BIAS IN REGRESSION COEFFICIENT ESTIMATES UPON DIFFERENT TREATMENTS OF SYSTEMATICALLY MISSING DATA


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**DETERMINISTIC CLAIMS RESERVING IN SHORT-TERM INSURANCE CONTRACTS**

PATRICK G. O. WEKE and ABEDNEGO T. MUREITHI

University of Nairobi, Kenya and Alexander-Forbes Financial Services, Nairobi.
Email: pweke@uonbi.ac.ke

Abstract Claims reserving for general insurance business has developed significantly over the recent past. This has been occasioned by the growth of the insurance market, with the risk underwriting process becoming more and more complex. New insurance products have been developed that cater for the more specific needs of the policyholder. Latent claims have also arisen in recent years, putting major strains on company resources. The case of asbestosis related claims testifies to this, having received widespread attention. Furthermore, recent disasters, such as the floods in Europe and the September 11th terrorist attacks on the U.S. have contributed to the need for more complex ways of analyzing claims experience. The suitability of the models used in claims reserving, have had to be reviewed to ensure that they do not give false impressions. The object of this paper, therefore, is to come up with a comparison of different methods of claims' reserving for a general insurer with a given claims' experience. The suitability of each of the estimates is noted to depend on the purpose of the reserving exercise. The paper discusses some of the methods (for instance, the basic chain ladder method, inflation adjusted chain ladder method, separation technique and Bornhuetter-Fergusson technique) used in claims' reserving, and for a particular claims experience, it gives an analysis of how well each of the methods models claims experience.

1. Introduction. General insurance refers to non-life insurance. Contracts of this type normally provide for cover for short periods of time, usually one year, as compared to life assurance contracts that run for many years. General insurance business can be classified into four basic insurance classes:
   - Liability insurance which includes employers' liability, motor third party insurance, product liability, public liability and professional indemnity.
   - Property damage insurance includes insurance covers on the following properties: Residential building, moveable property, commercial building, land vehicle, marine craft and aircraft.
   - Financial loss insurance
   - Fixed benefits – an example of this is where fixed medical benefits are paid to cover, at least in part, hospital costs that are incurred.

General insurance contracts are either long-tailed or short-tailed. Long-tailed insurance contracts tend to be settled many years after the insurance cover has expired. Liability insurance is normally of this nature. Short-tailed insurance contracts are settled relatively quickly. Most property damage claims are easily
verified and quantified, being settled within a few months or even days, and therefore short-tailed. The reserves of a general insurer comprise the provisions made for the purpose of meeting the uncertain claim payments. The settlement of these claims is the reason for the existence of insurance. The reserves of a general insurer are estimated for various reasons among them being:

- showing the liabilities of the insurer in its published accounts
- showing the liabilities of the insurer for supervision of solvency
- showing the liabilities of the insurer for internal management accounts
- as an intermediate step for the rating process
- to value the insurer for a purchase or a sale.

Claims reserves comprise of five elements, namely, reported claims, incurred but not reported (IBNR) claims, claim handling expenses, re-opened claims, and reserves for unexpired risks (though a claim may not have occurred, as long as the insured still has insurance cover, there is a possibility that a claim will arise and thus there is still the risk of a claim occurring). The claims process considers the stages between the occurrence of a claim event and the eventual settlement of the claim. Once a policy comes into force, then the risk of a claim occurring arises. For as long as the policy is in force, this risk remains, though it declines as the expiration of the insurance contract approaches. If a claim event occurs within the period of cover, then the insured party has the right to claim on any loss or damage suffered. However, there may be a time lapse between the occurrence of the claim event and the insured party notifying the insurer. This is a liability to the insurer although it is unknown. It is considered an IBNR claim. Upon notification of the claim, the insurer has to determine whether it is valid, and then try to estimate the cost of the claim, which is usually uncertain. Once the claim has been ascertained and payment made, it is considered closed, but in some circumstances, the insured party may make further claims after settlement has been made. Thus the insurer has to re-open the claim. The whole process of determining the validity of the claim, estimating its cost and making settlement thereof has expenses attached to it. The insurer has to pay intermediaries and incur other expenses in the process, which may lead to considerable expense amounts. Not all claims result in payment to the policyholder. These are referred to as nil or zero claims. They may still have handling expenses attached to them.

