MODELLING TRENDS IN HIV/AIDS PREVALENCE AND INCIDENCE IN KENYA

BY

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JULY 2014
DECLARATION

I Otieno Fredrick Ochola duly declare that this is my own original work and has not been presented for a degree in any other university or institution.

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<tr>
<td>AIDS</td>
<td>Acquired Immune Deficiency Syndrome</td>
</tr>
<tr>
<td>CDC</td>
<td>US Centers For Disease Control And Prevention</td>
</tr>
<tr>
<td>DHS</td>
<td>Demographic and Health Survey</td>
</tr>
<tr>
<td>HIV</td>
<td>Human Immunodeficiency Virus</td>
</tr>
<tr>
<td>HIV-1</td>
<td>Human Immunodeficiency virus type one</td>
</tr>
<tr>
<td>KDHS</td>
<td>Kenya Demographic and Health Survey</td>
</tr>
<tr>
<td>MSM</td>
<td>Men who have sex with men</td>
</tr>
<tr>
<td>NACC</td>
<td>National Aids Control Council</td>
</tr>
<tr>
<td>NASCOP</td>
<td>National Aids and STI Control Programme</td>
</tr>
<tr>
<td>PLHIV</td>
<td>People Living With HIV/AIDS</td>
</tr>
<tr>
<td>PEPFAR</td>
<td>US President’s Emergency Plan for AIDS</td>
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<tr>
<td>STD’s</td>
<td>Sexually Transmitted Diseases</td>
</tr>
<tr>
<td>UNAIDS</td>
<td>Joint United Nations Programme on HIV and AIDS</td>
</tr>
<tr>
<td>VCT</td>
<td>Voluntary Counseling and Testing</td>
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<td>WHO</td>
<td>World Health Organization</td>
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ABSTRACT

The trends in the evolution of HIV/AIDS epidemic in Kenya has been tracked through annual sentinel surveillance in antenatal clinics since 1990. Behaviors have been measured through national Demographic and Health Surveys in 1993, 1998, and 2003. The surveillance data indicates that prevalence has declined substantially starting in 1998. Nationally, adult prevalence has declined from 10% in the late 1990s to under 7% today.

In the analysis, the 2009 Estimation Projection Package (EPP) and Spectrum Packages customized using Kenya’s data from VCT, ART, and PMTCT programmes were entered into the EPP. Next entered into the EPP/Spectrum were birth rate, survival rate, adult mortality and population growth rate. EPP was used to fit epidemic model to the data yielding trend curves for the two sub groups: males and females. Next, the prevalence and incidence projections produced by EPP were imported into Spectrum which generated final trends and calculated the number of people living with HIV, new HIV infections, and AIDS related deaths.

From the results, the number of people living with HIV was increasing. This is due to the positive growth rate in the population. The prevalence trends increased to a pick value before decreasing steadily to the current values. The new infections of adults aged 15-49 constantly increased. This too is due to the ever increasing overall population in Kenya. The incidence rates decreased due to the interplay between several factors including the effects of ART and PMTCT.

In conclusion, there is need to scale up the roll out of the PMTCT and sensitization programmes on awareness as there is evidence that these would greatly serve to bring down the prevalence and incidence rates to the desired zero rates.
CHAPTER ONE

1.0 INTRODUCTION
This report looks at the trends in HIV incidence and prevalence structured in five chapters. First chapter entails introduction, the second chapter brings out the processes and the steps followed in generating the HIV estimates and trend curves. The third chapter highlights the methodology used in defining the epidemic and population groups, the tools applied, and details out the various inputs in terms of programme coverage, demographic projections and epidemiological assumptions. The fourth chapter focuses on results. The analysis is done based on the key indicators such as national adult HIV prevalence, the number of HIV infections, and the distribution of HIV infections by sex and age group. Additionally, estimates of HIV incidence, number of deaths due to AIDS related causes, and estimates of women requiring prevention of mother to child transmission (PMTCT) services were determined. Finally, the last chapter dwelt on conclusion and recommendations on way forward in the fight against HIV/AIDS.

1.1 Background of the problem
Kenya has a number of sources of information on HIV prevalence levels and trends. Three national surveys, the Kenya Demographic and Health Survey of 2003 (KDHS 2003), the Kenya AIDS Information Survey 2007 (KAIS 2007) and the Kenya Demographic and Health Survey of 2008/9 provide good estimates of national prevalence for those three years and the trend between those years. Antenatal clinic surveillance has been conducted since 1990. ANC surveillance provides information on trends at surveillance sites particularly in the period before the first survey in 2003. The new estimates for 2011 are based on the three national surveys and surveillance data through 2011. (NACC 2012).

The prevalence rates provide important information that enables planning for national response, evaluation of programme impact, and measurement of progress on the National HIV/AIDS Strategic Plan. The understanding of the distribution of HIV within the population and the analysis of social, biological, and behavioral factors associated with HIV infection provide new insights on the HIV epidemic in Kenya that may lead to more precisely targeted messages and interventions. (KDHS 2008-9).

1.2 HIV Prevalence by age
Results from the 2008-9 KDHS indicate that 6.3 percent of Kenyan adults aged 15-49 are infected with HIV. HIV prevalence in women age 15-49 is 8.0 percent, while for men aged 15-49, it is 4.3 percent. This female-to-male ratio of 1.9 to 1 is higher than that found in most population-based studies in Africa. Young women are particularly vulnerable to HIV infection compared with young men. For example, 3 percent of women age 15-19 are HIV infected, compared with less than one percent of men age 15-19, while HIV prevalence among women 20-24 is over four times that of men in the same age group (6.4 percent vs. 1.5 percent). Rates among women and men begin to converge as age increases, except for the unusually high level
among women age 40-44; prevalence among men rises gradually with age to peak at age 35-39. HIV prevalence is higher for women than men at all ages except for the 35-39 age groups.

