section 4.3.

4.3 CHOICE OF TEST SPECIMENS

All the specimens were rectangular non-prestressed reinforced concrete beams. The beams were simply-supported to yield a constant shearing
force (neglecting self-weight), over the outer regions (Fig. 4.1). The shear force is therefore negligible in the central region.

![Diagram of Loading, Shear Force, and Bending Moment](image)

**Fig. 4.1** LOADING, S.F and B.M DIAGRAMS

The cross-section of specimens were kept constant at 150 mm x 400 mm with an effective span of 2400 mm. The total length of the beams were 2700 mm. All the beams were over-reinforced to insure that shear distress would occur before flexural failure. The shear span was 1000 mm on either side.

The choice of test specimens was made to
incorporate the following:

(a) Type I beams - to verify the objective (a) stated in Sec. 4.1, for two identical beams, one with vertical stirrups and the other with an approximately equal amount of wave reinforcement.

(b) Type II beams - to verify the objective (a) stated in Sec. 4.1, for two beams, each with vertical stirrups in the right shear spans and 45 deg and 30 deg wave reinforcement respectively in the left shear spans.

(c) Type III beam - to verify the objectives (a) and (b) stated in Sec. 4.1, for a beam with 45 deg wave reinforcement.

The details of manufacture of the above test specimens are explained in Section 4.4.

4.4 TEST SPECIMENS

4.4.1 Manufacture of Test Specimens

As has been noted in Section 4.3, the test
specimens were grouped under three categories, and for which the details of manufacture are explained below.

(a) Type I beams (Fig. 4.2)

The two beams cast under this category were:

(i) BVS-1 (Beam with Vertical Stirrups—serial number 1) Only vertical stirrups were provided as shear reinforcement.

(ii) BWR-1 (Beam with Wave Reinforcement, serial number 1) Only 45 deg wave reinforcement was provided as shear reinforcement. But three vertical stirrups (two at the ends, one in the middle of the beam) were used to form the cage for handling purposes.

The beams BVS-1 and BWR-1 were identical, in that the reinforcing steel used were cut adjacent (from one single length) and used alternatively for the two beams. The amount of shear reinforcement was approximately equal while the amount of longitudinal reinforcement was exactly the same. The concrete produced in
each batch was placed in the beams alternatively in equal quantities. The vibration and curing were carried out under similar conditions.

(b) Type II beams (Fig. 4.3)
The two beams cast under this category were:

(i) BVWR-1 (Beam with Vertical Stirrups and Wave Reinforcement - serial number 1)
Vertical stirrups in the right shear span and 45 deg wave reinforcement in the left shear span were the types of shear reinforcement used.

(ii) BVWR-2 (Beam with Vertical Stirrups and Wave reinforcement - serial number 2)
Vertical stirrups in the right shear span and 30 deg wave reinforcement in the left shear span were the types of shear reinforcement used.

In the left shear spans of both the beams, vertical stirrups were used at all the sections where the vertical legs of the wave reinforcement existed.
(c) Type III beam (Fig. 4.4)

Under this category, a beam BWR-2 (Beam with Wave Reinforcement - serial number 2), was cast to have 45 deg wave reinforcement as shear reinforcement in addition to the vertical stirrups introduced at all sections where the vertical legs of the wave reinforcement existed.

The details of the general arrangement of steel and further beam properties are shown appropriately in Figures 4.2 to 4.4.
### FURTHER BEAM PROPERTIES

<table>
<thead>
<tr>
<th>PROPERTIES</th>
<th>BVS-1</th>
<th>BWR-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>shear span / depth ratio, $a_v/d$</td>
<td>2.71</td>
<td>2.71</td>
</tr>
<tr>
<td><strong>LONGITUDINAL STEEL</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>2Y8 ($A_s = 101$ mm$^2$)</td>
<td>2Y8 ($A_s = 1470$ mm$^2$)</td>
</tr>
<tr>
<td>Bottom</td>
<td>3Y25 ($A_s = 1470$ mm$^2$)</td>
<td>3Y25</td>
</tr>
<tr>
<td>$A_s/bd$</td>
<td>2.65%</td>
<td>2.65%</td>
</tr>
<tr>
<td><strong>SHEAR REINFORCEMENT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical stirrups</td>
<td>13Y9 - 200</td>
<td>3Y8</td>
</tr>
<tr>
<td>Wave reinforcement</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td><strong>Total mass, kg</strong></td>
<td>5.78</td>
<td>5.80</td>
</tr>
</tbody>
</table>

**NOTE:** ALL DIMENSIONS IN mm

---

Fig. 4.2 DETAILS OF TYPE I BEAMS
Fig 4.3 DETAILS OF TYPE II BEAMS

<table>
<thead>
<tr>
<th>PROPERTIES</th>
<th>BEWWR-1</th>
<th>BEWWR-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>shear span / depth ratio $a_y/d$</td>
<td>2.71</td>
<td>2.71</td>
</tr>
<tr>
<td><strong>LONGITUDINAL STEEL</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>2Y8 ($A_y = 101 \text{mm}^2$)</td>
<td>2Y8</td>
</tr>
<tr>
<td>Bottom</td>
<td>3Y25 ($A_y = 1470 \text{mm}^2$)</td>
<td>3Y25</td>
</tr>
<tr>
<td>$A_y/d$</td>
<td>2.65 %</td>
<td>2.65 %</td>
</tr>
<tr>
<td><strong>SHEAR REINFORCEMENT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear span</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Vertical stirrups</td>
<td>6R6-370</td>
<td>7R6-200</td>
</tr>
<tr>
<td>Wave reinforcement</td>
<td>2R6-370</td>
<td>3R6-400</td>
</tr>
<tr>
<td></td>
<td>- 5/8</td>
<td>- 8/16</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 45^\circ$</td>
<td>$\alpha = 45^\circ$</td>
</tr>
<tr>
<td>Total mass, kg</td>
<td>329</td>
<td>311</td>
</tr>
</tbody>
</table>

NOTE: ALL DIMENSIONS IN mm
(b) FURTHER BEAM PROPERTIES:

<table>
<thead>
<tr>
<th>PROPERTIES</th>
<th>BWR-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear span/depth ratio $a_s/d$</td>
<td>2.69</td>
</tr>
<tr>
<td>LONGITUDINAL STEEL Top</td>
<td>2Y8 ($A'_s = 10\text{mm}^2$)</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
</tr>
<tr>
<td></td>
<td>$A_s/ bd$</td>
</tr>
<tr>
<td>SHEAR REINFORCEMENT Vertical stirrups</td>
<td>8R6-370</td>
</tr>
<tr>
<td></td>
<td>Wave reinforcement</td>
</tr>
<tr>
<td></td>
<td>Total mass, kg</td>
</tr>
</tbody>
</table>

Fig. 4.4 DETAILS OF TYPE III BEAM
Each beam was cast in an wooden mould (plank thickness 19 mm) which was set on a vibrating table. The mould was properly sealed at the joints to prevent the escape of cement slurry. The steel cage was prepared to shape as rigidly as possible (reinforcement suitably tied using steel wires) and secured into position by 10 mm thick cover blocks.

Owing to the small capacity of the mixer, a total of nearly five batches were used for each beam (two cylinder moulds were also filled from each batch — refer Sec. 4.4.2.1). Proper vibration of the table was done at each of these casting stages to prevent honey-combing of the beam (and cylinders). Over-vibration was avoided to prevent segregation. The top surface of beam was finished smooth with a trowel and two lifting hooks were placed near the top ends of the beam.

The wooden mould was wrapped with polythene sheets immediately after casting and also wetted appropriately. Stripping was done after about forty eight hours and the beam kept wet, covered with hessian for 28 days. Thereafter the beam
was kept in laboratory temperature and during that time targets for strain measurements were fixed using 'araldite' quick fix. The beam was also painted white using slaked lime to facilitate the observation of cracks and for clear photography.

Beams BVS-1 and BWR-1 were cast at the same time. The beams BVWR-1, BVWR-2 and BWR-2 were cast at different times as per the planned laboratory schedule. The method of manufacture of all the specimens (placing, vibrating, curing and preparation in readiness for testing) were the same as explained earlier. Age at test varied between 30 to 45 days for all the specimens.

4.4.2 Materials

4.4.2.1 Concrete

Ordinary Portland cement, natural sand, and 9.5 mm (3/8") and 19 mm (3/4") crushed stone (normal weight aggregates), and water which was fit for drinking were used for concrete production. No admixtures were added. All concrete mix design was according to a method outlined by
Neville [17], the aggregates grading having been approximated to one of the four curves in Road Note No. 4.

Concrete compression test specimens were cast from actual batches used in casting the beams, using a standard cylinder of 150 mm (6") in diameter, 300 mm (12") long. Cylinder specimens were compacted in two layers using the bench vibrator used for the beam and the top surface finished with a trowel. For each batch of concrete, two cylinder specimens were made and hence a total of ten specimens for each beam. The specimens were standard-cured for 28 days and during that time targets were fixed at 200 mm apart on two side faces (diagonally opposite). Also the cylinders were capped with plaster-of-paris, a few hours before the test.

The cylinders were tested in the same 'as-cast' positions and 28-days stress-strain curves obtained. A standard compression testing machine was used and loads at 50 kN increments were applied without shock. The strains measured on the two pairs of targets were averaged in the plotting of
the compressive stress-strain curves (Figures 4.5 to 4.7). The parabolic stress-strain relationships used to predict the behaviour of beams (to approximate the actual stress-strain relationship) are also plotted alongwith. The peak compressive cylinder strength of concrete $f'_c$, the corresponding strain $\varepsilon_{co}$, and the modulus of elasticity of concrete $E_c$, are also marked appropriately. The various properties of the concrete cylinder specimens are summarised in Table 4.1.
Fig. 4.5 CONCRETE CYLINDER COMPRESSIVE STRESS - STRAIN CURVES

Stress (N/mm²)

Strain (mm/mm)

$E_c = 28.6 \text{ kN/mm}^2$

$E_s = 0.0022$

Concrete Cylinder Compressive Stress - Strain Curves for Beams BVS-1 & BWR-1
Fig. 4-6 CONCRETE CYLINDER COMPRESSIVE STRESS—STRAIN CURVES

(a) FOR BEAM BVWR-1

Idealised Parabola

Experimental

$\varepsilon_e = 0.00202$

$E_c = 275 \text{ kN/mm}^2$

$\varepsilon_e = 0.0022$

$E_c = 253 \text{ kN/mm}^2$

(b) FOR BEAM BVWR-2

Idealised Parabola

Experimental

$\varepsilon_e = 330 \text{ N/mm}^2$

$\varepsilon_e = 355 \text{ N/mm}^2$
Fig 4.3 CONCRETE CYLINDER COMPRESSIVE STRESS-STRAIN CURVES
<table>
<thead>
<tr>
<th>DESCRIPTION</th>
<th>TYPE I BEAMS</th>
<th>TYPE II BEAMS</th>
<th>TYPE III BEAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water-cement ratio</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Peak compressive strength</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f'_c ) N/mm(^2)</td>
<td>32.5</td>
<td>32.5</td>
<td>33.0</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>28.6</td>
<td>28.6</td>
<td>27.5</td>
</tr>
<tr>
<td>Strain corresponding to peak stress</td>
<td>2.2</td>
<td>2.2</td>
<td>2.02</td>
</tr>
<tr>
<td>( \varepsilon_{co} \times 10^{-3} ) mm/mm</td>
<td>2.2</td>
<td>2.2</td>
<td>2.1</td>
</tr>
</tbody>
</table>
4.4.2.2 Steel

The tensile testing of bars, which were representative pieces cut from the actual reinforcing bars used to make the steel cages for the specimens, consisted of straining test pieces by tensile stress to fracture. Non-proportional test pieces (or full section test pieces) with incised marking were used. At least three specimens were tested for each bar size and type used in the beam and the average was used in obtaining the tensile stress-strain curve.

The tests for Y25 bars were performed using Avery-Denison testing machine and for Y8, R8 and R6 bars using 'Instron' testing machine. Both plotted load-elongation curves from which the nominal stress-strain curves were prepared (Figures 4.8 to 4.10). The yield stress, the corresponding yield strain and the modulus of elasticity are also marked appropriately. For bars which did not have a distinct yield point, the yield stress was taken as 0.2% proof stress. The modulus of elasticity was obtained as the slope of the linear elastic portion of the curve. The various properties of reinforcing steel specimens are summarised in Table 4.2.
Fig 46  TENSILE  STRESS - STRAIN CURVES  FOR  Y25  BARS
Fig. 4.9 TENSILE STRESS-STRAIN CURVE - Y8 BARS
Fig. 4.10 TENSILE STRESS-STRAIN CURVES FOR ROUND BARS

(a) FOR TYPE II & III BEAMS-R8 BARS

\[ \sigma = 192 \text{ kN/mm}^2 \]

\[ \varepsilon_s = 0.00209 \]

\[ \varepsilon = 1.0 \times 10^{-3} \text{ mm/mm} \]

(b) FOR TYPE II & III BEAMS-R6 BARS

\[ \sigma = 342 \text{ N/mm}^2 \]

\[ \varepsilon_s = 0.0021 \]

\[ \varepsilon = 2.0 \times 10^{-3} \text{ mm/mm} \]
### TABLE 4.2 PROPERTIES OF REINFORCING STEEL SPECIMENS

<table>
<thead>
<tr>
<th>DESCRIPTION</th>
<th>BAR SIZES</th>
<th>Y25</th>
<th>Y8</th>
<th>R8</th>
<th>R6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used in beams</td>
<td>Types I &amp; III</td>
<td>505</td>
<td>517</td>
<td>455</td>
<td>342</td>
</tr>
<tr>
<td>Yield stress, N/mm²</td>
<td>Type II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ALL</td>
<td></td>
<td></td>
<td>430</td>
<td>342</td>
</tr>
<tr>
<td>Ultimate stress, N/mm²</td>
<td>Type II</td>
<td>574</td>
<td>602</td>
<td>642</td>
<td>342</td>
</tr>
<tr>
<td>Modulus of elasticity, kN/mm²</td>
<td>Types II &amp; III</td>
<td>225-</td>
<td>227</td>
<td>194</td>
<td>163</td>
</tr>
<tr>
<td>Yield strain x 10⁻³ mm/mm</td>
<td></td>
<td>2.24</td>
<td>2.28</td>
<td>2.35</td>
<td>2.09</td>
</tr>
</tbody>
</table>


4.5 TEST RIG

A testing machine transmitting loads to two hydraulic loading jacks was used to test the beam. The loading jacks were positioned 400 mm apart (centre to centre) and held in stable vertical positions by suitable tie rods. The verticality was checked using plumb-bobs attached to the sides of the loading jacks. The supports to the test beam were provided by two horizontal I-beams which were bolted to rectangular framework.