1.1 Literature Review. In recent years there has been increased reliance on actuaries to "sign off" on the claims reserves held by insurance companies. Currently the guidelines set forth by the American National Association of Insurance Commissioners, European associations and even the Kenyan government, require that a signed statement of actuarial opinion be included with the Annual Statement submitted to the Insurance Commissioner. Other regulators also make use of actuaries to help determine whether reserves are in accordance with legislation and regulations. Accounting firms use both in-house and consulting actuaries to determine whether reserves held by their insurance company clients make fair provision for claim-related liabilities. This increased reliance is based on the understanding that actuarial analysis provides an independent, scientific evaluation of contingent liabilities (see Blum and Otto, 1998).

One concept that appears frequently both in actuarial literature and in regulations pertaining to claims reserves is the concept of a "best estimate".

DETERMINISTIC CLAIMS RESERVING IN SHORT-TERM INSURANCE CONTRACTS

Guidance Notes 20 and 33 (Manual of Actuarial Practice, Institute of Actuaries), require that an actuary signs-off that the reserves of a Lloyd's syndicate are "at least as large as those implied by a "best estimate" basis without precautionary margins". Not only does this raise the question of what a best estimate is, but also raises the issue of prudential margins held in reserves (see Taylor 2000). According to Blum and Otto (1998), the term might provide a good foundation for clear communication, but the actuarial profession does not put forth any consistent definition. Quoting from their paper,

"In fact, there has been no clear terminology set forth by the profession which attempts to define any specific point, such as a "best estimate", within a range of "reasonable" estimates. A "reasonable" estimate is defined simply (and rather broadly) as an estimate based on reasonable methods and assumptions. Yet the frequent occurrence in the literature of terms like "best estimate", and the efforts by the consumers of actuarial science to provide their own definitions, suggest that something is lacking. This, concept of a best estimate of reserves for a general insurer and a range of best estimates, has thus received great attention in recent years (see Gibson, 2000 and Weke, 2003).

England and Verrall (2002) state in their paper,

"It is far from clear what this ('best estimate') means".

They further state that it can be argued that several different deterministic reserving methods applied to different data sets can provide a range of best estimates. When looking at a best point estimate, different methods may be used to arrive at this estimate. Blum and Otto (1998) propose a consistent definition of the term 'best estimate expected value':

"is the undiscounted, unmargin, unbiased best estimate of the probability weighted average of all possible unpaid loss amounts"

They additionally recognize that this does not imply that every actuary will arrive at the same number to satisfy this definition, since, in any statistical analysis, there may be several possible estimators of the expected value. Furthermore, each estimation process, applied by different analysts, may produce somewhat different results. The outcomes of these estimators, though varying, should converge to the desired number. While different actuaries may produce different "best estimate" numbers, the range of best estimates among these actuaries should be considerably narrower than the range of all "reasonable" estimates. What might be considered a "reasonable" estimate of unpaid claims may not be acceptable as a "best estimate" of those claims (e.g. "reasonable but optimistic" does not qualify as a "best estimate"). For a particular actuary, there should be only one "best estimate" as of a given reserve date. Claims are highly subject to uncertainty. Therefore, it is often appropriate to consider ranges of values in addition to point estimates. The following is extracted from the reserving principles adopted by the Casualty Actuarial Society:

"The uncertainty inherent in the estimation of required provisions for unpaid losses or loss adjustment expenses implies that a range of reserves can be actuarially sound. The true value of the liability for losses or loss adjustment expenses...can be known only when all attendant claims have been settled."

When interpreting the results of a claims reserving exercise, it is important to keep in mind that there is a certain level of variability included in the predictions. The most appropriate reserve within a range of actuarially sound
estimates depends on both the relative likelihood of estimates within the range and the financial reporting context in which the reserve will be presented. Different groups will rely on the claim reserves that are estimated. Among those who rely on reserve estimates, interests and priorities may vary. To company management the reserve estimate should provide reliable information in order to maximize the company's viability and profitability. To the insurance regulator, concerned with company solvency, reserves should be set conservatively and to reduce the chance of insurance company failure. To the tax agent charged with ensuring timely reporting of earned income, the reserves should reflect actual payments as "nearly as it is possible to ascertain them". The policyholder is most concerned that reserves be adequate to pay insured claims, but does not want to be overcharged for that assurance (Blum and Otto 1998).