1.3 Statement of the problem
HIV/AIDS remains to be one of the greatest present scourges facing mankind globally. The trends in HIV incidence and prevalence are determined by the interplay between several factors such as the effects of gender, age-groups, risk groups, ART, and PMTCT among others. In Kenya, many preventive measures based on these factors have been put in place to address the HIV/AIDS pandemic. Despite these measures, the HIV incidence and prevalence is still high compared to rates in the developed countries. There is therefore a need for precise information on the nature of trends given the past and present preventive measures so as to inform better planning and accurate evaluation of the impact of these preventive interventions and the general progress in the fight against HIV/AIDS - hence the study.

1.4 Research Questions
1. What are the trends in HIV/AIDS prevalence and incidence in Kenya over the last 20 years?
2. What will be the projected trends in HIV/AIDS prevalence and incidence in Kenya over the next 5 years?

1.5 Objectives

1.5.1 Main Objective:
To determine the trends in HIV/AIDS prevalence and incidence rates in Kenya between the year 1993 to date and the projected rates in the next 5 years given the interventions and suggest a way forward in the fight against HIV/AIDS.

1.5.2 Specific Objectives:
1. To determine which factors have influence on HIV/AIDS prevalence and incidence in Kenya
2. To determine the trends in HIV/AIDS prevalence and incidence over the last 20 years
3. To determine the projected HIV/AIDS prevalence and incidence rates over the next 5 years.

1.6 Significance and Justification of the Study
Knowledge on the nature of trends in HIV/AIDS prevalence and incidence rates provides useful information that can be used in designing the best preventive measures to curb its further spread. The best preventive measures then leads to reduced cases of deaths and sicknesses arising from HIV/AIDS thereby ensuring an active and healthy population that works efficiently for the socio-economic growth of the nation.
1.7 The Scope of the Study
The study encompasses the entire population of Kenya as a Country.

1.8 Limitations
The use of models to determine the projected trends in HIV prevalence and incidence rates may not be very accurate. This is because most models do not capture changes in the prevention efforts such as the programmes that result in reducing the number of sexual partners or increased condom use as they are difficult to accurately quantify and their impacts are difficult to model. These prevention programmes are thus not reflected in the estimates and projections. The values in this report reflect the situation if our prevention efforts remain constant over the next five years.
1.9 Conceptual Framework

Figure 1: Conceptual Framework
CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Meaning of HIV prevalence and HIV incidence
HIV prevalence is a measure of the proportion of people who are living with HIV in a given population at a particular point in time. Prevalence is typically measured in cross-sectional surveys. It is a useful measure for understanding the total burden of disease and for planning care and treatment needs. HIV incidence is the number of new HIV infections that occur in a given population over a given period of time. Incidence is usually expressed as a number or percentage of infections that occur over a given period of time. Knowing the current incidence of HIV in a population provides information on how fast the virus is spreading through a population. Measuring HIV incidence is much more complex than measuring prevalence. (UNAIDS, 2009)

2.2 Importance of the information on HIV incidence trends
According to global report UNAIDS, 2008, information on HIV incidence trends is vital for many reasons.

First, HIV incidence trends promotes better understanding on to how the epidemic is progressing and whether current interventions are having any impact on the spread of infections. In addition, information on HIV incidence trends helps in focussing intervention programmes to where they are needed most. It is important to know where and amongst which populations or group the recent HIV infections have occurred, as this will help refine and focus HIV prevention interventions. HIV incidence data can provide such information. In contrast, HIV prevalence data include the total of everybody living with HIV regardless of when the infections occurred. The people currently becoming infected with HIV, the patterns of new HIV infections and the risk factors may be different now from what was happening a year or more ago.

Similarly, HIV incidence trends help us understand the risk factors in the HIV transmission. These may be proximal (direct, e.g. low condom use or high use of contaminated needles) or distal (indirect or underlying, e.g. limited access to condoms, counselling/information or needle-exchange services). More insights into current risk factors for HIV transmission can be gained if, in a particular survey, investigators can identify which are recent (incident) cases and link this to information about assumed proximal and distal risk factors so as to determine the relative importance of those risk factors in transmission dynamics.

Finally, HIV incidence trends help us assess the effect of the interventions. A particular HIV prevention programme has had a positive impact if we can measure and demonstrate reductions in Modes of HIV transmission. The HIV epidemics in most of eastern and southern Africa are mature and generalised epidemics. Additionally, most of the southern African countries in this
region have been classified as having ‘hyper endemic’ HIV epidemics, where HIV prevalence amongst 15-49 year olds is 15% or higher. However, whilst the trends in prevalence of HIV in the general population and sub-populations is measured on a regular basis in most countries in eastern and southern Africa through a combination of population based surveys and sentinel surveillance of pregnant women at antenatal clinics, HIV incidence is not routinely measured. Hence, there is a dearth of HIV incidence data in the region. The main reasons for the lack of incidence data are the challenges with measuring or estimating incidence.