The loading from the jacks was transferred to the beams through rollers kept over steel plate. It was noted that the rollers permitted the loading jack to follow the curvature of the beam easily. The readings were taken in 'pounds per square inch' and then converted into kilonewtons (kN). The typical test arrangement is shown in Fig. 4.11 and in Plate 4.1. A load sensor introduced between the beam and the loading jacks (Plate 4.2) indicated that the jacks transmitted the loads equally to the beam and that the machine reading against the transmitted load compared well.
Fig. 4.11  TYPICAL TEST ARRANGEMENT
PLATE 4-1 TYPICAL TEST ARRANGEMENT
(a) with load sensors

(b) without load sensors

PLATE 4.2 TYPICAL LOADING
4.6 INSTRUMENTATION

The following instruments were used during the testing of beams:

(a) Hand-held strain gauges

Two types of hand-held strain gauges were used to measure strains between demec or target points. They are:

(i) strain gauge with 1 division = strain of $0.81 \times 10^{-5}$; and

(ii) strain gauge with 1 division = 0.00254 mm

(converted from 0.0001 inches)

The demec points were stuck using quick-fix adhesive pastes choosing a pattern of targets as shown in Fig. 4.12a, and left dry for a day before the test. The target pattern was maintained constant for all the beams. The regions of the beams approximately within a radius equal to the effective depth are normally affected by the effects of flexural moment and local disturbances of point loads [5] and hence such regions were neglected from strain.
measurements.

(b) Deflection gauges
Baty Push-off (magnetic base) deflection gauge with a least count of nearly 0.025 mm (converted from 0.001 inches) was used to measure deflection of the beam bottom, at selected locations. (refer Sec. 4.7).

(c) Hand-held crack microscope
A hand-held microscope with built-in reticule to read widths to the nearest 0.05 mm was used to measure crack widths. A hand-held magnifying glass was also used to locate the existence of the cracks which were not visible to the naked eye.

4.7 TESTING PROCEDURE

In order to check the general performance of the test arrangement, the beam was loaded a day earlier to the actual test to about 15% of the calculated flexural failure load and then unloaded. During the actual test, load was applied in increments of about 1/10th of the calculated flexural
failure load though the increment was reduced around the flexural cracking, shear cracking and failure load.

The following observations were made at zero load and thereafter at each increment of load:

(a) Strain observations
The longitudinal, transverse and diagonal strains were measured between the targets (refer Fig. 4.12b).

(b) Deflection observations
For each beam the deflection readings were taken at the bottom face, one at the centre and two more on either side in the shear spans but equidistant from the supports. Just prior to the ultimate failure the deflection gauges were removed to avoid any damage.

(c) Crack observations
The crack width at various levels was measured at 90 deg to the propagation direction and recorded. The propagation pattern of cracks was marked on both sides of the beam using a
black felt pen and recorded fairly accurately. The corresponding applied moment was also marked at the locations where the cracks formed first. Photographs were also taken at different stages of crack formation and on ultimate failure.

A period of about 45 minutes was spent at every load stage for the observations and recordings. Enough care was taken to maintain the load constant during this period. The average time for the complete test on a beam from zero load to ultimate failure load was about seven hours and with an active participation of about five technicians (Typical 'observation' activity is shown in Plate 4.3). Test on each beam was performed without any interruption. The ultimate failure load was recorded for all the beams.
(a) TARGET POSITIONS ON SIDE FACE

(b) DETAILS OF STRAIN MEASUREMENTS AT X FOR A PARTICULAR LOAD

LONGITUDINAL: E₁F₁, E₂F₂, E₃F₃, & C

TRANSVERSE: E₁E₃, F₁F₃, E₂E₄, F₂F₄, & C

DIAGONAL: E₁F₃, E₂F₄, F₁E₃, F₂E₄, & C

Fig. 4-12 TARGET ARRANGEMENT FOR BEAMS
PLATE 4-3  TYPICAL OBSERVATION ACTIVITIES
CHAPTER 5 ANALYSIS OF TEST RESULTS AND DISCUSSION

The chief items of behaviour of a beam which are of practical interest are the ultimate strength and deformations such as the extent of cracking and deflection under service conditions. Keeping this in mind, the data collected are appropriately analysed and discussed in this chapter. The experimental strain patterns and other related aspects are also compared.

Plates 5.1 to 5.6 provided at the end of this Chapter shows the arrangement of steel, the pattern of cracks and other details for the beams tested.

5.1 ULTIMATE STRENGTHS OF BEAMS

5.1.1 Flexural Strength

The ultimate flexural strengths of all the beams tested and their type of failures are indicated in Table 5.1. While the experimental ultimate flexural moment ($M_{ue}$) for each beam is obtained from the load recorded in the testing machine, the computed
ultimate flexural strength ($M_{uf}$) is based on the flexural formula using the actual material properties.

TABLE 5.1 COMPARISON OF EXPERIMENTAL AND COMPUTED FLEXURAL STRENGTHS AND TYPE OF FAILURES

<table>
<thead>
<tr>
<th>BEAMS</th>
<th>ULTIMATE STRENGTH (kN m)</th>
<th>$M_{ue}/M_{uf}$</th>
<th>Type of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental ($M_{ue}$)</td>
<td>Computed ($M_{uf}$)</td>
<td></td>
</tr>
<tr>
<td>TYPE I BEAMS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BVS-1</td>
<td>268.9</td>
<td>225.5</td>
<td>1.19</td>
</tr>
<tr>
<td>BWR-1</td>
<td>242.0</td>
<td>225.5</td>
<td>1.07</td>
</tr>
<tr>
<td>TYPE II BEAMS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BVWR-1</td>
<td>220.3</td>
<td>217.9</td>
<td>1.01</td>
</tr>
<tr>
<td>BVWR-2</td>
<td>239.2</td>
<td>222.2</td>
<td>1.08</td>
</tr>
<tr>
<td>TYPE III BEAM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BWR-2</td>
<td>240.9</td>
<td>212.0</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Based on the results of Type I beams (Table 5.1) the following observations can be made:
(a) beam BWR-1 with 45 deg wave reinforcement recorded lower experimental ultimate flexural strength than the identical beam BVS-1 with vertical stirrups.

(b) beam BWR-1 failed by shear-tension by splitting of the concrete at the level of longitudinal tensile steel by destruction of bond (Fig. 5.1b), while BVS-1 failed by shear-compression (Fig. 5.1a).

(c) the ratios $M_{ue}/M_{uf}$ are greater than unity in both cases.

From the observations (a) and (b) mentioned above, it can be noted that the lower ultimate strength of beam BWR-1 may be attributed to the absence of vertical stirrups and hence the reduced capacity of longitudinal bars in developing bond forces necessary to share in the shear mechanism. It can also be suggested that the high concentration of inclined compression stresses would have attributed to the concrete failure near the joints of the transverse reinforcement and the longitudinal tensile steel leading to the type of failure mentioned above.
The tests on beam BVS-1 and BWR-1 have confirmed that beam BWR-1 with wave reinforcement has satisfied the ultimate flexural strength criteria and hence wave reinforcement is functionally a viable alternative type of shear reinforcement. The remedy for the splitting failure and the associated lower ultimate strength of beam BWR-1 was thought of by placing vertical stirrups at all sections where the vertical legs of the wave reinforcement exist. This combination of vertical stirrups and wave reinforcement was adopted in Type II and Type III beams. It should be noted that the vertical stirrups facilitate fabrication of a rigid cage and this eliminates the necessity of providing additional vertical stirrups exclusively for forming a cage.

The following observations may be made from the results of beam BVWR-1 and BVWR-2 (for details of beams refer to Sec. 4.4.1):

(a) both beams have satisfied the ultimate strength criteria.

(b) both beams failed by shear-compression and not by splitting at the level of
longitudinal tensile steel, thus supporting the necessity of vertical stirrups along with wave reinforcement.

Results of beam BWR-2 confirmed that the 45 deg wave reinforcement combined with vertical stirrups has satisfied the ultimate flexural strength criteria. Also, the absence of splitting of the longitudinal tensile steel is a noteworthy feature.

5.1.2 Shear Strength

For the beams with wave reinforcement the nominal shear strength $V_n$ is computed from $V_n=V_c+V_s$ (refer equation 2.1), where the values of $V_c$ and $V_s$ are worked out from the equations 7.1 and 7.5 respectively. It is also assumed that $\theta$ is equal to 45 deg. The ultimate shear strengths $V_{ue}$ attained in all the tests for the beams with wave reinforcement are shown in Table 5.2.
TABLE 5.2 COMPARISON OF CALCULATED NOMINAL SHEAR STRENGTH $V_n$ AND EXPERIMENTAL ULTIMATE SHEAR $V_{ex}$

<table>
<thead>
<tr>
<th>BEAMS</th>
<th>CALCULATED VALUES (kN)</th>
<th>EXPERIMENTAL</th>
<th>RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_C$</td>
<td>$V_S$</td>
<td>$V_n$</td>
</tr>
<tr>
<td>BWR-1</td>
<td>52.66</td>
<td>110.50</td>
<td>163.16</td>
</tr>
<tr>
<td>BVWR-1</td>
<td>53.07</td>
<td>129.86</td>
<td>182.93</td>
</tr>
<tr>
<td>BVWR-2</td>
<td>55.04</td>
<td>129.86</td>
<td>184.90</td>
</tr>
<tr>
<td>BWR-2</td>
<td>52.21</td>
<td>136.25</td>
<td>188.46</td>
</tr>
</tbody>
</table>

It can be noted from Table 5.2 that the ratio $V_{ex}/V_n$ is greater than unity for the beams BVWR-1 and BWR-2 (beams with 45 deg wave reinforcement combined with vertical stirrups). Thus the shear strength calculated using Eq. 3.5 compared well with the actual shear strength of the specimens listed. This confirms the validity of equations derived for the design of wave reinforcement.
5.2 CRACKING

The vital stages of formation of cracks for all the beams tested are as follows:

(a) Initially, flexural cracks developed and they were almost vertical.

(b) Subsequently, with the addition of loads, inclined shear cracks developed. Further widening of these cracks and flexural cracks led to secondary cracks.

(c) Finally, with the exception of beam BWR-1, the failure of beams occurred with the extension of critical shear crack into the compression zone. The concrete above the upper end of the crack crushed, resulting in shear-compression failure. But the beam BWR-1 failed by the splitting of the concrete at the level of longitudinal tensile steel and destruction of bond, resulting in shear-tension failure.

The cracks in the pure moment section were vertical whereas the cracks in the shear spans curved
towards the applied load points. It is also worth noting that the inclined cracks formed at a distance larger than the depth, \( d \) of the beams from the support and this may be attributed to the effect of vertical compression due to support reactions in the immediate vicinity of the supports.

The crack pattern for Type I to III beams along with the critical crack and the load in kN, at which a particular crack first formed, are illustrated in Fig. 5.1 to 5.3.

Referring to Fig. 5.2a and b, it can be seen that the critical crack in beam BVWR-1 formed in the shear span with vertical stirrups whereas in beam BVWR-2, it formed in the shear span with 30 deg wave reinforcement. Hence it may be suggested that 45 deg wave reinforcement is superior to either 30 deg wave reinforcement or vertical stirrups. The number of cracks are nearly the same in both the shear spans of beams BVWR-1 and BVWR-2.

Referring to Fig. 5.3 it can be seen that the beam BWR-2 developed fewer cracks than Type I and II beams.
From the typical maximum crack width (flexural and shear) against the ratio of applied load to ultimate load plotting (Fig. 5.4), it is evident that maximum crack width was the least in beam BWR-2.

Based on the extent of cracks and the formation of critical cracks in all the test beams, it can be suggested that the 45 deg wave reinforcement with the combination of vertical stirrups (beams BVWR-1 and BWR-2) is an acceptable type of shear reinforcement and is also a better type than vertical stirrups or 30 deg wave reinforcement.
Fig. 5.1 CRACK PATTERN FOR TYPE I BEAMS
Fig. 5.2 CRACK PATTERN FOR TYPE II BEAMS
Fig. 5.4 MAXIMUM CRACK WIDTH AGAINST RATIO OF APPLIED LOAD TO ULTIMATE LOAD
5.3 DEFLECTION

The study of deflections of flexural members under service load is of greater sensory importance. Such members should be designed to have adequate stiffness to limit deflection that may adversely affect strength or serviceability of a structure at service loads. Thus for all the beams tested, deflections at various load levels were measured and plotted (Fig. 5.5 and Fig. 5.6). From the graphs it can be observed that the deflection behaviour is closely the same for all the beams tested.