A consistent definition of the "best estimate" should therefore:

• identify a target for a point estimate
• define that target unambiguously
• be soundly based in actuarial science
• be explainable to a wide audience

'Best estimate', used in a general sense will only fulfill part of these requirements. While the concept of "best estimate" may elicit an intuitive understanding, this understanding will not necessarily be the same from one person to the next.

This research paper looks at the issue of a best estimate in the light of the purpose of the reserving exercise. It seeks to determine what claims reserve estimate given by the different deterministic methods of reserving produces the 'best estimate' for a particular set of circumstances surrounding the reserving exercise.

2. DETERMINISTIC METHODS OF CLAIMS RESERVING

2.1 Data Presentation. The methods for estimating claims reserves that are discussed require data to be presented in the form of a run-off triangle. This presentation cross classifies the data according to the period of origin and the period of development. The period of origin may be the year when the claim was incurred, reported or when the policy relating to the claim was underwritten, while the development period refers to the length of time since the period of origin in which the claims were incurred, reported or paid. By convention, the development year relating to the year of origin is denoted as development year zero. A claim cohort is defined depending on the definition used for claims from each origin period and development period. For example, we could have each entry in the triangle as being the value of the claim paid in development year \( j \) for period of origin \( i \). The general form of the run-off triangle is given by:

\[
\begin{array}{cccccc}
C_{0,0} & C_{0,1} & C_{0,2} & \ldots & C_{0,n-1} \\
C_{1,0} & C_{1,1} & C_{1,2} & \ldots & C_{1,n-1} \\
& \vdots & \ddots & \ddots & \vdots \\
C_{i,0} & C_{i,1} & C_{i,2} & \ldots & C_{i,n-1} \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
C_{n,0} & & & & \\
\end{array}
\]

2.2 Statistical Methods. Statistical methods in essence

(i) Attempt to find a consistent claim run-off pattern which applied in the past; and
(ii) Apply that pattern to estimate the run-off of claims that have been incurred but are still outstanding.

A general model of claims can be represented in the form:

\[ C_y = F\left(N_y, S_y, R_y, X_y\right) + e_y \]  

(2.1)

Where \( C_y \) is the observed value in development year \( j \) for period of origin \( i \) and is a function of: \( N_y \) a parameter applicable to period of origin \( i \) and development period \( j \), for example the number of claims settled in the period. \( R_y \) a parameter varying by the development period and assumed to be independent of the year of origin, for example, the proportion of claim payments by period \( j \). \( S_y \) a parameter varying by exposure period for example, the number of claims incurred in period of origin \( i \). \( X_y \) a parameter reflecting effects varying by calendar period, for example, the effect of claims inflation on claims. \( e_y \) an error term. This statistical model is analogous to the analysis of variance.

2.3 The Basic Chain Ladder Method The basic chain ladder method is the simplest statistical model used and assumes that all external factors, for example, claims inflation of claim costs, change in the mix of business, change in the rate of settlement of claims, can be effectively ignored and the model assumes the form:

\[ C_y = S_j R_j + e_y \]  

(2.2)

and with known parameters takes the following realization:

\[ C_y = s_j r_j + e_y \]  

(2.3)
Where $s_i$, the ultimate total is cost of the claims in the period of origin $i$ and $r_j$ is the proportion of total payments made by the end of development period $j$. $C_y$ is the cumulative amount of payments to the end of period $j$. The method assumes that the $r_j$'s are constant for each period of origin, i.e. the claims development pattern is constant.