According to Stover John (2009), a key aspect of the policy process is recognizing that a problem exists and placing that problem on the policy agenda. Trends in HIV/AIDS projections can illustrate the magnitude of the AIDS epidemic and the demographic, social and economic consequences. This illustration also can show policymakers the impacts on other areas of development and the size of the impacts that could be expected without effective action. HIV/AIDS projections are also needed to plan the response. For example, AIM can project the number of people needing antiretroviral therapy at a given time, which can serve as the basis for planning expanded access to treatment. It can be used to estimate the number of orphans in order to develop support programs.

### 2.3 Methods to estimate trends in HIV incidence

According to the global report UNAIDS, 2008, there are three main methods through which the evolution of HIV incidence can be traced, namely by cohort studies, mathematical models or laboratory assays.

#### 2.3.1 Trends in HIV incidence through cohort studies

Cohort studies involve a specific group of people being followed up over time (longitudinal follow-up), tested for HIV at regular intervals, and their HIV status recorded. The rate of new infections over time in the study group can then be determined directly.

#### 2.3.2 Trends in HIV incidence through mathematical models

Mathematical models are used to trace the incidence from age-specific prevalence rates, and estimating HIV incidence from repeat cross-sectional population-based surveys.

#### 2.3.3 Trends in HIV incidence using laboratory assays (tests)

Laboratory assays generated from tests on dried blood samples are used to differentiate between recent and established infections. The data obtained can then be used to trace the trends in HIV incidence.
2.4 Using models to estimate HIV incidence rates

According to global report UNAIDS, 2008, mathematical and statistical models have been used to produce trends curves of incidence from prevalence data. Over time, as our knowledge of the HIV epidemic has increased, these models have become more reliable. Indirect strategies for estimating HIV incidence include the combination of the Estimating and Projection Package (EPP) and Spectrum mathematical modeling software tools used to generate the epidemiological estimates reported annually by UNAIDS. EPP/Spectrum combines available HIV surveillance data with data from programmes for antiretroviral therapy and prevention of mother-to-child HIV transmission to calculate HIV prevalence, HIV incidence, AIDS mortality, number of AIDS orphans and HIV treatment needs.

Another mathematical model that has been developed uses successive rounds of national cross-sectional HIV prevalence data to estimate HIV incidence by age in the general population (Hallett et al., 2008). Still other dynamic models have been developed to generate HIV incidence estimates (Williams et al., 2001; Gregson et al., 1996).

Trends in HIV prevalence among young people are less subject to changes over time due to mortality and the effect of antiretroviral therapy than are the trends in prevalence among people of all ages. Therefore, trends in prevalence among antenatal clinic attendees aged 15–24 years old have been used to assess trends in incidence in countries with a high prevalence (UNAIDS, 2008). Similarly, differences in age-specific prevalence in national surveys have been used to assess trends in incidence (Shisana et al., 2009). The ‘incidence by modes of transmission’ approach, developed by the UNAIDS Reference Group on Estimates, Modeling and Projections, estimates the number of new infections in a given year. Unlike the previously described methods, the mode of transmission analysis does not aim to identify incidence trends over time. The model assumes that the risk of infection for any individual is a function of the HIV prevalence among partners, the number of partners and the number of contacts with each partner, with additional weight given to the presence or absence of sexually transmitted infections and to circumcision status. The model allows for estimates of new infections to be developed by population and transmission source. The ‘incidence modes of transmission’ model is already proving useful in detecting dissonance between national prevention programmes and epidemiological patterns. (UNAIDS, 2008).

Population-level HIV incidence can be calculated from repeat cross-sectional population-based surveys. Changes in the prevalence of HIV in a population are dependent on the incidence of HIV, the mortality rates of those infected and whether there is out- or in-migration. In theory, therefore, incidence rates could be calculated from repeat cross-sectional population-based studies in which HIV prevalence is determined, and where mortality and migration rates are known. (UNAIDS, 2008)

There have been a variety of methods proposed to estimate HIV incidence from measures of prevalence but none has been widely used, either because of the restricted applicability of the methods or because of the complexity of the approach. A mathematical method can also be used...
to determine incidence in the population of young people aged 15-24 years using HIV prevalence data in this age group. This approach is based on the assumption that very few 15 year olds are HIV positive. (UNAIDS, 2008)

2.5 Using the Epidemic curves and the Spectrum model to generate the trends in HIV

The epidemic curve produced by EPP is entered into the Spectrum Projection Package to generate estimates of national HIV prevalence, incidence, mortality and treatment needs. In addition to the input from EPP, the Spectrum model is based on assumptions about the epidemiology of HIV in the country, including the ratio of female to male prevalence, distribution of HIV infection by age, the survival distribution from HIV infection to treatment eligibility and to death, and the effect of HIV on fertility. In addition, data are required on adult and child treatment coverage and on PMTCT services and infant feeding practices. In order to make projections on the impact on the overall population, we need reliable information on population size and age structure, rates of birth, fertility, death and migration and estimates of how these rates change over time. These demographic projections can be directly obtained from the UN Population Division database which can be accessed in Spectrum. (UNAIDS, 2008)

2.6 Generalized Epidemics

According the UNAIDS 2008 global report, this is how a six-step process works for projections in countries with a generalized epidemics.