It can also be noted that the maximum deflection at 0.5 times the ultimate load was about L/450 for beams with wave reinforcement and about L/350 for beams with vertical stirrups (where L is the effective span of the member).
(a) FOR TYPE I BEAMS

(b) FOR TYPE II BEAMS

(i) BVWR-1

(ii) BVWR-2

Fig. 5.5 MEASURED LOAD — VERTICAL DEFLECTION BEHAVIOUR
Fig. 5-6 MEASURED LOAD—VERTICAL DEFLECTION BEHAVIOUR - BWR-2

Applied Load (kN)

Central Deflection (mm)

0.5 to 0.6 of ultimate load
5.4 STRAIN PATTERNS

The average longitudinal and transverse strain and diagonal compressive strain profiles at two load levels, namely

(a) at nearly half capacity load level, and

(b) at nearly capacity load level

are plotted for typical cases (Fig. 5.7a to d). They are compared with the corresponding profiles predicted using compression field theory (CFT).

(a) Variation of longitudinal strains

Fig. 5.7a illustrates the average longitudinal strain profile at nearly half capacity loads and full capacity loads. From the profiles for beams BVWR-1 and BVWR-2, in the shear span A with wave reinforcement the average longitudinal strain curves are linear over the depth of the beam at both the load levels.

For the beam BWR-2 (Fig. 5.7b) the experimental and predicted profiles are fitted for half capacity and full capacity load levels. Certain disparities exist between the two profiles, which may be due
to the non-uniform cracking behaviour or due to the influence of loading. But from Fig. 5.10 it can be noted that longitudinal strain profile at pure moment section is linear with correlation coefficient close to unity and provides a clear picture of strain variation over the depth of beam.

(b) Variation of transverse strains:

A typical transverse strain profile for the beam BWR-2 is illustrated in Fig. 5.7c. Again the influence of loading system and possible inaccuracy in demec gauges would have caused the disparity in shapes but at approximately half capacity load level the curves are identical.

(c) Variation of diagonal compressive strains:

The diagonal compression strain profile (Fig. 5.7d) at the above mentioned two load levels has a similarity in shape. Again the predicted curve agreed closely with the experimental results.
(a) Beam BVWR-1:

(i) At $= 1/2$ capacity load level

(ii) At $= 0$ capacity load level

(b) Beam BVWR-2:

(i) At $= 1/2$ capacity load level

(ii) At $= 0$ capacity load level

Legend: Compressive strains $+$; Tensile strains $-$

Figures against plots: X-axis - average longitudinal strain $x 10^3$ (experimental) - Region EF

$r$ = linear correlation coefficient

Fig. 5.7 (a) AVERAGE LONGITUDINAL STRAIN PROFILES
(a) At ≤ 1/2 capacity load level

(b) At ≥ full capacity load level

Legend:
- Compressive strains +ve
- Tensile strains -ve
- EXPERIMENTAL
- X-axis - average longitudinal strain x 10^3
- PREDICTED By CFT
- r - linear correlation coefficient

Fig. 5.7 (b) AVERAGE LONGITUDINAL STRAIN PROFILES
Fig. 5.7 (c) TRANSVERSE STRAIN PROFILE

Beam BWR-2

Legend:
- Compressive strains +ve
- Tensile strains -ve
- At ≈ 1/2 capacity level load (111 kN)
- At ≈ capacity level load (211 kN)
- Predicted by CFT (114 kN)
Beam BWR-2

Legend:
- ▲ At 1/2 capacity level load (111 kN m)
- □ At full capacity level load (211 kN m)
- ● Predicted by CFT (114 kN m)

Fig. 5.7 (d) DIAGONAL COMPRESSION STRAIN PROFILES
5.5 RELATIONSHIPS BETWEEN THE SHEAR FORCE AND PRINCIPAL ANGLES OF COMPRESSION AND MAXIMUM SHEAR STRAIN

The experimental angles of principal compression are deduced from measured strains using Mohr's circle (Appendix D). Typical plotting (Fig. 5.8a) for beam BWR-2 indicates that the angles of principal compression at the level of neutral axis agreed quite closely between the experimental values and the values predicted using the compression field theory. Again the values of $\theta$ varied nearly between 20 and 40 deg and the average is around 31 deg. Comparing the value of $\theta$ at bottom longitudinal steel level, it can be said that $\theta$ varied between 22 and 62 deg and the average is around 40 deg. It can also be mentioned that the variation between the plotted curves may be due to effect of non-uniform cracking of beams.

Typical shear force-maximum shear strain relationships are plotted in Fig. 5.8b. Within the range of experimental data a close similarity exists.
between the experimental and predicted curves, though the predicted shear strains are generally greater than those obtained from experiment. Hence the theory predicts maximum shear strains conservatively.

5.6 RELATIONSHIP BETWEEN MOMENT-CURVATURE  
(M - \( \phi \) curves)

Based on the test results, the experimental M-\( \phi \) curves are obtained for the test beams BVS-1, BWR-1 and BWR-2 (Fig. 5.9a and b). Using the strain profiles (some of which are shown in Fig. 5.10) the curvature \( \phi \) for a given section of the beam corresponding to the moment \( M \) is calculated as,

\[
\phi = \left( \varepsilon_{ct} + \varepsilon_t \right)/d
\]

where \( \varepsilon_{ct} \) is the concrete strain at the extreme compression fibre and \( \varepsilon_t \) is the strain at longitudinal tensile steel.

The theoretical M-\( \phi \) curves for flexure are fitted by calculating the moment and curvature corresponding to a range of \( \varepsilon_{ct} \) values. For an
assumed $\varepsilon_{ct}$, the neutral axis depth is adjusted until the stresses in the concrete and steel result in internal forces to balance. The material properties are used in computing the stresses. Strains are calculated on the assumption that the plane sections before bending remain plane after bending. Another $M-\phi$ curve based on the compression field theory (Appendix B and C) for the beam BWR-2 is also fitted. It can be noted that the experimental and the theoretical curves compared quite closely and the ultimate loads predicted correspond with the experimental results.
Beam BWR-2

Fig. 5-8(a) - RELATIONSHIP BETWEEN SHEAR FORCE AND PRINCIPAL ANGLE OF COMPRESSION
Fig. 5.8 (b) RELATIONSHIP BETWEEN SHEAR FORCE AND MAXIMUM SHEAR STRAIN AT NEUTRAL AXIS
Fig. 5.9 (a) RELATIONSHIPS BETWEEN MOMENT & CURVATURE
Fig. 5.9(b) RELATIONSHIP BETWEEN FLEXURAL MOMENT AND CURVATURE FOR BEAM BWR-2
Fig 5.16 LONGITUDINAL STRAIN PROFILE IN PURE MOMENT SECTION

Legend: - Compressive strains: -ve  - Tensile strains: +ve  - r is linear correlation coefficient
PLATE 5-1 ARRANGEMENT OF REINFORCEMENT FOR TYPE I BEAMS (BVS-1 and BWR-1)
(a) For beam BVS-1 (old name VSB-1)

(b) For beam BWR-1 (old name DSB-1)

PLATE 5-2  CRACK PATTERNS
FOR TYPE I BEAMS
(a) Arrangement of reinforcement

(b) Crack pattern

PLATE 5-3 BEAM BVWR-1
(a) Arrangement of reinforcement

(b) Crack pattern
PLATE '5-5 CRACK PATTERN IN BEAM BWR-2
PLATE 5-6 BEAM BWR-2:

REINFORCEMENT PARTLY EXPOSED
To evaluate the 'best' choice between vertical stirrups and wave reinforcement the efficiency concept with respect to function and economy is applied. The details of which are explained in the following sections.

6.1 FUNCTIONAL EFFICIENCY

Function refers to the use or uses performed, including the esteem features provided. Any shear reinforcement should play an effective role in resisting the opening or widening of inclined cracks (Sec. 2.2) and accordingly the vertical stirrups and wave reinforcement are studied as under:

(a) percentage chance in not intercepting inclined cracks, and
(b) crack interception efficiency.

6.1.1 Percentage Chance in Not Intercepting Inclined Cracks

The Code [4] requires that the stirrups must be spaced such that every 45 deg line representing
a potential inclined crack must be crossed in the tension side of the beam by at least one stirrup which limits the maximum spacing to $d/2$.

Fig. 6.1a to c show the arrangement of steel for a beam with vertical stirrups, 45 deg wave reinforcement and 30 deg wave reinforcement respectively.
A typical case of 45 deg line (assumed inclined crack angle) extending towards the reaction from middepth of member d/2 to longitudinal tension reinforcement is also indicated.

It may be noted that the cracks between AA and BB are intercepted by at least one leg of shear reinforcement whereas the cracks between BB and CC are intercepted by more than one leg. But the cracks between A'A' and AA (Fig. 6.1c) are not intercepted by shear reinforcement at all. The following observations may also be made:

(a) From Fig. 6.1a, it can be seen that for \( \theta \leq 45 \text{ deg} \) the vertical stirrups will definitely intercept the inclined crack at any one point. But for 45 deg < \( \theta \leq 60 \text{ deg} \) varying chances do exist for the cracks not intercepting the vertical stirrups.

(b) From Fig. 6.1b, it can be seen that for \( \theta \leq 45 \text{ deg} \), the 45 deg wave reinforcement will definitely intercept the inclined crack at any one point. But for 45 deg < \( \theta \leq 60 \text{ deg} \) varying chances do exist for the cracks not intercepting the 45 deg wave reinforcement.
(c) From Fig. 6.1c, it can be seen that for $\theta \leq 30$ deg the 30 deg wave reinforcement will definitely intercept the inclined crack at any one point but for $30 \text{ deg} < \theta \leq 60$ deg varying chances do exist for the cracks not intercepting the 30 deg wave reinforcement.

Table 6.1 shows the percentage chance that the transverse reinforcement not intercepting the inclined cracks developing at an assumed constant angle $\theta$ deg.

**TABLE 6.1 PERCENTAGE CHANCE OF TRANSVERSE REINFORCEMENT NOT INTERCEPTING THE INCLINED CRACKS**

<table>
<thead>
<tr>
<th>TYPES OF SHEAR REINFORCEMENT</th>
<th>VALUE OF $\theta$ (in deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Vertical stirrups</td>
<td>NIL</td>
</tr>
<tr>
<td>45 deg wave reinforce-ment</td>
<td>NIL</td>
</tr>
<tr>
<td>30 deg wave reinforce-ment</td>
<td>NIL</td>
</tr>
</tbody>
</table>

**NOTE:** The above values of percentage chance are calculated from $(A'/A/A'C) \times 100$
It is evident from Table 6.1 that 45 deg wave reinforcement is more effective in intercepting the inclined cracks (for the ranges of $\theta$ indicated) than the vertical stirrups and also 30 deg wave reinforcement.

6.1.2 Crack Interception Factor

Defining crack interception factor, $k_{CI}$ as the ratio of the areas of the portion where the cracks cut more than one leg (BBCC) to the area of the portion where the cracks cut at least one leg (AABB), different values of $k_{CI}$ are obtained for various values of $\theta$. The results are shown in Table 6.2.

**TABLE 6.2 CRACK INTERCEPTION FACTOR, $k_{CI}$**

<table>
<thead>
<tr>
<th>TYPES OF SHEAR REINFORCEMENT</th>
<th>VALUE OF $\theta$ (in deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Vertical stirrups</td>
<td>0</td>
</tr>
<tr>
<td>45 deg wave reinforcement</td>
<td>6.0</td>
</tr>
<tr>
<td>30 deg wave reinforcement</td>
<td>1.0</td>
</tr>
</tbody>
</table>
It is evident from Table 6.2, for $0 \leq 45$ deg, the values of $k_{CI}$ are greater than 1.0 for 45 deg wave reinforcement whereas for vertical stirrups, $k_{CI} = 0$ and for 30 deg wave reinforcement $k_{CI} = 1.0$. Even for $45$ deg $< \theta < 60$ deg, $k_{CI}$ value is higher for 45 deg wave reinforcement than for 30 deg wave reinforcement. Fig. 6.2 (based on Table 6.2) shows the variation of $k_{CI}$ against $\theta$.

![Fig. 6.2 CRACK INTERCEPTION FACTOR AGAINST ANGLE OF INCLINED CRACK](image-url)
This again confirms that 45 deg wave reinforcement is more efficient in intercepting the inclined cracks than either vertical stirrups or 30 deg wave reinforcement.

6.1.3 Other Pertinent Facts

The experience of placing and fixing transverse reinforcement to the various test beams (Fig. 4.2 to 4.4) has highlighted certain facts which lead to the understanding of the following advantages of wave reinforcement over vertical stirrups:

(a) the number of cutting of steel involved was far fewer; and hence lesser wastage of material, if any.

(b) the bar bending (using the arrangement as shown in Fig. 3.2) was much easier and quicker particularly after some experience.
(c) the arrangement of reinforcement provided lesser congestion and hence better access for the pouring of concrete.

With poker vibrators, this advantage may lead to a better compaction.

(d) the time taken for bending, placing and fixing was much shorter.

It can also be observed that larger diameter bars can be bent and used for wave reinforcement which was otherwise a limitation for vertical stirrups (due to number of hooks and/or bends involved).

Summarising, it can be said that 45 deg wave reinforcement is functionally more efficient than the vertical stirrups.