The method then consists of estimating the successive values of the $r_j$ factors. If $b_j$ is the ratio of the expected amount paid by the end of period $j$ to the expected amount paid by period $(j-1)$, then $b_j$ may be estimated by

$$b_j = \frac{\sum_{i=0}^{n-j} C_{ij}}{\sum_{i=0}^{n-1} C_{i,i-1}}$$

(2.4)

and

$$r_j = \prod_{i=1}^{j} b_i$$

(2.5)

By combining $b_j$ ratios, an estimate $B_j$ of the total payments ultimately paid for a year of origin to the value of the payments by year $j$ is obtained. Thus

$$B_j = \prod_{i=2}^{n} b_i$$

(2.6)

The estimated claims still outstanding as at the end of year $i+j$ in respect of origin $i$ is therefore given by

$$C_y = s_i r_j \frac{\lambda_{i+j}}{A_{i+j}} + e_y$$

(2.7)

where $C_y$ is the cumulative claims paid for year of origin $i$ as at the end of year $(i+j)$.

**Assumptions:**
- The claims development pattern is stable between each year of origin
- Future claims inflation is implicitly averaged in by past inflation

One major drawback of the basic chain ladder method is that it fails to make explicit numerous features which can affect the assumptions of a stable run-off pattern. One such feature is claims inflation. In a stable claims inflation environment, the basic chain ladder method may be argued to average out the past claims inflation and project it into the future forecasts of outstanding claims. However, for volatile claims inflation, the basic chain ladder method fails to make reliable estimates.

**2.4 Inflation Adjusted Chain Ladder Method.** This method adopts the general model in the form:

$$C_y = S_i R_j \frac{X_{i+j}}{A_{i+j}} + e_y$$

and the parameters thus become:

$$C_y = s_i r_j \lambda_{i+j} + e_y$$

(2.9)

Where $C_y$ are the payments made in development year $j$ of year of origin $i$, (i.e., non-cumulative)

$s_i$ is the ultimate total cost in real terms of claims incurred in the period of origin $i$.

$r_j$ is the proportion of total payments in real terms made in development year $j$.

$\lambda_{i+j}$ is an assumed index of claims cost

Under the inflation adjusted method, the run-off triangle has to be presented as incremental claims for each year of origin and development. Using a claims inflation index, the past values are brought to current monetary values. Incremental claims along the same diagonal (moving from bottom left to top right) arise from the same year and hence the same inflation index value is applied on them. The adjusted incremental claims are then accumulated and the normal procedures of the basic chain ladder method are applied. These estimated claims reserves are also in current monetary terms. In order to estimate the cash value of future claim payments, an assumption has to be made about the likely level of future claim inflation.

**Assumptions:**
- The claims development pattern is stable
- Claims inflation will be at the assumed future rate

**2.5 The Separation Technique.** It has the form of equation (2.7):

$$C_y = S_i R_j \frac{X_{i+j}}{A_{i+j}} + e_y$$

with parameters

$$C_y = n_i r_j \lambda_{i+j} + e_y$$

where $n_i$ is the number of claims incurred in the year of origin $i$ and $\lambda_{i+j}$ is related to the year of payment. In this case $\lambda_{i+j}$ is derived from the data rather than assumed from external sources. The derived factors will be related to increases in claim costs but will also be affected by other external factors and by random fluctuations in the claim size. As a result, they are likely to correspond to any assumed index considered suitable for use with the chain ladder method.

The method for analysing the run-off triangle is as follows. In respect of each year of origin $i$, the claim payment $C_y$, made in each development year $j$ are divided by some exposure index $S_i$, attributable to the period of origin. These exposure measures may be vehicle years or earned premiums. However, they may not accurately reflect the differences in the risks underwritten (a particular
Assumptions: - the given loss ratio is correct
- the claims development pattern is stable
- the past claims development does not provide any additional information on
  the future development of claims.

If \( b_j \) is the ratio of the expected amount of claims paid by the end of period
\((j - 1)\), then \( b_j \) can be estimated by:

\[
b_j = \frac{\sum_{i=0}^{n-j} C_j}{\sum_{i=0}^{n-j} C_{i,j-1}}
\]

which is the same parameter as that of the chain ladder method and defining

\[
B_j = \prod_{i=0}^{n-j} b_i
\]

The estimated claims still outstanding at the end of year \( i + j \) with respect to
the origin year \( i \) is given by:

\[
S_i^{BF} (1 - B_j)
\]

The Bornhuetter-Ferguson technique assumes that there is prior knowledge
about the parameters of the model, making it analogous to a Bayesian approach.
The B-F method may also be applied on inflation adjusted claims data (as is the
case with the inflation adjusted chain ladder method), and then future claims
reserves estimated on an assumption of the future rate of claims inflation.