![Diagram of generalized epidemics]

*Figure 2: Overview of generalized epidemics. Source: UNAIDS 2008 global report on HIV/AIDS.*
2.7 The Aids Impact Model (AIM)
According to Stover, John (2009), the AIDS Impact Model is a computer program for projecting the trends in the impact of the AIDS epidemic. It can be used to project the future number of HIV infections, AIDS cases, and AIDS deaths, given an assumption about adult HIV prevalence. It can also project the demographic and social impacts of AIDS. These projections then can be used in graphic policy presentations intended to enhance knowledge of AIDS among policymakers and to build support for effective prevention and care. AIM requires an assumption about the past and future course of adult HIV incidence and treatment coverage. Assumptions about other HIV/AIDS characteristics can also be entered for such variables as the survival period from HIV infection to AIDS death, the age and sex distribution of new infections, and the perinatal transmission rate. A demographic projection (DemProj) must be prepared first, before AIM can be used. DemProj is one of the Spectrum systems of policy models used to make the demographic projection. The demographic projection is modified by AIM through AIDS deaths and the impact of HIV infection on fertility. The Epidemiology section of AIM calculates the number of HIV infections, AIDS cases, and AIDS deaths.

2.8 Strengths and limitations of models for estimating HIV incidence
The main advantage to using models is that HIV incidence estimates can be made for large populations including regions, countries and provinces/states with reasonable levels of confidence. Provided the required input data are available, models are relatively straightforward to use, and they are not costly to implement as they require no primary data collection. Mathematical models do, however, require skilled personnel to run, calibrate and interpret the model results. (UNAIDS, 2008)

The biggest drawback to applying these models is that, often, not all the data required for the models are available, are not representative or are not of sufficient quality. In some cases the model is not sensitive to regional dynamics (e.g. the UNAIDS incidence model does not include a separate category for persons with concurrent sexual partners), and the model also allows the use of data from different years if not available from the same year. However, the model could be adapted to specific regional needs should the required data be available in the region. Except for the Asian Epidemic Model (which requires extensive data over time) and demographic models such as Spectrum, some models do not provide a comprehensive picture over time. Even if the model is run in consecutive years, it would need new data every year in order to produce estimates over time. Levels of uncertainty around how reliable the model assumptions are in different settings also adds to the uncertainty in regards to the model outputs (UNAIDS 2008).

Small variations in any of the inputs in the spectrum data can affect the population size years later. Perhaps the most important difference is that the UN Population Division estimates may use a different HIV prevalence trend than the one you are using in Spectrum. In that case, the population projected by Spectrum may not match the UN Population Division estimate or the latest census estimate. Variations in the age distribution of mortality or migration can also cause small difference in the population size today. If this problem occurs you can get a better match to the census population by adjusting some of the inputs to the demographic projection. Changes to
the fertility rate, life expectancy, the model life table chosen and the migration inputs can help to fine tune the projection. Changes to the HIV prevalence curve could also make a difference (Stover John, 2009).

A particular challenge to the use of models is the roll out of ARVs, which is changing survival times. In response to this, some models are now building in the capacity to input and process data on the extent of ARV roll out. Finally, incidence estimates from models are made only for the general population or for specific subpopulations. Unlike data from cohort or certain laboratory testing strategies, the data from modelling exercises cannot be used to investigate associations between risk factors and HIV transmission at the individual level (UNAIDS 2008).
CHAPTER THREE

3.0 RESEARCH METHODOLOGY

This section focuses on research design, target population, data sources, and data analysis. To allow comparisons across results obtained with different methods, we restricted the analysis to adults aged 15–49 years.

3.1 Research Design
The study adopted secondary research approach. Available literature on the current HIV/AIDS situation in Kenya and globally was reviewed. The review provided valuable information on the current situation of HIV/AIDS in Kenya as far as development of programs that can be clearly planned, executed and implemented in taming the HIV/AIDS incidence is concerned. The review also helped in selecting issues that are relevant and efficient for use in designing the research instruments.

3.2 Target Population
The study looks at the entire country of Kenya.

3.3 Data sources
The sources of data used in this study included document review on previous research report, published data, Global and Kenyan Government reports on HIV and AIDS - mainly secondary data. The secondary data used was sourced from Antenatal care clinic (ANC) surveillance, DHS 2003, DHS 2008/9, and from the UNAIDS. Both the DHS and ADI Health related Indicators surveys shared comparable survey methodology and questionnaires, allowing for comparisons.

3.4 Description of Data Analysis Procedure
First, the estimates in this study were generated using 2009 Estimation Projection Package (EPP) and Spectrum Packages which are informed by the Global Reference Group on Estimates, Modeling and Projections, and customized using data from Kenya.

The four – step – process applied in the methodology was as follows: First, the Kenya data from various categories were entered into the EPP. These categories included data from VCT, ART and PMTCT. The demographic parameters such as birth rate, survival rate, and adult mortality and population growth rate also entered into the EPP. Secondly, EPP was used to fit epidemic model to the data yielding trend curves for the two sub groups: males and females. Next, the prevalence and incidence projections produced by EPP were imported into Spectrum to generate final trends and calculate the number of people living with HIV, new HIV infections, AIDS related deaths and treatment needs.
3.5 The Estimation Projection Package (EPP) procedure

3.5.1 Epidemiological model used in EPP

The model used in this study is as shown below:


\[ \frac{dN}{dt} = X + Y + Z \] \hspace{2cm} \text{[1]}

Where
- \( N \) = total population
- \( X \) = at-risk population,
- \( Z \) = not at-risk population,
- \( Y \) = infected,

\[ \frac{dX}{dt} = (1 - f(X/N))E_t - \mu X \] \hspace{2cm} \text{[2]}

\[ \frac{dZ}{dt} = f(X/N)E_t - (\mu + rY/N + t)Z \] \hspace{2cm} \text{[3]}

\[ \frac{dY}{dt} = (rY/N + t)Z - \int_0^t (rY_{x/N} + t_x)Z_x g(t-x) \, dx \] \hspace{2cm} \text{[4]}