6.2 ECONOMIC EFFICIENCY

Though the consideration of function will be a dominant feature in the whole exercise of evaluating the "best" choice between vertical
stirrups and wave reinforcement, the cost advantages technique may be used to identify the economic efficiency. Economic efficiency is studied for:

(a) Case I: beam with vertical stirrups as shear reinforcement

(b) Case II: identical beam with 45 deg wave reinforcement combined with vertical stirrups (as explained in Sec. 5.1.1) as shear reinforcement

The details of which are explained below.

6.2.1 Economy of Material Used for Shear Reinforcement

For a typical case of 45 deg wave reinforcement and $\theta = 45^\circ$, it can be seen that (Eq. 3.7),

nominal shear strength provided by an inclined leg of wave reinforcement

$\approx 1.414$ times the nominal shear strength provided by vertical stirrups
This proves that the 45 deg wave reinforcement can provide more shear strength than the vertical stirrups having same area and hence economical.

Consider a case loading for a simply-supported doubly-reinforced rectangular concrete beam with a central point load as shown in Fig. 6.3.

\[ V_u = 110.5 \text{ kN} \]

The beam is to be designed for shear using

(a) ACI 318M code \[4 \] for vertical stirrups
(b) BS8110 code \[3 \]
(c) Proposed design recommendations (Chap. 7) - for 45 deg wave reinforcement

Assume

\[ f_c' = 25 \text{ N/mm}^2 \]
\[ f_{ys} = 250 \text{ N/mm}^2 \] (for transverse steel)
(a) ACI 318M Code:

factored shear force $V_u = 110.5 \text{ kN}$
nominal shear strength $V_n = 110.5/0.85 = 130 \text{ kN}$
permissible

$$V_C = (\sqrt{f'_c / 6})bd$$

$$= (\sqrt{25/6}) \times 150 \times 378 \text{ N}$$

$$= 47.25 \text{ kN}$$

Since $V_u > V_C$, stirrups must be designed for,

$$V_s = V_n - V_C = 130 - 47.25 = 82.75 \text{ kN}$$

For vertical stirrups,

$$V_s = A_v f_y d / s$$

where $A_v$ is the area of shear reinforcement within a distance $s$.

Try R8 two-legged stirrups

$$s = 101 \times 250 \times 378 / (82.75 \times 10^3) \text{ mm}$$

$$= 115.3 \text{ mm}$$

Choose 115 mm spacings which is less than $d/2 = 378/2 = 189 \text{ mm}$, permissible maximum.

Provide 20 R8 - 115 two-legged stirrups

Total length of transverse steel per stirrup

$$= 2(A+B) + 20d$$

$$= 2(370 + 114) + 20 \times 8 \text{ mm}$$

$$= 1128 \text{ mm}$$
1. Assuming a standard mass of 0.395 kg/m, total mass of vertical stirrups
   \[ = 20 \times \left( \frac{1128}{1000} \right) \times 0.395 \text{ kg} \]
   \[ = 8.91 \text{ kg} \]

(b) BS 8110 Code:

Design concrete shear stress \( (v_c) \) is calculated as follows:

\[ 100A_s/\text{bd} = 100 \times \frac{943}{(150 \times 378)} \]
\[ = 1.66 \]

The relevant table from the code is partially extracted and given below (Table 6.3).

**TABLE 6.3 DESIGN CONCRETE SHEAR STRESS \( (v_c) \)**

<table>
<thead>
<tr>
<th>100A_s/\text{bd}</th>
<th>Effective depth (mm)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>300</td>
<td>&gt;400</td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td>0.78</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>0.86</td>
<td>0.80</td>
<td></td>
</tr>
</tbody>
</table>

By suitable interpolation, \( v_c = 0.76 \text{ N/mm}^2 \)

The design shear stress \( v = 110.5 \times 10^3/(150 \times 378) \text{ N/mm}^2 \)
\[ = 1.949 \text{ N/mm}^2 \]
Since \((v_c+0.4) v 5 \text{ N/mm}^2\), shear reinforcement should be provided.

\[
\frac{A_{SV}}{s_v} > \frac{b(v-v_c)}{0.87f_{yV}} \\
\geq \frac{150(1.949-0.76)}{0.87 \times 250} \\
\geq 0.820 \text{ mm}
\]

Try R8 two-legged stirrups.

\[s_v = 123.2 \text{ mm}\]

Choose 120mm spacings which is less than

\[0.75d = 0.75 \times 378 = 283.5 \text{ mm}, \text{ permissible maximum.}\]

Provide 19 R8 -120 two-legged stirrups.

Using the procedure given in (a) above, the total mass of vertical stirrups can be calculated as 8.47 kg.

(c) Proposed design recommendations for 45 deg wave reinforcement:

Shear reinforcement in the form of 45 deg wave reinforcement should be placed perpendicular to the axis of member in combination with vertical stirrups placed at sections where the vertical legs of the wave reinforcement exist (refer Sec. 7.2.1).
permissible $V_C = (\sqrt{f'_c/6})bd$

$$= (\sqrt{25}/6) \times 150 \times 378 \text{ N}$$

$$= 47.25 \text{ kN}$$

$V_S = V_n - V_C = (110.5/0.85) - 47.25 \text{ kN}$

$$= 82.75 \text{ kN}$$

Since $\alpha = 45$ deg, $\theta = 45$ deg and $d-d' = s_w$, and same diameter for wave reinforcement and vertical stirrups, from Eq. 3.8,

$$V_S = 4.414 \ A_{ss} f_{ys}$$

Therefore,

$$A_{ss} = 82.75 \times 10^3/(4.414 \times 250) \text{ mm}^2$$

$$= 74.99 \text{ mm}^2$$

which is for one 45 deg wave reinforcement and one vertical stirrup (two-legged).

Provide R10 (area 78.5 mm$^2$)

as $1R10 - 355 - 500$

and

$8R10 - 355$

Alternatively,

provide two 45 deg wave reinforcement and one vertical stirrup (two-legged). From Eq. 3.10

$$A_{we} = A_{ss} (2N + 2.414n)$$

where $N=1$ and $n=2$,

$$A_{we} = 6.828 \ A_{ss}$$
From Eq. 3.9, $V_s = A_{wef_{ys}}$ leading to

$$= 6.828 \ A_{ssf_{ys}}$$

Therefore $A_{ss} = \frac{82.75 \times 10^3}{(6.828 \times 250)} \ mm^2$

$$= 48.48 \ mm^2$$

Provide 4R8 - 355 - 500 and 8R8 - 355 (Area provided = 50.3 mm$^2$)

The latter is preferable. Refer Sec. 7.2.2.2.

![Fig. 6.4 DETAILS OF TRANSVERSE STEEL](image)

Assume a standard mass of 0.395 kg/m.

Mass of wave reinforcement provided

$$= 4(4 \times 355 + 3 \times 500 + 2 \times 100) \times 10^{-3} \times 0.395 \ kg$$

$$= 4.93 \ kg$$

Mass of vertical stirrups

$$\geq 8[2(370+114)+20 \times 6] \times 10^{-3} \times 0.395 \ kg$$

$$= 3.44 \ kg$$

Hence the total mass provided

$$= 8.37 \ kg$$
Table 6.4 shows the various design features for shear reinforcement.

**TABLE 6.4 COMPARISON OF SHEAR REINFORCEMENT REQUIRED**

<table>
<thead>
<tr>
<th>S.No</th>
<th>DESIGN METHOD</th>
<th>SHEAR REINFORCEMENT</th>
<th>MASS REQUIRED, kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ACI 318M Code</td>
<td>Vertical stirrups 21R8-100</td>
<td>8.91</td>
</tr>
<tr>
<td>2</td>
<td>BS 8110 Code</td>
<td>vertical stirrups 25R8-90</td>
<td>8.47</td>
</tr>
<tr>
<td>3</td>
<td>Proposed design</td>
<td>45 deg wave reinforcement</td>
<td>8.37</td>
</tr>
<tr>
<td></td>
<td>recommendations</td>
<td>4R8-355-500</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>and</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>vertical stirrups 8R8-355</td>
<td></td>
</tr>
</tbody>
</table>

Based on the above calculations it can be worked out that the proposed 45 deg wave reinforcement combined with vertical stirrups requires:
(a) 93.9% of the amount of shear reinforcement required by the ACI method.

(b) 98.8% of the amount of shear reinforcement required by the BS method.

Thus the proposed type of shear reinforcement is relatively more economical, from the point-of-view of material usage, than the vertical stirrups. Additional example given in Appendix A also confirms the above statement.

6.2.2 Cost Advantages

The costs associated (direct and indirect) can be calculated for case I and case II as shown in Table 6.5.
### TABLE 6.5 ASSOCIATED COSTS

<table>
<thead>
<tr>
<th>ITEMS</th>
<th>COSTS FOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case I</td>
</tr>
<tr>
<td>(a) material for shear reinforcement</td>
<td>$C_1$</td>
</tr>
<tr>
<td>(b) labour for bending of shear reinforcement, their placing and fixing</td>
<td>$C_2$</td>
</tr>
<tr>
<td>(c) cost associated with vibration of concrete</td>
<td>$C_3$</td>
</tr>
<tr>
<td>(d) others</td>
<td>$C_4$</td>
</tr>
</tbody>
</table>

Note: 1. Value of $K_1$ is less than 1.0 (refer Sec. 6.2.1)  
2. Values of $K_2$ and $K_3$ are less than 1.0, though they are too general to be identified specifically (refer Sec. 6.1.3)

Since the beams in case I and II serve exactly the same purpose, the benefits ($B$) may be assumed to be same for both cases.

The benefit-cost ratio ($B/C$) can thus be calculated as follows:

1. Case I: $(B/C)_I = B/(C_1 + C_2 + C_3 + C_4)$
(2) Case II: \((B/C)_{\text{II}} = B/(K_1C_1 + K_2C_2 + K_3C_3 + C_4)\)

It can be seen that \((B/C)_{\text{I}} < (B/C)_{\text{II}}\). Also, the net difference between benefits and costs for Case II is more than that of Case I. Hence it may be stated that 45 deg wave reinforcement combined with vertical stirrups is economically justifiable.

Based on Sec. 6.1 and 6.2, it may be concluded that the 45 deg wave reinforcement combined with vertical stirrups is more efficient than vertical stirrups alone from the point-of-view of functional and economic efficiency and hence it is the 'best' choice.
CHAPTER 7 PROPOSED DESIGN RECOMMENDATIONS FOR SHEAR USING WAVE REINFORCEMENT

In this chapter specific design recommendations are proposed for the shear resistance of non-prestressed concrete beams with wave reinforcement subjected to flexure and shear. Existing code specifications [3] and [4] adopted for the design of wave reinforcement are also used accordingly.

7.1 SHEAR STRENGTH PROVIDED BY CONCRETE (V_c)

7.1.1 - The shear strength V_c should be computed by the provisions made in the approved design codes. In the absence of a more detailed calculation, the following equation may be considered adequate:

\[ V_c = (\sqrt{f'_c}/6)bd \quad \text{--- (SI units)} \]  \hspace{1cm} (7.1)

or

\[ V_c = \left[ (\sqrt{f'_c} + 120 \rho_w \frac{V_u}{M_u}) : 7 \right] bd \]  \hspace{1cm} (7.2)

but not greater than \( 0.3\sqrt{f'_c} \) bd and \( \frac{V_u}{M_u} \) should
not be greater than 1.0 where $M_u$ is factored moment occurring simultaneously with factored shear force $V_u$ at section considered and $\rho_w = A_s/bd$.

7.2 SHEAR STRENGTH PROVIDED BY WAVE REINFORCEMENT ($V_s$)

7.2.1 Wave Reinforcement

7.2.1.1 Shear reinforcement in the form of wave reinforcement should be placed perpendicular to axis of member in combination with vertical stirrups, placed at sections where the vertical legs of the wave reinforcement exist preferably of same diameter bars.

7.2.1.2 The inclined leg of the wave reinforcement should make an angle of 45 deg with longitudinal tension reinforcement. Any angle more than 45 deg may be considered, if verified suitable.

7.2.1.3 The wave reinforcement should end with a vertical leg and a standard hook and this should be continued at least a distance $d$, past the point where they theoretically are no longer required to restrain
inclined cracks starting at that point.

7.2.1.4 End anchorages in the form of hooks or bends should comply to specific code requirements.

7.2.1.5 Design yield stress \( f_{yw} \), of shear reinforcement should not exceed 400 N/mm\(^2\) or yield stress of wave reinforcement or yield stress of vertical stirrups whichever is the lesser.

7.2.2 Design of Wave Reinforcement

7.2.2.1 For \( v < \frac{v_c}{2} \), a minimum of two wave reinforcement combined with vertical stirrups in accordance with Section 7.2.1.1, should be provided in all beams of structural importance.

7.2.2.2 For \( \frac{v_c}{2} < v < (v_c + 0.4) \) N/mm\(^2\), a minimum of two wave reinforcement combined with vertical stirrups in accordance with Section 7.2.1.1, should be provided for the whole length of beam to serve as a minimum reinforcement. In such cases,

\[
\begin{align*}
\nu_{sw} &= 0.4 \text{ N/mm}^2 \\
A_{we} &= 0.4 \frac{bs_w}{f_{yw}}
\end{align*}
\]
The value of $A_{we}$, the equivalent area of minimum shear reinforcement (refer Eq. 3.10a) assuming same diameter for the 45 deg wave reinforcement and vertical stirrups, is given by,

$$A_{we} = 6.828 A_{ss}$$

(7.4)

where $A_{ss}$ is the cross-sectional area of each bar of shear reinforcement required at the section.

7.2.2.3(a) For $(v_c + 0.4) < v < 5 \text{ N/mm}^2$, a maximum number of wave reinforcement equal to the number of longitudinal tensile steel should be used.