2.7 Signs Test
It is a simple test for overall bias. If the estimated incremental claims at each
development year for each year of origin do not tend to be higher or lower than
those observed at that time period, then we would expect that roughly half the
estimates would be above the observed values, and half below. Therefore an
excessively high number of positive or negative residual errors will indicate that
the rates are biased. Defining

\[
P = \text{number of positive deviations},
m = \text{number of total deviations calculated for the run-off triangle}.
\]

Then we have the hypotheses

\[
H_0 : P \sim \text{Binomial } (m,1/2) \quad \text{and hence the model is not biased.}
H_1 : \text{The model is biased.}
\]

The test will be two tailed and we find \( k \), such that

\[
\sum_{j=0}^{n} \left( \frac{m}{j} \right) \left( \frac{1}{2} \right)^m \geq 0.025
\]

where \( \alpha = 0.05 \), the level of significance.

The test would be satisfied if \( k \leq P \leq m - k \). Alternatively, the \( p \)-value can
be determined and used to accept or reject the null hypothesis. Zero values are
ignored in the analysis.

Assumptions: The residual errors are independent.

2.8. The Grouping of Signs Test
This test detects the clumping of deviations of the same sign. Defining

\[
G = \text{number of groups of positive deviations}
\]
\[
n_1 = \text{number of positive deviations}
\]
\[
n_2 = \text{number of negative deviations}
\]

and \( m = \text{number of deviations calculated} \), we have the hypothesis

\[
H_0 : G \sim \left( \frac{n_1 - 1}{g - 1} \right) \left( \frac{n_2 + 1}{g} \right)
\]

\[
\left( \frac{m}{n_1} \right)
\]

and hence the model does not cluster residual errors of the same sign together.

\[
H_1 : \text{The model tends to clusters residual errors of the same sign}
\]

and find the smallest \( k \) such that

\[
\sum_{i=1}^{k} \left( \frac{n_1 - 1}{t - 1} \right) \left( \frac{n_2 + 1}{t} \right) \geq 0.05
\]

where \( \alpha = 0.05 \), the level of significance.

The test is one tailed as the number of positive or negative groups will be small
or large alike. We say that the test has failed if \( G \leq k \). Alternatively, the
\( p \)-value can be obtained.
Assumptions: The residual errors are independent
The grouping of signs test would be performed after the signs test as it depends on the number of positive residual errors. Zero values are ignored in the analysis.

3. DATA ANALYSIS AND RESULTS

The data in Appendix I was analyzed using Excel to come up with the relevant values for each of the models discussed. The parameters for each model were first determined before estimating the claims reserves. The results of the analysis were as follows.

3.1. Model Parameters

3.1.1. The Basic Chain Ladder Method: The parameters obtained were:

\[
\begin{align*}
b_1 & \quad 1.542121 \\
b_2 & \quad 1.101987 \\
b_3 & \quad 1.075744 \\
b_4 & \quad 1.047183 \\
b_5 & \quad 1.030069
\end{align*}
\]

3.1.2. The Inflation Adjusted Chain Ladder Method: In this case, the values of the run-off triangle were first adjusted for past claims inflation using the values of past claims inflation shown in Table 1-3 of Appendix 1. The parameters were then estimated, with the following results:

\[
\begin{align*}
b_1 & \quad 1.4863 \\
b_2 & \quad 1.0842 \\
b_3 & \quad 1.0557 \\
b_4 & \quad 1.0314 \\
b_5 & \quad 1.0185
\end{align*}
\]