Where;
- \( \mu \) = the non-AIDS death rate,
- \( g \) = function describing the proportion progressing to AIDS death by the number of years since HIV infection,
- \( r \) = the force of infection. Where a large value of “\( r \)” will cause prevalence to increase rapidly while a small value will cause it to increase slowly,
- \( f(X/N) \) = the fraction of those individuals entering the adult population (\( E_t \)) who enter the at-risk group \( Z \),
- \( t \) = the year of the epidemic.

\( f(X/N) \) is given by;
\[
f(X / N) = \frac{\exp[\phi(X / N - (1 - f_0))] - 1}{\exp[\phi(X / N - (1 - f_0))] + 1 - f_0} \tag{5}
\]

Where:
- \( t_0 \) = the start year of the HIV epidemic
- \( f_0 \) = the initial fraction of the adult population at risk of infection. \( f_0 \) determines the peak level of the epidemic curve, and
- \( \phi \) = the behavior adjustment parameter which determines how the proportion of new entrants in the adult population who are at risk of HIV infections changes over time.

The population not at risk (X) is increased by new entrants and reduced by non-AIDS deaths (\( \mu_X \)). The population at risk (Z) is increased by new entrants and reduced by non-AIDS deaths and new HIV infections (\( r_{Y/N} \)). The infected population (Y) is increased by new infections (\( r_{Y/N} \) and i) and decreased by progression to AIDS death. The function \( f(X/N) \) determines the proportion of new entrants to the adult population that enter the at-risk population. Initially this proportion is set by \( f_0 \). The epidemic progresses those in the at-risk category become infected with HIV and die. Since the death rate will be higher in the at-risk category than among those not at risk, the proportion of the population at risk will gradually decline. This will produce a prevalence curve that rises to a peak value and then declines rapidly to low levels.

These equations produced a prevalence curve that can fit a wide variety of epidemic shapes by adjusting the four parameters: \( t_0 \), \( f_0 \), \( r \), and \( \phi \). EPP uses this model to find prevalence curves that fit available surveillance data. The parameter \( \mu \), the non-AIDS death rate, is estimated for Kenya through the population estimates and projections of the United Nations Population Division. The progression to AIDS death (\( g \)) is assumed to be constant throughout the projection.

New entrants to the adult population at time \( t \), \( E_t \), are calculated from the births of HIV negative children \( B_{t-15}^- \) occurring 15 years previously and the probability of surviving to age 15, \( l \). The number of births is simply the birth rate multiplied by the size of the adult population. However, some children will be born infected. We assume that they do not survive to age 15. Thus the number of children born without HIV infection is determined by calculating births to HIV-negative adults (\( b(X+Z) \)) and HIV-negative births to HIV-positive adults (\( b'Y(1-v) \)) where \( v \) is the perinatal transmission rate and \( b' \) is the birth rate adjusted for the reduction in fertility caused by HIV infection, \( \varepsilon \).

\[
E_t = B_{t-15}^- \cdot l \tag{6}
\]
\[ B_{i-15}^- = b \left[ X_{i-15} + Z_{i-15} + (1 - v) \varepsilon Y_{i-15} \right] \quad \cdots \cdots \{7\} \]

This approach is implemented in the EPP model by assuming that the parameters \( l \) (survival to age 15), \( b \) (birth rate), \( \varepsilon \) (fertility reduction caused by HIV), \( v \) (perinatal transmission rate) and the distribution \( g \) (progression from infection to AIDS death) are fixed. EPP searches for the best values of the four remaining parameters \( t_0, f_0, r, \phi \). The best values are defined as those that produce the prevalence curve that best fits the surveillance data.

### 3.5.2 The UNAIDS Maximum Likelihood Procedure

EPP works by fitting the simple epidemiological model, shown in equation 1, developed by the UNAIDS Reference Group on Estimates, Modeling and Projections to observed surveillance data using a maximum likelihood method. The model uses multiple sets of values for these four parameters: \( t_0, f_0, r, \phi \) to fit the data with similar likelihoods.

The large inherent uncertainties in existing surveillance data, means that many possible parameter combinations could produce epidemic trends with approximately equally valid fits to a given set of data. The spread observed in these statistically similar curves gives one indication of the uncertainty in fitting the Reference Group model to observed data. Each curve represents a different level of the epidemic, but the overall temporal trend of rise and fall is the same.

To make level fits, it was assumed that the national epidemic was being modeled by the sum of a number of curves for individual sites with different levels, \( \lambda_i \), that is, the overall regional epidemic curve, \( p(t) \), is given by:

\[ p(t) = \sum_i \lambda_i \tilde{p}(t) \]

Level fits assume that all surveillance sites in a region follow a similar pattern of rising and falling prevalence, but that the individual sites are at different prevalence levels, with some being high prevalence, while others low prevalence. Sites with long runs of data will contribute substantially to determining the shape of the underlying curve, \( \tilde{p}(t) \), while the lower prevalence sites recently added with relatively short data runs will help in bringing down the overall regional prevalence level when they are summed to form the regional epidemic.