(b) The shear strength $V_s$ is calculated from (Eq. 3.7),

$$V_s = [A_{sv} f_{yv} \cot \theta + A_{sw} f_{yw} \cot \theta + A_{sw} f_{yw} \cos \alpha + \sin \alpha \cot \theta] \frac{d - d'}{s_w}$$

(7.5)

(c) (i) In lieu of more exact analysis, assuming $\theta = 45$ deg, for a 45 deg wave reinforcement combined with two-legged vertical stirrups, from Eq. 3.8,

$$V_s = A_{sv} f_{yv} + 2.414 A_{sw} f_{yw}$$

(7.6)
(ii) Again, if same diameter shear steel is used, \( A_{ss} = A_{sv}/2 = A_{sw} \) and \( f_{ys} = f_{yv} = f_{yw} \), which leads to (Eq. 3.8a),
\[
V_s = 4.414 A_{ss} f_{ys} \quad (7.7)
\]
where \( A_{ss} \) is the cross-sectional area of each bar of shear reinforcement and \( f_{ys} \) is the characteristic strength of shear reinforcement used.

(iii) In a general form, Eq. 7.7 can be stated as
\[
V_s = A_{we} f_{ys} \quad (7.8)
\]
The equivalent area of shear reinforcement \( A_{we} \) is then given by (Eq. 3.10),
\[
A_{we} = A_{ss} (2N + 2.414 n) \quad (7.9)
\]
where \( N \) is the number of two-legged vertical stirrups and \( n \) the number of wave reinforcement, at the section considered.

7.2.2.4 Additional shear requirement, if necessary after satisfying the requirements of Section 7.2.2.3(a),
may be provided in the form of additional vertical stirrups with a spacing not exceeding $s_{w}/2$; or the wave reinforcement may be staggered (Fig. 7.1) midway between the vertical stirrups.

![Diagram of wave reinforcement staggered midway between vertical legs]

Fig. 7.1 WAVE REINFORCEMENT STAGGERED MIDWAY BETWEEN VERTICAL LEGS
CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

Based on the results of experimental and analytical study it may be concluded that the proposed wave reinforcement is an acceptable type of shear reinforcement. Again, for the 45 deg wave reinforcement combined with vertical stirrups at sections where the vertical legs of wave reinforcement exist, the following conclusions may be made:

(a) It is comparable to the provision of vertical stirrups alone as shear reinforcement in ensuring that the ultimate strengths are governed by flexure rather than by shear.

(b) It is a better alternate type of shear reinforcement than only the vertical stirrups from the point-of-view of controlling the number and widths of shear cracks.
(c) It is more efficient functionally and economically, than the vertical stirrups provided alone as shear reinforcement.

(d) The 45 deg truss analogy can be used to compute the shear strength provided by wave reinforcement.

(e) The compression field theory predicts the response characteristics of a beam reasonably well.

Finally, it is the hope of the author that the contents of this report will help the designer in assessing the acceptability of 45 deg wave reinforcement combined with vertical stirrups, as explained earlier, to non-prestressed rectangular concrete beams.
8.2 RECOMMENDATIONS

Though the present study has resolved the acceptability and efficiency of wave reinforcement, the test results would not be considered as conclusive since no duplication tests were made. Also, the idea of efficiency to the totality of a situation requires a knowledge of all the facts and involving not only 45 deg wave reinforcement but with varying angles as well.

Again further research work should be conducted to verify the effects of wave reinforcement to:

(a) prestressed concrete members

(b) resist torsion

(c) fluctuating loads

(d) reversal of loadings
(e) include all reinforced concrete structures such as arches and shells.

It is therefore recommended that a sufficient number of specimens incorporating all possible situations should be tested to permit statistical interpretation of the test results and to formulate comprehensive design requirements using wave reinforcement.
REFERENCES


4. "Building Code Requirements for Reinforced Concrete", ACI 318M-83 (metric version), American Concrete Institute, Detroit, 1985.

No. ST4, April 1978, pp. 649-666, Discussion
ASCE, ST7, July 1979, pp. 1567-1570.


The use of proposed design recommendations (Chapter 7) is illustrated by means of a design example and material used for the proposed shear reinforcement is also compared with the B.S. code design [3] involving only vertical stirrups.

It is required to design shear reinforcement for the left-end span of a three-span continuous beam. The beam has a uniform rectangular section of breadth $b = 300$ mm and an effective depth of 740 mm. The material properties are:

- $f'_c = 25$ N/mm$^2$
- $f_{yw} = f_{ys} = 280$ N/mm$^2$

The shear force envelope is as shown in Fig. A1. Suitable assumptions are also made.

![Shear Force Envelope](image)
Using Section 7.1.1, the following are worked out:

The shear strength provided by concrete \((V_c)\),

\[
V_c = \left(\sqrt{f'_c}/6\right)bd
\]

\[
= \left(\sqrt{25}/6\right) (300 \times 740) \text{ N}
\]

\[
= 185 \times 10^3 \text{ N}
\]

The design concrete shear stress \((v_c)\),

\[
v_c = 0.833 \text{ N/mm}^2
\]

The shear stress diagram is shown in Fig. A2.

Wave reinforcement to be provided should be in the form of:
45 deg wave reinforcement combined with vertical stirrups (refer Section 7.2.1.1 and 7.2.1.2).

Referring to Fig. A2, the following may be stated:

(a) For the inner portions 2.18 m and 2.14 m combining Sections 7.2.2.1 and 7.2.2.2, a minimum of two 45 deg wave reinforcement combined with vertical stirrups should be provided.

Assuming same diameter bars, using Eq. 7.3 and 7.4, namely

\[ A_{we} \geq 0.4b_s w/f_{yw} \]

\[ A_{we} = 6.828 A_{ss} \]

the minimum shear reinforcement can be designed.

\[ 6.828 A_{ss} = 0.4 \times 300 \times 740/280 \]

\[ A_{ss} = 46.45 \text{ mm}^2 \]

Choose 8 mm bars

(area provided 50.3 mm²)
Assuming a clear cover of 30 mm and top longitudinal steel of Y16,

\[ d-d' = 740 - (30+8+8) \text{ mm} \]

\[ = 694 \text{ mm} \]

\[ = 695 \text{ mm (multiples of 5 mm)} \]

Use two 45 deg wave reinforcement

\[ 2R8 - 695 - 980 \]

and vertical stirrups

\[ R8-695 \]

(b) For the regions 1.04 m from the left support and 2.64 m from the right support (shown hatched), \((v_c+0.4)<v<5 \text{ N/mm}^2\) and hence design shear reinforcement using Section 7.2.2.3.

(i) Region marked 1.04 m (near left support):

\[ V_S = v_n - v_c \]

\[ = 1.824 - (0.833+0.4) \text{ N/mm}^2 \]

\[ = 0.591 \text{ N/mm}^2 \]

Hence the shear strength provided by wave reinforcement combined with vertical stirrups \((V_S)\),

\[ V_S = 0.591 \times 300 \times 740 = 131.20 \times 10^3 \text{ N} \]
Assume same diameter bars for 45 deg wave reinforcement and vertical stirrups. From Eq. 7.7,

\[ V_s = A_{we} f_{ys} \]

\[ A_{we} = 131.20 \times 10^3 / 280 \text{ mm}^2 \]

\[ = 468.57 \text{ mm}^2 \]

From Eq. 7.9, \( A_{we} = A_{ss}(2N+2.414n) \)

Try three 45 deg wave reinforcement \((n=3)\)

two 2-legged vertical stirrups \((N=2)\).

\[ 468.57 = A_{ss}(2\times2+2.414\times3) \]

\[ A_{ss} = 41.68 \text{ mm}^2 \]

Choose 8 mm bars

(area provided 50.3 mm²)

Provide three 45 deg wave reinforcement

3R8 - 695 - 980

and

2 x 2-legged vertical stirrups

2 x 3R8-695

(ii) Region marked 2.64 m (near right support):

\[ V_s = V_n - V_c \]

\[ = 2.748 - (0.833+0.4) \text{ N/mm}^2 \]

\[ = 1.515 \text{ N/mm}^2 \]
Hence the shear strength provided by wave reinforcement combined with vertical stirrups \( (V_s) \),

\[
V_s = 1.515 \times 300 \times 740 \text{ N} \\
= 336.33 \times 10^3 \text{ N}
\]

\[
A_{we} = \frac{V_s}{f_{ys}} \\
= \frac{336.33 \times 10^3}{280} \text{ mm}^2 \\
= 1201.18 \text{ mm}^2
\]

Try three 45 deg wave reinforcement and two 2-legged vertical stirrups.

\[
1201.18 = A_{ss} (2 \times 2 + 2 \times 1.414 \times 3) \\
A_{ss} = 106.85 \text{ mm}^2
\]

Choose 12 mm bars

(area provided 113 mm\(^2\))

Provide three 45 deg wave reinforcement

3R12-695-980

and

2x2-legged vertical stirrups

2x5R12-695

The arrangement of shear reinforcement is shown in Fig. A3.
Using similar procedure shown in the example in Chapter 6, the following can be obtained:

mass of wave reinforcement $= 35.5 \text{ kg}$

mass of vertical stirrups $= 27.5 \text{ kg}$

Hence, the total mass of transverse steel required using the proposed shear reinforcement $= 35.5 + 27.5 \text{ kg}$

$= 63.0 \text{ kg}$
Using B.S Code [3], the following arrangement of transverse steel as shown in Fig. A4 can be obtained.

Fig. A4 ARRANGEMENT OF VERTICAL STIRRUPS

Hence, the total mass of vertical stirrups required is approximately equal to 80.0 kg.

Again, it can be seen that the proposed shear reinforcement requires only 79% of that required by the vertical stirrups alone.
Based on the compression field theory, Shitote [24] developed the necessary equations in order to predict the post-cracking response of a structural concrete beam for a combined loading case of flexure and shear. It was originally derived for the case of beams with vertical stirrups as transverse reinforcement. In the present study, a few of the equations are suitably modified for the case of wave reinforcement and shown in Sec. 3.4. But for easy reference all the equations used are provided below in a concise manner along with the procedure.

B.1 SOLUTION TECHNIQUE

Assuming that the beam cross-section, the stress-strain characteristics of concrete and steel are given, then for a specified extreme compression fibre strain, $\varepsilon_{ct}$, and a moment to shear ratio of loading, $a_v$, the following solution technique is proposed.
A: Pure Flexure case

**STEP 1:** To compute rectangular stress block factors, using parabolic stress-strain relationship for the doubly-reinforced section and $y_p$, $M_p$, $\phi_p$ for pure flexure case.

Choose top face strain, $\varepsilon_{ct}$ and obtain $\Omega_{ct}$

Compute stress block factors $\alpha$, $\beta$ and $\alpha\beta$

Establish strain profile based on depth of NA, $y$

Note: for pure flexure case $y=y_p$ and $\theta=\theta_p$

$$\Omega_{ct} = \frac{\varepsilon_{ct}}{\varepsilon_{co}}$$

$$\alpha = 6(1-\Omega_{ct}/3)^2/(4/\Omega_{ct}-1)$$

$$\beta = (4-\Omega_{ct})/(6-2\Omega_{ct})$$

$$\alpha\beta = \Omega_{ct}-\Omega_{ct}^2/3$$

$$\varepsilon' = (y-d')\varepsilon_{ct}/y$$

$$\theta = \varepsilon_{ct}/y$$

$$\varepsilon_l = (d-y)\varepsilon_{ct}/y$$

STRAIN PROFILE
Compute depth of neutral axis, \( y_p \), for pure flexure case using equilibrium of forces conditions.

*Note:* \( f'_{cb} = E'_{cb} \)

\( f'_c = E'_c \)

Either (a) all steel in elastic range

\[ \alpha_{fp} f'_{by} + \alpha'_{fp} f'_c = A_s f'_c \]

or (b) only tension steel yielding

\[ \alpha_{fp} f'_{by} + \alpha'_{fp} f'_c = A_s f'_{by} \]

or (c) only compression steel yielding

\[ \alpha_{fp} f'_{by} + \alpha'_{fp} f'_c = A_s f'_{cy} \]

or (d) all steel yielding

\[ \alpha_{fp} f'_{by} + \alpha'_{fp} f'_c = A_s f'_{cy} \]

Compute moment of resistance, \( M_p \), curvature, \( \phi_p \), for the above pure flexure case.

\[ M_p = \alpha_{fp} f'_{by} (d - \beta y_p / 2) + A_s f'_{cy} (d - d') \]

Equivalent Stresses

To STEP 2
B: Combined Flexure and Shear Case

**STEP 2:**

(a) Assume \( y_n = y_p \) and establish strain profile using,

\[
\epsilon_n = \frac{(y_n-d')c_t}{y_n} \quad \text{and} \quad \epsilon_y = \frac{(d-y_n)c_t}{y_n}
\]

(b) Compute the magnitude and the position of resultant longitudinal forces using the pure flexure moment \( M_p \).

\[
\begin{align*}
C_s & \quad \text{compression force in top steel} \\
F_{Cf} & \quad \text{Resultant concrete compression force in the longitudinal section} \\
T & \quad \text{Tension force in bottom longitudinal steel} \\
F_{Sf} & \quad \text{Resultant tension force in all the longitudinal steel}
\end{align*}
\]

\[
\begin{align*}
\text{If } \epsilon_n &< 0, \quad f_n = -\epsilon_n E_s \\
\text{If } \epsilon_n &= \epsilon_y, \quad f_n = \epsilon_n E_s \\
\text{If } \epsilon_n &> \epsilon_y, \quad f_n = f_{ky} \\
\text{If } \epsilon_y &< \epsilon_y, \quad f_y = \epsilon_y E_s \\
\text{If } \epsilon_y &= \epsilon_{ky}, \quad f_y = f_{ky} \quad T - C_s = A_s f_n - A_{s} f_{l} \\
\text{If } \epsilon_y &> \epsilon_{ky}, \quad f_y = f_{ky} \\
Y_{Sl} &= \frac{(A_s f_n d - A_{s} f_{l} d')}{F_{Sf}}
\end{align*}
\]
(F_{c1} and y_{c1} are calculated in Step 4)