3.1.3. The Separation Technique: In addition to the run-off triangle data, the data on the number of claims in the first development year of each year of origin was required to estimate the claims reserves. This data on the number of claims is contained in Table 1-2 of Appendix 1, and is given by year of development 0 of each year of origin. Dividing the run-off triangle data by the relevant number of claims and then estimating the parameters produced the following set of estimates:

\[
\begin{align*}
b_1 & \quad 1.486265 \\
b_2 & \quad 1.08422 \\
b_3 & \quad 1.055687 \\
b_4 & \quad 1.031435 \\
b_5 & \quad 1.018539
\end{align*}
\]

3.1.4. The Bornhuetter-Ferguson Method: The first case of this method considered the claims reserving process without any explicit assumptions on inflation. The loss ratio was taken to be 70% of the written premiums which are shown in Table 1-4 of Appendix 1. The parameters estimated were:

\[
\begin{align*}
B_1 & \quad 1.971936 \\
B_2 & \quad 1.278717 \\
B_3 & \quad 1.160374 \\
B_4 & \quad 1.078671 \\
B_5 & \quad 1.030069
\end{align*}
\]

With inflation adjustments on both the observed claims and the premiums written using the claims inflation data in Table 1-3 of Appendix 1, the parameters obtained were:

\[
\begin{align*}
B_1 & \quad 1.486265 \\
B_2 & \quad 1.08422 \\
B_3 & \quad 1.055687 \\
B_4 & \quad 1.031435 \\
B_5 & \quad 1.018539
\end{align*}
\]

3.2 The Claims Reserve Estimates

Table 2-1 shows the claims reserves estimates produced by each method under different scenarios of assumed future rates of inflation of 8%, 10% and 12%. The inflation adjusted chain ladder method, the separation technique and the B-F method with inflation adjustment (i.e., the methods that made explicit assumptions on future claims inflation) produced higher claims reserve estimates as the assumed future rate of inflation increased.

The B-F method with inflation adjustment gave the largest total claims reserves at each assumed level of future inflation followed by the B-F method with no inflation assumptions. The separation technique gave the lowest total claims reserves followed by the chain ladder method with inflation adjustment.
The basic chain ladder method produced intermediate reserves among all the methods.

4. CONCLUSIONS AND RECOMMENDATIONS
The main focus of this paper was to determine the 'best estimate' of claims reserves for a particular set of circumstances by comparing the reserve estimates produced by the different methods. The study revealed that for the particular data, the separation technique generally tended to give the best fit to the observed claims experience. It gave the lowest mean, median, range and inter-quartile range for the percentage residual errors. It gave the lowest total claims reserves. The method would thus be suitable in the case where it is important to have a fair picture of the reserves without being pessimistic or optimistic. This would generally be so when the reserving exercise is being carried out for management review purposes and when determining premium rates.

The inflation adjusted chain ladder also gave a reasonably good fit on the observed claims experience. However a trend for it to overestimate the claims in later years of origin was observed. This may explain why the estimated claims reserves of this method tend to be higher than those given by the separation technique. The basic chain ladder method gave similar results but did give higher total claims reserves. The overestimation was however not large in both cases. The methods would thus seem appropriate where a conservative approach is taken in the claims reserving exercise. Determining claims reserves for the published accounts of the company and also for supervision of solvency may be done using either of these two methods. Furthermore, in the case that the insurance company is being valued for a purchase, a conservative value of the reserves is appropriate and either of the two methods could be used.

The B-F methods gave poor fits to the observed claims data. The inflation-adjusted method was observed to consistently overestimate the claims at all years of origin. It thus would not be considered an appropriate model to estimate claims reserves for this class of business. The B-F method without inflation adjustment would also not be appropriate for estimating the claims reserves.

This paper only considered some deterministic methods of claims reserving. In some cases, the distribution of the reserve estimate is important in the decision making process. Stochastic methods then become more appropriate in such situations. P. D. England and R. J. Verrall (2002) give a review of some of the suggested stochastic methods. The use of claims reserving computer packages (e.g. Prophet, Winsbug) make the application of such methods relatively easy.

A further consideration in the claims reserving process is discounting of the reserves. Though not considered in this paper, discounting may be significant in the long-tailed classes of insurance. The written premiums are expected to be invested for some period of time before claims are made and subsequently paid. The short-tailed classes are however not greatly affected by the discounting of claims reserves.