Mathematically, EPP implements this by leaving the individual \( \lambda_i \) as free parameters that are fit along with the usual Reference Group model parameters \( t_0, f_0, r, \phi \) using a maximum likelihood method. During the fitting procedure within EPP a final value for \( \lambda_i \) for each site is calculated to maximize the likelihood function. These \( \lambda_i \) then say how high or low a given site
is above the average best fit prevalence curve. EPP then sums the levels for each site and applies them to the prevalence pattern automatically to give the overall prevalence nationally.

3.5.3 Estimating adult HIV prevalence using EPP

The following data sets were entered into the EPP to enable estimation and projection of HIV prevalence and HIV incidence in Kenya from surveillance data: the annual population for HIV+ men, the annual population for HIV+ women in Kenya, and the sentinel surveillance data from 1993 to 2014 on HIV prevalence among antenatal care clinic (ANC) attendees.

The Estimation Projection Package (EPP) was developed to fit to multiple points with the four parameters \( t_0, r, f_0, \phi \) which also describes the dynamics in the model used.

When \( \phi \) is negative, the people reduce their risk in response to the HIV epidemic and the curve shows a sharper HIV prevalence decline after the peak. When \( \phi \) is zero the proportion at risk remains constant and the prevalence declines after the peak. If \( \phi \) is positive, risk actually increases over time and HIV prevalence falls less quickly or stabilizes at a high level. A positive value of \( \phi \) means that, risk actually increases over time and prevalence falls less quickly or stabilizes at a high level. When \( \phi \) is zero, then the proportion at risk remains constant and the HIV prevalence declines after the peak as people die. If \( \phi \) is positive, risk actually increases over time and HIV prevalence falls less quickly or stabilizes at a high level.

3.5.4 Estimating HIV prevalence for males and females aged 15-49

The adult HIV prevalence for males and females was calculated from 1993 to 2014 by fitting an epidemiological model to HIV Sentinel Surveillance data for antenatal clinic attendees in the EPP. This is because antenatal clinic attendees are considered as proxy for the general population. It was ensured that data outliers were excluded as the sentinel surveillance data was fitted to an epidemiological model for generating adult HIV prevalence.

3.5.5 Estimating numbers infected, new infections and AIDS deaths using the Spectrum.

Source of Equations used:

The number of adults 15-49 infected with HIV in any year is simply the number of adults multiplied by the HIV prevalence provided by EPP.

\[ \text{HIV}_t = \text{adult population}_t \times \text{prevalence}_t \quad \text{........... [8]} \]

New infections are calculated as the number of infections expected in year \( t \) minus the number of infections surviving from the previous year. Surviving infections are the number of infections in the previous year minus deaths occurring during the previous year.
New HIV infections\( t \) = HIV\( t \) – (HIV\( t -1 \) – AIDS deaths\( t -1,t \) – non-AIDS deaths to HIV+\( t -1,t \)) \[9\]

AIDS deaths are a function of the number of new infections in previous years and the rate of progression from AIDS to death.

AIDS deathst - 1, t = \( \sum_{i=1}^{t} \) [ New HIV infections\( t - i \) x Proportion progressing to death\( i \) ] \[10\]

Child infections occur when an HIV positive mother passes the infection to her child during gestation or birth or after birth through breastfeeding.

New child infections\( t \) = HIV+WRA\( t \) x TFR\( t \) x (1-TFR reduction) x PTR\( t \) \[11\]

Where:
- HIV+WRA = the number of HIV positive women of reproductive age
- TFR = total fertility rate
- TFR reduction = the reduction in fertility caused by HIV infection
- PTR = the perinatal transmission rate

### 3.5.6 Demographic input

The demographic inputs to EPP include the proportion of male population, adult birth rate (15+ years), survival to age 15 (I\( _{15} \)), adult mortality in 15+ (\( \mu \)) and adult population growth rate.

### 3.5.7 Generating gender-specific prevalence curves

The HIV prevalence curves were generated by inputting HIV Sentinel Surveillance data to EPP. As a first step, prevalence curves were generated independently for males and females and subsequently, their curves were cumulated to form a prevalence curve for Kenya.

Initial guesses were made using four parameters \( t_0 \), \( r \), \( f_0 \), and \( \phi \) for each adult HIV prevalence epidemic curve that was generated for males and females. Using a maximum likelihood procedure, the EPP model fits curves to HIV epidemics by varying the four parameters.
CHAPTER FOUR

4.0 RESULTS AND DISCUSSIONS

This section highlights the output, both graphical and tabular, from the analysis of HIV data. It gives the description of the nature of trends of each curve as well as the interaction of the curves.

4.1 Trends and projected trends in HIV population and HIV prevalence

The number of persons aged 15 years and above infected with HIV increased sharply from the year 1993 to the peak in 1997 followed by a steady decline to date. The HIV prevalence for males, females and total showed a sharp increase to pick values before decreasing steadily to date as shown in figure 3.

![Figure 3: Trends in HIV population and HIV prevalence](image)

The projected trends in number of HIV+ persons aged 15 and above was estimated (using equation 8) within the spectrum which generated best values of the four parameters which was then used to plot the smooth curve shown in figure 4. The curve predicts a gradual decline in trend beyond the year 2014 through to 2019.
From figure 4 above, HIV prevalence increased rapidly from onset to its highest peak ever of 10.8 percent. The peak year according to the estimates, adjusted using the DHS 2003 and DHS 2008/9, (dotted line on the graph in figure 4) was in the year 1995. After the peak, the prevalence has been decreasing steadily to date giving the current prevalence at about 6.3%. The graph indicates that the projected prevalence will be fairly constant for the next few years. The graph also shows that the surveillance data was over estimating prevalence (see the “gr3 result” curve in figure 4 above). With the population parameters assumed (table 2), the final fitting parameter values used to project the HIV prevalence were calculated as shown in column 2 of the table 2 below.