The internal moment lever arm,

\[ j_d = \frac{M_p}{T} \]

(c) Obtain shear flow distribution

<table>
<thead>
<tr>
<th>Levels</th>
<th>Shear flow</th>
<th>Longitudinal strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of beam</td>
<td>( q_t = 0 )</td>
<td>( \varepsilon_{ct} )</td>
</tr>
<tr>
<td>( y_n )</td>
<td>( q_4 = 0.75 q_n )</td>
<td>( \frac{1}{4} \varepsilon_{ct} )</td>
</tr>
<tr>
<td>3</td>
<td>( q_3 = q_n )</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( q_2 = \frac{1}{4}(q_b + 3q_n) )</td>
<td>( \varepsilon_{k} )</td>
</tr>
<tr>
<td>1</td>
<td>( q_1 = q_b )</td>
<td>( \varepsilon_{k} )</td>
</tr>
</tbody>
</table>

Note: \( q_b = \frac{V}{j_d} \)

\[ q_n = 1.5(V/d)(1-(d-y_n)/3j_d) \]

**STEP 3:** Compute compatible strains at bottom steel level (longitudinal strain = \( \varepsilon_{k} \))

Let \( c = \frac{A_{we} f_{yw}}{(s_w q_b)} \)

\[ \Omega_{wy} = \varepsilon_{wy} / \varepsilon_{co} \quad \text{and} \quad \Omega_{k} = \varepsilon_{k} / \varepsilon_{co} \]

\[ \kappa_1 = \left( \frac{\Omega_{k}}{\Omega_{wy}} \cdot \frac{1}{c^2} \right) \quad \text{and} \quad \kappa_2 = 1/c \]

If \( \kappa_1 < 1.0 \) and \( \kappa_2 < 1.0 \), GO TO Step 3(b)

(a) Assume \( f_{c}^{*} = f_{c}^{'} \)

\[ f_{cp} = q_b (1+c^2)/bc \]
\[ \Omega_{cp} = \frac{\varepsilon_{cp}}{\varepsilon_{co}} = 1 + \sqrt{1 - f_{cp}/f_c} \]

(+ve when \( \varepsilon_{ct} > \varepsilon_{co} \))

\[ \varepsilon_w = \frac{(\varepsilon_{cp}(1-c^2) + \varepsilon_c)}{c^2} \]

If \( \varepsilon_w < \varepsilon_{wy} \) GO TO step 3(b); otherwise

\[ \Omega_{ct} = \frac{\varepsilon_t}{\varepsilon_{co}} = \frac{\varepsilon_c + \varepsilon_{cp} + \varepsilon_w}{\varepsilon_{co}} \]

Re-evaluate \( f^* = \frac{f_c'}{0.80 + 0.34\Omega_t} \)

Repeat step 3(a) until convergence in \( f^* \) is obtained

Compute strains \( \varepsilon_w, \varepsilon_t, \varepsilon_{cp} \) and angle \( \theta_b \) from

\[ \tan \theta_b = c \]

(b) Assume \( \eta = (\eta_1 + \eta_2)/2 \)

\[ \Omega_{cp} = \frac{\eta_3c^2\Omega_{wy} - \Omega_c}{(1 - \eta^2c^2)} \]

\[ f(\eta) = \Omega_{cp}z - \frac{1}{3} \Omega_{cp}^2 z - k \]

where \( z = \frac{\eta c}{(1 + \eta^2c^2)} \)

\[ k = q_b/f_{cb} \]

(assume \( f^*_c = f_c^* \))

\[ f'(\eta) = \frac{d\Omega_{cp}}{d\eta} z + \frac{d\Omega_{cp}}{d\eta} \frac{dz}{d\eta} - \frac{1}{3} (2\Omega_{cp}) \frac{d\Omega_{cp}}{d\eta} z - \Omega_{cp}^2 \frac{dz}{d\eta} \]

where \( \frac{d\Omega_{cp}}{d\eta} = \eta^2c^2(3\Omega_{wy} + 2\Omega_{cp}/\eta)/(1 - \eta^2c^2) \)

and \( \frac{dz}{d\eta} = c(1 - \eta^2c^2)/(1 + \eta^2c^2)^2 \)

If \( f(\eta) \) is not close enough to zero,

use the Newton-Raphson iteration method,

\[ \eta_{n+1} = \eta_n - f(\eta_n)/f'(\eta_n) \]
where $u_{n+1}$ is a better approximation to the root of $f(u) = 0$

- Re-evaluate $f^*$ (refer Step 3a)
  - until convergence in $f^*$ is achieved.
- Compute strains $\varepsilon_w$, $\varepsilon_f$, $\varepsilon_{cp}$ and $\theta_B$ from $\tan \theta_B = \eta_C$

(c) Repeat Step 3 to compute compatible strains and also angle of principal compression at all other levels by altering the longitudinal strains as shown in Step 2(c).

**STEP 4:** Compute the longitudinal compression stresses in concrete and obtain $F_{Ct}$ and $y_{Ct}$

<table>
<thead>
<tr>
<th>Levels</th>
<th>Longitudinal compression stresses in concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of beam</td>
<td>$Q_{ct} = bf'<em>c (2\Omega</em>{ct} - \Omega_{ct}^2)$</td>
</tr>
<tr>
<td></td>
<td>$Q_{c4} = q_4 \cot \theta_4$</td>
</tr>
<tr>
<td></td>
<td>$Q_{c3} = q_3 \cot \theta_3$</td>
</tr>
<tr>
<td></td>
<td>$Q_{c2} = q_2 \cot \theta_2$</td>
</tr>
<tr>
<td></td>
<td>$Q_{c1} = q_1 \cot \theta_1$</td>
</tr>
</tbody>
</table>

- $F_{Ct} = (Q_{ct} + 4Q_{c4} + Q_{c3}) y_n / 6 + (Q_{c3} + 4Q_{c2} + Q_1) (d - y_n) / 6$
- $y_{Ct} = [(2Q_{ct}y_n + Q_{c3}y_n) (y_n / 6)]$
  $+(Q_{c3}y_n + 2Q_{c2} (y_n + d) + Q_{c1}d) (d - y_n) / 6$
Equilibrium in the longitudinal direction requires that the magnitudes of $F_{Cl}$ and $F_{Sl}$ (Step 2b) be equal and that these internal resultant forces constitute a couple which equilibrates the applied moment, $M$, so that:

$$F_{Sl} = F_{Cl}$$

$$M = F_{Sl}(y_{Sl} - y_{Cl})$$

If $F_{Cl} \neq F_{Sl}$, revise $y_n$ and repeat Steps 2, 3 and 4 till convergence is obtained. Use the latest evaluated value of $j_d (=M/T)$ for each new iteration.

**STEP 5:**

A new extreme compression fibre strain $\varepsilon_{ct}$ can then be selected and the steps 2, 3 and 4 are repeated.

Obtain the results $M$, $y_n$, $\phi$ for the section and also $\theta$, $q$, $\varepsilon_x$, $\varepsilon_\omega$, $\varepsilon_t$, $\varepsilon_{cp}$, $\gamma_{lt}$ and $\gamma_m$ for various levels considered.
B.2 SAMPLE CALCULATION

Some of the equations shown in B.1 are utilized hereunder for a beam having the following cross-section and material properties:

\[ b = 150 \text{ mm} \]
\[ h = 400 \text{ mm} \]
\[ d = 371.5 \text{ mm} \]
\[ d' = 20 \text{ mm} \]

\[ A_s = 1470 \text{ mm}^2 \]
\[ A'_s = 101 \text{ mm}^2 \]
\[ A_{we} = 344 \text{ mm}^2 \]
\[ s_w = 371.5 \text{ mm} \]
\[ f_{ly} = 505 \text{ N/mm}^2 \]
\[ f_{ly} = 455 \text{ N/mm}^2 \]
\[ f_{yw} = 455 \text{ N/mm}^2 \]
\[ f_c = 31.5 \text{ N/mm}^2 \]
\[ \varepsilon_{co} = 2.1 \times 10^{-3} \]
\[ E_S = 225 \times 10^3 \text{ N/mm}^2 \]
\[ E'_S = 194 \times 10^3 \text{ N/mm}^2 \]
\[ E_w = 194 \times 10^3 \text{ N/mm}^2 \]
\[ E_C = 20.5 \times 10^3 \text{ N/mm}^2 \]

Consider a case loading with top face strain \( \varepsilon_{ct} = 0.0005 \).

**Step 1**

Pure flexure case:

\[ \Omega_{ct} = \varepsilon_{ct}/\varepsilon_{co} = 0.2381 \]
\[ \alpha = 0.2192 \]
\[ \beta = 0.6810 \]
Assume all steel in elastic range.

\[ Y_p = 173.618 \text{ mm} \]

check:

\[ \epsilon_y = 0.5699 \times 10^{-3} < \epsilon_y \]
\[ \epsilon_y = 0.4424 \times 10^{-3} < \epsilon_y \]

check correct.

Hence the moment of resistance for a pure flexure case, \( M_p \) is calculated as,

\[ M_p = 59.219 \times 10^6 \text{ N mm} \]

**Step 2** Combined flexure and shear case:

Guess \( Y_n = Y_p \)

\[ = 173.618 \text{ mm} \]
\[ \epsilon_y = 0.5699 \times 10^{-3} \]
\[ \epsilon_y = 0.4424 \times 10^{-3} \]

The magnitude and the position of resultant longitudinal forces are calculated as follows:

\[ T = 188.49 \text{ kN} \]
\[ c_s = 8.668 \text{ kN} \]
\[ F_{sl} = T - c_s \]
\[ = 179.82 \text{ kN} \]
\[ Y_{sl} = 196.62 \text{ mm} \]
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Assuming $a_v = 1000 \text{ mm}$,

$V = 59.219 \times 10^3 \text{ N}$

$jd = 314.176 \text{ mm}$

The shear flows can be obtained as follows:

$q_b = 188.49 \text{ N/mm}$

$q_n = 188.91 \text{ N/mm}$

**Step 3:** To compute compatible strains and principal angles of compression, etc. at bottom steel level:

$c = 2.235$

$\omega_{wy} = 1.1168$

$\omega_s = 0.2714$

$\lambda_1 = 0.3651 < 1.0$

$\lambda_2 = 0.4474 < 1.0$

Therefore the following can be calculated:

$n = 0.4063$

$\Omega_{cp} = 0.5860$

Assuming $f_c^* = f_c'$, $f(n) = 0.1947$

$f'(n) = 8.9451$

Using the iteration formula,

$n_{n+1} = 0.3845$

and hence $f(n) = 0.0415$
The step 3 is now repeated until convergence in \( f(n) = 0 \) is obtained and then, \( n = 0.3775 \) (approximately). Therefore, \( \tan \theta_b = n \) leading to \( \theta_b = 40.11^\circ \).

Also the following are calculated:

\[
\begin{align*}
\varepsilon_W &= 0.8871 \times 10^{-3} \\
\varepsilon_{cp} &= 0.2092 \times 10^{-3} \\
\varepsilon_i &= 0.5699 \times 10^{-3} \\
\Omega_t &= 0.7934 \times 10^{-3}
\end{align*}
\]

Re-evaluate \( f^*_c \) as \( f^*_c = 29.4457 \text{ N/mm}^2 \) and step 3 is again repeated until convergence in \( f^*_c \) is obtained.

**Step 4:** Evaluation of shear flow for the levels considered and \( F_{ck} \) and \( y_{ck} \) and also the convergence in \( F_{ck} = F_{sk} \) using the new value of \( j_d \) for each new iteration, is carried out and results obtained. This warrants for convenience—a computer-aided design.

The solution technique outlined above has been programmed to enable a computer-aided design in predicting the behavioural response of rectangular reinforced concrete beams loaded in flexure and shear. The steps of the program, the full listing and output of results are given in Appendix C.
A computer program for the analysis of solid rectangular reinforced concrete beams under combined flexure and shear based on the compression field theory is outlined below. The equations used are already provided in Appendix B. For an assumed top face strain, the sequence of operations followed in the program (for the known geometric and material properties of a beam) are:

1. Input beam data
2. Input the analysis control parameters
3. Function to compute pure moment
4. Work out stress block factors
5. Compute depth of neutral axis, moment and curvature for pure moment case
6. Assume depth to neutral axis for combined flexure and shear case and compute the forces and shear flow
7. Compute strains, principal angle of compression at chosen points
8. Repeat steps (6) and (7) to obtain convergence
9. Output results
C.1 BEAM INPUT DATA

It includes the section and material properties of the beam (the symbols used in the program listing are shown on the left and the corresponding symbols as explained in 'NOTATION' are shown on the right).