REFERENCES


APPENDIX

<table>
<thead>
<tr>
<th>Year of Origin</th>
<th>Claim Numbers in Development Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>963</td>
</tr>
<tr>
<td>1</td>
<td>1010</td>
</tr>
<tr>
<td>2</td>
<td>991</td>
</tr>
<tr>
<td>3</td>
<td>932</td>
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<tr>
<td>4</td>
<td>915</td>
</tr>
<tr>
<td>5</td>
<td>1029</td>
</tr>
</tbody>
</table>

Table 1-2: Number of Claims Data

<table>
<thead>
<tr>
<th>Year of Origin</th>
<th>Claim Numbers in Development Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1368</td>
</tr>
<tr>
<td>1</td>
<td>1455</td>
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<tr>
<td>2</td>
<td>1421</td>
</tr>
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<td>1302</td>
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<tr>
<td>4</td>
<td>1309</td>
</tr>
<tr>
<td>5</td>
<td>1029</td>
</tr>
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</table>
The assumed past rates of claims inflation are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0 / Year 1</td>
<td>7.40%</td>
</tr>
<tr>
<td>Year 1 / Year 2</td>
<td>17%</td>
</tr>
<tr>
<td>Year 2 / Year 3</td>
<td>16.90%</td>
</tr>
<tr>
<td>Year 3 / Year 4</td>
<td>10.90%</td>
</tr>
<tr>
<td>Year 4 / Year 5</td>
<td>8.20%</td>
</tr>
</tbody>
</table>

Table 1-3: Past Rates of claims inflation for each year of payment

The premiums written in each year of origin were:

<table>
<thead>
<tr>
<th>Year of Origin</th>
<th>Premiums Written</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>153605</td>
</tr>
<tr>
<td>1</td>
<td>194595</td>
</tr>
<tr>
<td>2</td>
<td>218003</td>
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<tr>
<td>3</td>
<td>243786</td>
</tr>
<tr>
<td>4</td>
<td>282342</td>
</tr>
<tr>
<td>5</td>
<td>371848</td>
</tr>
</tbody>
</table>

Table 1-4: Premiums written for each year of origin

### Abstract

Insect counts data arising from ecological or pest management studies usually exhibit a high degree of clustering (aggregation) which present special problems for regression modeling. In this paper, we have investigated three different methods that can be used to analyze such data. These are power transformation of response variable based on Taylor’s Power Law (1961) and Quasi-likelihood modeling with variance functions based on the power law, $V(Y) = a\mu^r$ and the variance function of the Negative Binomial, $V(Y) = (\mu + \mu^2)/k$. We apply the methods to some counts data collected on fruit fly species *Bactocera zonata*. The response variable transformation based on Taylor’s Power Law was effective in stabilizing the variance but did not achieve normality of the transformed variable. On the other hand, Quasi-likelihood models appeared to fit the data fairly well for both variance function forms. Overall, the results show that response variable transformation of raw data is not appropriate for the fruit fly counts data used in this study or more general data of similar kind, but quasi-likelihood modeling with variance forms $V(Y) = \phi \mu^r$ or $V(Y) = \phi (\mu + \mu^2)/k$ appears to be a sensible approach.

### Key words

Aggregated insect counts data, Transformation of raw data, Quasi-likelihood, Taylor’s Power Law, Negative binomial distribution

### 1. Introduction

In populations where the individuals are spatially distributed at random, that is independent of each other, implying a poisson process, the variance ($V$) of counts from a number of samples sampled simultaneously at each site is equal to the mean ($\mu$). However, individuals in natural populations are not distributed strictly at random more often than not. Mutual attraction leads to aggregation which makes variance greater than the mean and occasionally mutual repulsion leads to regularity which makes the variance less than the mean.

Insect counts data arising from ecological or pest management studies usually exhibit a high degree of clustering (aggregation). Aggregated data present special problems for regression modeling as the relationship between the mean and the variance is inconsistent with the distributional assumptions of the Poisson distribution which is normally used for counts. In this paper we compare alternative methods namely Quasi-likelihood methods and more traditional, the power transformation of raw data. The transformation of raw