**Table 2: Results For National Curve fit in figure 2.**

<table>
<thead>
<tr>
<th>Population parameters:</th>
<th>Fitting parameters:</th>
<th>Epidemiological parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>b= 0.07933</td>
<td>r= 5.19828</td>
<td>Vert tran= 0.32</td>
</tr>
<tr>
<td>l15= 0.84441</td>
<td>f0= 0.15197</td>
<td>Fert red= 0.7</td>
</tr>
<tr>
<td>mu= 0.008</td>
<td>t0= 1972</td>
<td>alpha= 2</td>
</tr>
<tr>
<td>gr= 0.0363</td>
<td>phi= 6.01199</td>
<td>beta= 13.2123</td>
</tr>
<tr>
<td>% ale= 0.5</td>
<td>lnL= 42.3799</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5: The uncertainty graph of HIV population for persons aged 15-49 years: the fit, the upper and lower bounds, and the resampled curves.

Table 3: Results for Uncertainty Fit in figure 5.

<table>
<thead>
<tr>
<th>Population parameters:</th>
<th>Fitting parameters:</th>
<th>Epidemiological parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>b= 0.079327</td>
<td>r= 50</td>
<td>Vert tran= 0.32</td>
</tr>
<tr>
<td>l15= 0.844413</td>
<td>f0= 0</td>
<td>Fert red= 0.7</td>
</tr>
<tr>
<td>mu= 0.007999</td>
<td>t0= 1970</td>
<td>alpha= 2</td>
</tr>
<tr>
<td>gr= 0.036301</td>
<td>phi= 100</td>
<td>beta= 13.21235</td>
</tr>
<tr>
<td>% male= 0.5</td>
<td>lnL= -1</td>
<td></td>
</tr>
</tbody>
</table>

From figure 5 above, the uncertainty gap in the estimates of HIV prevalence significantly decreased from the year 1993 to date. This is because of the DHS surveys and KAIS survey which are now available for calibrating the HIV surveillance data from the ANCs in Kenya. With the population parameters assumed as shown in table 3, the final fitting parameter values, used to project the uncertainty in HIV prevalence were calculated as shown in column 2 of the table 3.
4.2 Trends and projected trends in new HIV infections


![New HIV infections among persons aged 15-49](image)

*Figure 6: New HIV infections by gender and total among persons aged 15-49 years.*

From figure 6, the trend in new HIV infections was found to decrease exponentially from the base year 1993 to date. The trend in males was fairly equal to that in females.

The EPP adjusted the guesses of the four parameter values to best estimates (fitting parameters, table 4) that produced the smooth curve (figure 7). New HIV infections for females first reduced steadily then gradually up to the year 2013. The decrease can be suggested by suggest the effect of ART, PMTCT, increased condom use and increased awareness programs on fight against HIV/AIDS.
The year 1988-1999 was the period when Kenyan population was at highest risk of HIV infection which was about 2.7% persons infected per year (figure 7). The rate of occurrence of new infections increased sharply to the peak value before reducing sharply until the year 2000. Thereafter the rate of new HIV infections has been decreasing at a very low rate. The projected trend in HIV will continue to decrease at a slow pace beyond 2014 (figure 7).
Figure 8: The uncertainty graph of new HIV infections for persons aged 15-49 years: the fit, the upper and lower bounds and the resampled curves.

Table 5: Results For National Curve fit in figure 8.

<table>
<thead>
<tr>
<th>Population parameters:</th>
<th>Fitting parameters:</th>
<th>Epidemiological parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>b= 0.079327</td>
<td>r= 3.704207</td>
<td>Vert tran= 0.32</td>
</tr>
<tr>
<td>l15= 0.844413</td>
<td>f0= 0.132785</td>
<td>Fert red= 0.7</td>
</tr>
<tr>
<td>mu= 0.007999</td>
<td>t0= 1971</td>
<td>alpha= 2</td>
</tr>
<tr>
<td>gr= 0.036301</td>
<td>phi= 32.31467</td>
<td>beta= 13.21235</td>
</tr>
<tr>
<td>% male= 0.5</td>
<td>lnL= 35.1211</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8 shows that the uncertainty gap in the estimates of new HIV infections greatly shortened especially from around the year 1993. The effect can be explained by the fact that the DHS surveys and KAIS survey became more available and was used to adjust HIV surveillance data on new HIV infections from the ANC's data in Kenya.
4.3 Trends and projected trends in HIV population given the effect of PMTCT

The results, from figure 9, indicates that the number of pregnant women needing PMTCT services was increasing until the year 1998 which was the peak year. This trend then started decreasing to date and will continue decreasing for the next five years.

The number of new infections without PMTCT was very high compared to the case with PMTCT which was near zero values. This indicates that the roll out of PMTCT was very effective in taming new infections among the new born babies.
Figure 11: The projected number of new infection, in children, with PMTCT.

The trend in number of new child infections due to PMTCT increased reaching its peak in the year 1998 before declining steadily until year 2010. Thereafter the trend has been decreasing rather slowly. The projected trend will also be decreasing slowly (figure 11).