(a) Section geometry:
- B
- b
- D
- d
- D1
- d'

(b) Material properties:
- As
- A_s
- A1
- A'_s
- AW
- A_we
- SW
- s_w
- Fl
- f'_ly
- F2
- f'_y
- FW
- f_yw
- ES
- E_s
- El
- E'_s
- EC
- E_c
- YW
- c_wy
- EO
- c_co
(b) Cont.d

FC $f'_c$

EW $\epsilon_v$

(c) Control parameters:

$AV$ shear span $a_v$

$ET$ top fibre compression strain, $\epsilon_{ct}$

C.2 NOTATION USED IN PRINTING RESULTS

Levels considered:

1 - level of bottom steel
2 - midway between levels 1 and 3
3 - level of neutral axis
4 - midway between level 3 and top of beam

THETA principal angle of compression in deg

SHEAR FLOW shear flow

ELC average longitudinal strain

EW average transverse strain

ECP principal compressive strain in concrete

FCP principal compressive stress

GAML T shear strain

GAM-M maximum shear strain
The program listing and the printout of output are given in C.3 and C.4 respectively.
C.3 PROGRAM LISTING AND INPUT DATA FOR BEAMS

BWR-2 AND BWR-1

10 LPRINT TAB(20)"TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR"
20 LPRINT TAB(19)"CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION"
30 LPRINT TAB(29)"BY COMPRESSION FIELD THEORY"
40 LPRINT : LPRINT
50 LPRINT TAB(17)"SOURCE : P. GANAPATHI, UNIVERSITY OF NAIROBI (1988"
60 LPRINT : LPRINT
70 LPRINT TAB(21)"BEAM : BWR-2"

200 LET B=150
210 LET D=371.5
220 LET D1=20
230 LET A1=1470
240 LET AW=101
250 LET SW=299
260 LET SW=371.5
270 LET F1=455
280 LET F2=505
290 LET FW=455
300 LET ES=225000!
310 LET E1=194000!
320 LET EC=205000
325 LET YW=.00224
330 LET EO=.0021
340 LET FC=31.6
350 LET AV=1000
16 LPRINT TAE(19)"TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR:
20 LPRINT TAE(19)"CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION"
30 LPRINT TAE(20)"EI COMPRESSION FIELD THEORY"
40 LPRINT :LPRINT
50 LPRINT TAE(17)"SOURCE: P. GANAPATHI, UNIVERSITY OF NAIROBI (1984)"
60 LPRINT :LPRINT
70 LPRINT TAE(21)"BEAM I"BWF=1" 
200 LET E=150
210 LET D=371.5
220 LET E1=20
230 LET A5=1470
240 LET A1=101
250 LET Aw=243
260 LET Sw=389.5
270 LET F1=455
280 LET F2=505
290 LET Fm=455
300 LET ES=225000
310 LET E1=194000
320 LET EC=28600
325 LET YM=.00235
330 LET ED=.0022
340 LET FC=.325
350 LET AV=1000
360 DIM El(4),Es(4),C(4),CS(4),L1(4),L2(4)
370 DIM TH(4),F1(4),FF(4),EF(4),Em(4),DG(4)
380 DIM DF(4),DT(4),F2(4),A(4),Z1(4)
390 DIM ZG(4),Z4(4),FT(4),EZ(4),EE(4),Z5(4)
395 DIM F2(4),E4(4),Em(4),EC(4)
397 DIM F3(4),FA(4),FS(4)
400 FOR ET=.0005 TO .0005 STEP .00025
405 Z1=1
410 DM=E1/EO
420 BE=(4-CM)/(6-2*OM)
430 AE=DM/5*OM-.5
440 DE=AE*FCFG
450 REM TOP & BOTTOM STEEL YIELDING
460 Y2=F2/EE
470 Y1=F1/E1
480 YF=(AS*F2-F1*F1)/EE
490 EOSUB 20000
500 IF S1=x1 AND S2=x2 THEN 510 ELSE 550
510 CS=AI*F1
520 CC=BE*FCF*YFB
530 M=CS*(0.1)+(C-EM*FC5)*CC
540 PF=ET/YP :GOTO 795
550 REM BOTTOM STEEL YIELDING
560 XI=AI*E1*EI
570 X2=(0.1-AS*F2)/E
580 YF=-X2+(X2*2*(4*11111/EE))-.5)1.5
590 EOSUB 20000
600 IF S1+Y1 AND S2+Y2 THEN 610 ELSE 650
610 CS=AI*F
620 CC=AIFIYF
630 M=(CS*(D-D1))+(CC*(D-2*YFYF.5))
640 FH=ET/YF :GOTO 795
650 REM NONE OF THE STEEL YIELDING!
660 X3=(AI*FET+AI2*FET)/2
670 YF=-13*(X2+2*4444).5+.5
680 GOSUB 20000
700 IF (S1-Y1)X AND S2-Y2) Then GOTO 750 ELSE 710
710 CS=AI*F
720 CC=AIFIYF
730 M=(CS*(D-D1))+(CC*(D-2*YFYF.5))
740 FH=ET/YF :GOTO 795
745 REM TCF STEEL YIELDING!
750 ES=AI*FET
755 X6=(15+AI*F1)/2
760 YF=-13*(X6+2*(44444+D))/2.5+.5
765 GOSUB 20000
770 IF S1+Y1 AND S2+Y2 THEN 775 ELSE 750
775 CS=AI*F
780 CC=AIFIYF
785 M=(CS*(D-D1))+(CC*(D-2*YFYF.5))
790 FH=ET/YF
795 PRINT ET,M,FH,YF
796 LFPRINT TAB(25)'TOP FACE STRAIN*TAЕ(48)=**TAЕ(50)ET
797 LFPRINT TAB(25)'PURE MOMENT*TAЕ(48)=**TAЕ(50)M
798 LFPRINT TAB(25)'DEPTH OF NA BELOW TOP*TAЕ(48)=**TAЕ(50)YF
799 LFPRINT TAB(25)'PURE CURVATURE*TAЕ(48)=**TAЕ(50)FH
800 YN=YF
810 S1=(1-D1)/YN*ET
820 S2=(G/YN-D1)*ET
830 IF S1<0 GOTO 660
840 IF S1=Y1 GOTO 870
850 IF S1+Y1 GOTO 660
860 CS=AI*F1 GOTO 890
870 CS=AI*F1 :GOTO 890
880 CS=AI*F1
890 IF S2+Y2 GOTO 910
900 IF S2+Y2 GOTO 920
910 T2=ASIF2:GOTO 930
920 T2=ASIF2
930 FL=T2-C5
940 YL=(T2*GCS1*O)/FL
950 JB=M/T2
960 V=MYAV
970 EQ=V/JE
980 GM=1.504/[(1-1D-YMI/13)&(1)]
990 TM=AM/SW
995 EZ=FW/YW
1000 DT=FW/YW/ED
1150 EL (1)=52
1160 Q (1)=0
1170 EL (2)=52/2
1180 Q (2)=(1Q*34QN)/4
1190 EL (3)=0
1200 Q (3)=0
1210 EL (4)=EL/2
1220 Q (4)=.75*Q2
1300 FOR I=1 TO 4
1340 C (1)=TWFW/G(1)
1250 ES(I)=EL(1)/EO
1260 L1(I)=[(ES(I)/4+1XI(1)^2)]^1/3
1270 LS(I)=1/C(I)
1280 IF L1(I)<1 AND LS(I)<1 THEN 1500
1290 TH(I)=ATN(I)
1279 LET F5(I)=FC
1300 IF F5(I)<0 THEN 1520 ELSE 1310
1310 CF(1)=1+(1-FF(1)/ES(1))^5 :EDTO 1330
1320 DF(1)=1-(1-FF(1)/ES(1))^5
1330 EF(1)=OF(1)=EO
1340 EM(I)=(EP(I))<1+(C(I)^2)-EL(1)/C(I)^2
1350 YW=FW/EZ
1360 IF EM(I)<YW THEN 1500
1370 DT(I)=EL(1)+EP(I)/EO
1375 IF F4(I)>0 THEN 1410
1380 FI(I)=FC/LE,+34UTG(1))
1385 F4(I)=F5(I)-F1(I)
1390 IF F4(I)<.5 THEN 1410 ELSE 1400
1395 IF F4(I)<0 THEN 1400
1400 F5(I)=F1(I) :EDTO 1300
1410 PRINT EM(I),EL(I),EP(I),TH(I)
1430 GOTO 1770
1500 FC(I)=FC
1510 KA(I)=Q(I)/[FC(I)*B]
1520 Z1(I)=(L1(I)+L2(I))/2
1530 Z3(I)=1-(Z1(I)+C(I))^2
1540 Z4(I)=1+(Z1(I)+C(I))^2
1550 DF(I)=(Z1(I)^3+C(I)^2)*DFT-05(I))/Z3(I)
1560 EZ(I)=(Z1(I)+C(I))/Z4(I)
1570 FT(I)=DF(I)*EZ(I)*[DP(I)-2EZ(I)]/3-A(I)
1580 DE(I)=C(I)/Z3(I)/Z4(I)^2
1590 DF(I)=(3DFT+2DF(I)/Z1(I))XZ(I)^2*X(I)^2/Z3(I)
1600 ZS(I)=(2*DF(I)*[EP(I)+EZ(I)+105(I)]*2EDE(I))/3
1610 F3(I)=(EP(I)*EZ(I)*DF(I)+EDE(I)-ZS(I)
1620 IF FT(I)<.1 THEN 1630 ELSE 1650
1630 Z1(I)=Z1(I)-FT(I)/F3(I)
1640 GOTO 1530
Towards an efficient shear reinforcement for concrete rectangular beams—response prediction by compression field theory

Source: P. Ganapathi, University of Nairobi (1989)

Beam: BWR-2

- Top Face Strain = 0.0005
- Pure Moment = 5.934791E+07
- Depth of NA below Top = 173.4329
- Pure Curvature = 2.88296E-06
- Top Face Strain = 0.00075
- Pure Moment = 8.617271E+07
- Depth of NA below Top = 176.0106
- Pure Curvature = 4.261107E-06
- Top Face Strain = 0.001
- Pure Moment = 1.110202E+08
- Depth of NA below Top = 178.7204
- Pure Curvature = 5.95332E-06
- Top Face Strain = 0.00129
- Pure Moment = 1.33826E+08
- Depth of NA below Top = 181.5751
- Pure Curvature = 6.884203E-06
C.4 PRINTOUT OF OUTPUT OF RESULTS:

(a) Beam BWR-2:
| TOP FACE STRAIN | = 0.0015 |
| PURE MOMENT   | = 1.545191E+08 |
| DEPTH OF NA BELOW TOP | = 184.5884 |
| PURE CURVATURE | = 8.126187E-06 |
| TOP FACE STRAIN | = 0.00175 |
| PURE MOMENT   | = 1.73022E+08 |
| DEPTH OF NA BELOW TOP | = 187.7762 |
| PURE CURVATURE | = 9.319604E-06 |
| TOP FACE STRAIN | = 0.002 |
| PURE MOMENT   | = 1.892491E+08 |
| DEPTH OF NA BELOW TOP | = 191.157 |
| PURE CURVATURE | = 1.04626E-03 |
| TOP FACE STRAIN | = 0.00225 |
| PURE MOMENT   | = 2.031057E+08 |
| DEPTH OF NA BELOW TOP | = 194.7526 |
| PURE CURVATURE | = 1.153312E-03 |
| TOP FACE STRAIN | = 0.0025 |
| PURE MOMENT   | = 2.14857E+08 |
| DEPTH OF NA BELOW TOP | = 198.5873 |
| PURE CURVATURE | = 1.238891E-05 |
TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR
CONCRETE RECTANGULAR BEAMS-RESPONSE PREDICTION
BY COMPRESSION FIELD THEORY


BEAM: BWP-2

| TOP FACE STRAIN | = | .0005 |
| PURE MOMENT | = | 5.934791e+07 |
| DEPTH OF NA BELOW TOP | = | 173.4329 |
| PURE CURVATURE | = | 2.88296e-06 |

COMBINED FLEXURE AND SHEAR

| MOMENT | = | 2.029701e+07 |
| DEPTH OF NA BELOW TOP | = | 187.3075 |
| CURVATURE | = | 4.006126e-06 |
| SHEAR | = | 20297.01 |

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### TOP FACE STRAIN

- Pure Moment = 8.617271E+07
- Depth of NA Below Top = 176.0106
- Pure Curvature = 4.261107E-06

### COMBINED FLEXURE AND SHEAR

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**Combined Flexure and Shear**

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<th>Curvature</th>
<th>Shear</th>
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**Top Face Strain**

- Pure Moment: 1.110202E+08
- Depth of NA Below Top: 178.7204
- Pure Curvature: 5.595332E-06
35.6056
285.759
.0005
1.251652E-03
2.910398E-04
7.209521
1.562262E-03
2.333732E-03
TOP FACE STRAIN = 0.00125
PURE MOMENT = 1.33826E+08
DEPTH OF NA BELOW TOP = 181.5751
PURE CURVATURE = 6.884203E-06

COMBINED FLEXURE AND SHEAR

MOMENT = 4.389097E+07
DEPTH OF NA BELOW TOP = 201.3484
CURVATURE = 9.928664E-06
SHEAR = 43890.97

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TOP FACE STRAIN = .0015
PURE MOMENT = 1.545191E+08
DEPTH OF NA BELOW TOP = 184.5884
PURE CURVATURE = 8.126187E-06

COMBINED FLEXURE AND SHEAR

MOMENT = 4.730981E+07
DEPTH OF NA BELOW TOP = 204.8932
CURVATURE = 1.279076E-05
SHEAR = 47309.81

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Towards an Efficient Shear Reinforcement for Concrete Rectangular Beams—Response Prediction by Compression Field Theory

Source: P. Ganapathi, University of Nairobi (1989)

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<th>Pure Moment</th>
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<th>Pure Moment</th>
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TOWARDS AN EFFICIENT SHEAR REINFORCEMENT FOR CONCRETE RECTANGULAR BEAMS—RESPONSE PREDICTION BY COMPRESSION FIELD THEORY


BEAM BMR-1

| TOP FACE STRAIN | = | .0005 |
| PURE MOMENT | = | 5.876878E+07 |
| DEPTH OF NA BELOW TOP | = | 174.2814 |
| PURE CURVATURE | = | 2.868924E-06 |

COMBINED FLEXURE AND SHEAR

| MOMENT | = | 2.002E+07 |
| DEPTH OF NA BELOW TOP | = | 188.2239 |
| CURVATURE | = | 4.021274E-06 |
| SHEAR | = | 20022 |

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**TOP FACE STRAIN**  
- Pure Moment: \(8.546926 \times 10^7\)  
- Depth of MA Below Top: \(176.7334\)  
- Pure Curvature: \(4.243681 \times 10^{-6}\)

**Combined Flexure and Shear**

- Moment: \(2.89063 \times 10^7\)  
- Depth of MA Below Top: \(194.4067\)  
- Curvature: \(5.863012 \times 10^{-6}\)  
- Shear: \(28906.3\)

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APPENDIX D  EVALUATION OF AVERAGE STRAINS
FROM EXPERIMENTAL DATA OF STRAINS

The average longitudinal, transverse and diagonal strains were calculated as explained below for the test beams, using the experimental data of strains tabulated in Appendix E.

D.1 AVERAGE LONGITUDINAL STRAINS ($\varepsilon_{l}$)

For beams BVWR-1 and BVWR-2, the regions EF and TU, and GH and RS had similar load conditions. But the pattern of shear reinforcement was different. Using the method of 3-level moving average, the average longitudinal strains were computed for all the regions and levels considered (Tables D1 and D2). For example, the average longitudinal strain ($\varepsilon_{l2}$) for the region EF at level 2 was calculated as,

$$\varepsilon_{l2} = \frac{1}{3} (\varepsilon_{l1} + \varepsilon_{l2} + \varepsilon_{l3})$$

where $\varepsilon_{l1}$, $\varepsilon_{l2}$ and $\varepsilon_{l3}$ are the longitudinal strains E1F1, E2F2 and E3F3 respectively.

For beam BWR-2, the load conditions and the reinforcement pattern were similar for regions
EF and TU, and GH and RS. Applying again the 3-level moving average method, for example, the average longitudinal strain ($\varepsilon_{l2}$) for the region EF or TU at level 2, was calculated as,

$$\varepsilon_{l2} = \frac{1}{3} (\varepsilon_{l1} + \varepsilon_{l2} + \varepsilon_{l3})_{EF} + \frac{1}{3} (\varepsilon_{l1} + \varepsilon_{l2} + \varepsilon_{l3})_{TU}$$

This procedure was repeated for all other levels for the region EF or TU and also for GH or RS (Table D3).

D.2 AVERAGE TRANSVERSE STRAINS ($\varepsilon_v$)

For beams BVWR-1 and BVWR-2, the average transverse strains were calculated for the regions EF and GH (regions with vertical stirrups) by a simple arithmetic mean (Tables D4 and D5). For example, the average transverse strain ($\varepsilon_v$) for the region EF at level 2 was calculated as,

$$\varepsilon_v = \frac{1}{2} (\text{strain E1E3} + \text{strain F1F3})$$

Considering that the beam BWR-2 had similar load conditions and pattern of shear reinforcement, the average transverse strain was assumed to represent the regions EF or TU and GH or RS. For example, the average transverse strain ($\varepsilon_v$) for the
region EF (or TU) at level 2 was computed as,

\[ \varepsilon_{v2} = \frac{1}{2}(\text{strain } E1E3 + \text{strain } F1F3) + \frac{1}{2}(\text{strain } T1T3 + \text{strain } U1U3) \]

This procedure was repeated for all other levels for the regions EF or TU and GH or RS (Table D6).

**D.3 AVERAGE DIAGONAL STRAINS (\( \varepsilon_d \))**

For beams BVWR-1 and BVWR-2 the average diagonal strains \( \varepsilon_{d1} \) and \( \varepsilon_{d2} \), which are mutually perpendicular, were taken as the strain values calculated from the experimental data (Tables D7 to D10). For example, in the region EF at level 2,

\[ \varepsilon_{d1} = \text{strain } E1F3 \]
\[ \varepsilon_{d2} = \text{strain } F1E3 \]

Since the shear reinforcement pattern and loading conditions were similar for the regions EF and TU, and GH and RS for the beam BWR-2, the average diagonal strains were computed by averaging the corresponding values (Table D11). For example, in the region EF (or TU) at level 2,

\[ \varepsilon_{d1} = \frac{1}{2}(\text{strain } E1F3 + \text{strain } U1T3) \]
\[ \varepsilon_{d2} = \frac{1}{2}(\text{strain } F1E3 + \text{strain } T1U3) \]
<p>| Moment (kN m) | Regions EF and TU | | | | | | Levels | Regions GH and RS | | | | | | Levels |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 55.6 | 4.883 | 2.467 | 2.392 | 0.650 | -1.025 | -1.125 | 1.392 | 1.975 | 2.658 | -3.317 | -3.050 | -2.492 |
| | 0.508 | 0.658 | 0.600 | -1.375 | -0.492 | -0.653 | 0.517 | 0.217 | 0.083 | -0.133 | -0.216 | -0.058 |
| 89.0 | 2.792 | 2.992 | 2.683 | -3.000 | -2.925 | -2.400 | 1.892 | 2.233 | 2.050 | -2.500 | -3.325 | -3.258 |
| | 0.550 | 0.833 | 0.742 | -1.383 | -0.533 | -0.792 | 0.250 | 0.008 | 0.158 | -0.008 | -0.100 | -0.233 |
| 111.3 | 2.058 | 1.975 | 2.242 | 1.283 | -1.667 | -1.825 | 2.300 | 2.025 | 3.383 | -3.150 | -4.200 | -3.617 |
| | 0.300 | 1.008 | 0.700 | -0.950 | -0.208 | -0.417 | 0.450 | 0.008 | 0.058 | -0.083 | -0.008 | -0.525 |
| 133.5 | 2.358 | 0.558 | 2.250 | -2.292 | -1.667 | -2.600 | 1.083 | 1.125 | 1.650 | -1.758 | -2.858 | -2.917 |
| | 1.025 | 1.583 | 1.158 | -0.475 | -0.458 | -0.325 | 0.517 | 0.042 | 0.075 | -0.283 | -0.008 | -0.550 |
| | 0.942 | 1.958 | 1.100 | -0.292 | -0.592 | -0.192 | 0.408 | 0.008 | 0.050 | -1.550 | -0.225 | -0.267 |
| 178.0 | 1.208 | 1.375 | 0.650 | -0.125 | -2.408 | -1.927 | 1.617 | 1.900 | 2.125 | -2.142 | -2.567 | -3.000 |
| | 1.775 | 2.567 | 1.517 | -0.833 | -0.183 | -0.358 | 0.242 | 0.256 | 0.233 | -0.383 | -0.117 | -0.617 |</p>
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**TABLE D2**

BEAM: BVWR-2

AVERAGE LONGITUDINAL STRAINS x 10^-3 (ε₁)
## TABLE D3 BEAM: BWR-2

**AVERAGE LONGITUDINAL STRAINS \( \times 10^{-3} (z) \)**

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**BEAM UVWR-1**

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**TABLE E.1**

**HELM HEAT-1**

**LONGITUDINAL STRAINS**
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**TABLE E.9**

**BEAM BVHR-2**

**LONGITUDINAL STRAINS**

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**TABLE E.10**

**BEAM BVHR-2**

**LONGITUDINAL STRAINS**
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**TABLE E.12**

**BEAM BMD-2 TRANSVERSE STRAINS**
### TABLE E.13 BEAM BVWR-2
#### DIAGONAL STRAINS

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### TABLE E.15 BEAM BVWR-2
#### DIAGONAL STRAINS

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<th>0.0577</th>
<th>0.0678</th>
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### TABLE E.16 BEAM BVWR-2
#### DIAGONAL STRAINS

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BEAM BWR-2
LONGITUDINAL STRAINS


### TABLE E.19 BEAM BWR-2
#### TRANSVERSE STRAINS

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<th>Moment (KN m)</th>
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<th>123.4</th>
<th>164.4</th>
<th>205.4</th>
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#### TRANSVERSE STRAINS

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<th>82.4</th>
<th>123.4</th>
<th>164.4</th>
<th>205.4</th>
</tr>
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<tbody>
<tr>
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### TABLE E.21 BEAM BWR-2
#### DIAGONAL STRAINS

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<th>Moment kN m</th>
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<th>89.0</th>
<th>111.3</th>
<th>122.4</th>
<th>133.5</th>
<th>155.8</th>
<th>178.0</th>
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<td>0.636</td>
<td>0.919</td>
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<td>0.713</td>
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<td>0.698</td>
<td>0.992</td>
<td>0.368</td>
<td>0.772</td>
<td>1.089</td>
<td>1.225</td>
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<td>0.958</td>
<td>1.008</td>
<td>1.078</td>
<td>1.146</td>
<td>1.255</td>
<td>1.367</td>
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<tr>
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<td>0</td>
<td>0.955</td>
<td>1.008</td>
<td>1.078</td>
<td>1.146</td>
<td>1.255</td>
<td>1.367</td>
<td>1.506</td>
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<td>0.955</td>
<td>1.008</td>
<td>1.078</td>
<td>1.146</td>
<td>1.255</td>
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</tr>
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<td>0</td>
<td>0.955</td>
<td>1.008</td>
<td>1.078</td>
<td>1.146</td>
<td>1.255</td>
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<tr>
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<td>0</td>
<td>0.955</td>
<td>1.008</td>
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### TABLE E.22 BEAM BWR-2
#### DIAGONAL STRAINS

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<th>89.0</th>
<th>111.3</th>
<th>122.4</th>
<th>133.5</th>
<th>155.8</th>
<th>178.0</th>
</tr>
</thead>
<tbody>
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<td>0.00</td>
<td>0</td>
<td>0.449</td>
<td>0.538</td>
<td>0.654</td>
<td>0.449</td>
<td>0.538</td>
<td>0.654</td>
<td>0.760</td>
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<tr>
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<td>0</td>
<td>0.448</td>
<td>0.535</td>
<td>0.650</td>
<td>0.448</td>
<td>0.535</td>
<td>0.650</td>
<td>0.760</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>0.448</td>
<td>0.535</td>
<td>0.650</td>
<td>0.448</td>
<td>0.535</td>
<td>0.650</td>
<td>0.760</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0.477</td>
<td>0.599</td>
<td>0.732</td>
<td>0.477</td>
<td>0.599</td>
<td>0.732</td>
<td>0.868</td>
</tr>
<tr>
<td>0.20</td>
<td>0</td>
<td>0.477</td>
<td>0.599</td>
<td>0.732</td>
<td>0.477</td>
<td>0.599</td>
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<tr>
<td>0.25</td>
<td>0</td>
<td>0.477</td>
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<td>0.868</td>
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<td>0.477</td>
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<td>0.732</td>
<td>0.868</td>
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<tr>
<td>0.35</td>
<td>0</td>
<td>0.477</td>
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<td>0.477</td>
<td>0.599</td>
<td>0.732</td>
<td>0.868</td>
</tr>
</tbody>
</table>
### TABLE E.23 BEAM BWR-2
#### DIAGONAL STRAINS

<table>
<thead>
<tr>
<th>Moment</th>
<th>(\text{kn m})</th>
<th>(\text{F1E3})</th>
<th>(\text{F1E4})</th>
<th>(\text{F1E5})</th>
<th>(\text{F1E6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
</tr>
<tr>
<td>55.6</td>
<td>0</td>
<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
</tr>
<tr>
<td>111.1</td>
<td>0</td>
<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
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<tr>
<td>122.4</td>
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<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
</tr>
<tr>
<td>153.5</td>
<td>0</td>
<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
</tr>
<tr>
<td>183.0</td>
<td>0</td>
<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
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<tr>
<td>211.4</td>
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<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
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</tr>
<tr>
<td>240.0</td>
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<td>0.953</td>
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<td>282.4</td>
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<td>0.953</td>
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</table>

### TABLE E.24 BEAM BWR-2
#### DIAGONAL STRAINS

<table>
<thead>
<tr>
<th>Moment</th>
<th>(\text{kn m})</th>
<th>(\text{F1E3})</th>
<th>(\text{F1E4})</th>
<th>(\text{F1E5})</th>
<th>(\text{F1E6})</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
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<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
</tr>
<tr>
<td>55.6</td>
<td>0</td>
<td>0.495</td>
<td>0.953</td>
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<td>0.778</td>
</tr>
<tr>
<td>111.1</td>
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<td>0.495</td>
<td>0.953</td>
<td>1.380</td>
<td>0.778</td>
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<tr>
<td>122.4</td>
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<td>0.495</td>
<td>0.953</td>
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<td>0.953</td>
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**LIBRARY**
### TABLE E.25 BEAM BWR-2

**LONGITUDINAL STRAIN**

<table>
<thead>
<tr>
<th>Moment (kN m)</th>
<th>L1 N1</th>
<th>L2 N2</th>
<th>L3 N3</th>
<th>L4 N4</th>
<th>L5 N5</th>
<th>L6 N6</th>
<th>L7 N7</th>
<th>L8 N8</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
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<td>-0.112</td>
<td>-0.133</td>
<td>-0.322</td>
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### Table E.25: Beam BWR-2

**Longitudinal Strain**

<table>
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<tr>
<th>Moment (kN m)</th>
<th>L1 N1</th>
<th>L2 N2</th>
<th>L3 N3</th>
<th>L4 N4</th>
<th>L5 N5</th>
<th>L6 N6</th>
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