4.4 Comparing the HIV prevalence by age groups

Figure 12: Comparing the HIV prevalence by age groups

The 2008 survey data showed that the distribution of HIV prevalence was found to be concentrated among people in the aged 15-49 years. The pick prevalence occurred among those of age group 35-45. This is explained by the fact that more persons are reproductive in the above age groups hence were more at risk of infection by HIV.
CHAPTER FIVE

5.0 CONCLUSIONS AND RECOMMENDATIONS

This chapter summarizes the findings from the study and briefly suggests what can be done to realize the objectives stated.

The use of EPP and Spectrum was very appropriate and successful in this study. EPP helped to produce smooth curves by varying the four parameter values, $t_0$, $r$, $f_0$, and $\phi$. The spectrum then used data from the EPP to generate estimates which were then used for further analysis to produce trend curves and projected curves as shown in the chapter on results above.

The rate of decline in new infections since 1994 has been low for both males and females aged 15 and above (figure 6 & 7). There is therefore a need to accelerate the decreasing trends towards the zero rates. The AIDS deaths, though on the decline, are still relatively high compared to the situation in developed countries.

To bring about the desired rapid decline in AIDS deaths, there is a need for improved accessibility of treatment and care for people living with HIV/AIDS. The number of HIV pregnant women needing is still quiet high putting the unborn children at risk. The awareness and provision of services through PMTCT should hence be prioritized. This would greatly reduce on HIV incidence and prevalence.

The uncertainty ranges in the estimates of HIV prevalence and incidence have greatly reduced. This indicates improved accuracy in the information on the HIV situation and hence promotes better planning of preventive and care-based measures on the HIV situation.

Figure 12 shows that the distribution of HIV prevalence rate is dominated by persons aged between 25 – 45 years. This is because persons in this age group are in the reproductive age and thus sexually active making them more susceptible to HIV infections compared to those under 25, and over 49 years of age. It is therefore recommended that the preventive programmes, such as VCT services and provision of Condoms, and care programmes, such provision of ART, be mostly targeted to persons aged 25-49 years.

Finally the latest estimates of HIV situation should not be compared directly with estimates published in previous years. This is because the assumptions, methodologies and data used to produce the estimates are gradually changing as a result of on-going enhancement of our knowledge of the epidemic. New trends are based on new assumptions and enhancements and the latest surveillance data. Therefore the latest estimates will be more accurate and reliable than those produced in previous years, since they are based on improved methods and more data than earlier estimates.
REFERENCES:


7. http://nrs.harvard.edu/urn-3:HUL.InstRepos:10587317


### Table 6: Summary of impacts due to ART and PMTCT

<table>
<thead>
<tr>
<th>Impacts</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deaths averted by ART (Thousands)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Infections averted by PMTCT</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Life years gained by ART and PMTCT</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Deaths averted by cotrimoxazole (0-4)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Deaths averted by PMTCT (0-4)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Source: Output from EPP
Table 7: An Example of a complete EPP output

<table>
<thead>
<tr>
<th>UNAIDS Maximum Likelihood Fitter: beginning the fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNAIDS SMLFitter called - doing a non-level fit with phi not fixed</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 8.771929824561402 0.05700000000000005 1970.0 100.0 LL: 212.65529961729771</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 8.77192995273343 0.05700000000000005 1970.0 100.0 LL: 212.65529834933113</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 8.771929824561402 0.05700000000000005 1970.0 100.0 LL: 212.65529961729771</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 8.771929824561402 0.05700000000000005 1970.0 100.0 LL: 212.65529961729771</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 0.3877227471587586 0.8662682189760174 1970.0 100.0 LL: 1514.1426942001854</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 10.44 5830955789233 0.06613733196030652 1970.0 100.0 LL: 153.8480001965654</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 10.445831111444244 0.06613733196030652 1970.0 100.0 LL: 153.84799971841858</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 10.445830955789233 0.06613733281546985 1970.0 100.0 LL: 153.84799592968193</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 10.445830955789233 0.06613733196030652 1970.0 100.0 LL: 153.8480001965654</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 11.852491673039086 0.10029671107747884 1970.0 100.0 LL: 57.349658189341426</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 11.852491849654974 0.10029671107747884 1970.0 100.0 LL: 57.349658064660616</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 11.852491673039086 0.10029671227222192 1970.0 100.0 LL: 57.34965708255186</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 11.852491673039086 0.10029671107747884 1970.0 100.0 LL: 57.349658189341426</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 24.585991573058706 0.2756851629219999 1970.0 100.0 LL: 504.5983885017631</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 12.022322849709195 0.11199205905022619 1970.0 100.0 LL: 51.41406276871567</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 12.022323028855766 0.11199205905022619 1970.0 100.0 LL: 51.41406275329064</td>
</tr>
<tr>
<td>Trying r, f0, t0, phi: 12.022322849709195 0.111992060345250567 1970.0 100.0 LL: 51.41406262645614</td>
</tr>
</tbody>
</table>
UNAIDSMLFitter: Run complete: final values:

\[
\begin{align*}
& r = 12.043305383358986 \\
& f_0 = 0.11379975501582001 \\
& t_0 = 1970.0 \\
& \phi = 99.99999999998899
\end{align*}
\]
<table>
<thead>
<tr>
<th>Log Likelihood = 51.3147005996143</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total elapsed time for fit in seconds: 0.172</td>
</tr>
</tbody>
</table>